

Reliability sensitivity analysis of frame structures controlled by external fractional viscoelastic dampers

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ABSTRACT

Installing seismic devices, such as energy dissipation devices and seismic isolators, is one of the most effective approaches to protect new and existing structures against earthquake effects in seismic areas. An efficient strategy to improve the calibration of the main parameters of seismic devices is the *design sensitivity analysis* (DSA). Design sensitivity relies on a linear approximation of the selected performance measure in terms of the design variables. It provides a quantitative estimate of design changes due to variations in the available design variables. This paper focuses on frame structures connected to much more rigid walls (the exoskeleton) by means of a set of linear energy dissipation devices with fractional viscoelastic behaviour, subjected to seismic excitation modelled as a zero-mean stationary Gaussian random process. To assess the seismic performance of the controlled structure, the *reliability function* of the *extreme value* response process is evaluated by adopting Vanmarcke's approximation. Then, explicit analytical expressions of the sensitivities of the *reliability function* to the main parameters of fractional viscoelastic devices are derived. Numerical results demonstrate that such expressions are highly valuable in the context of the DSA of controlled structures, as they may assist engineers in the optimal placement and design of seismic devices.

1. Introduction

One of the most effective approaches to protect existing structures in seismic areas against earthquake effects is to install seismic protective devices, such as energy dissipation devices and seismic isolators (see e.g., Refs. [1,2]). Recently, it has been recognized that a promising approach to seismic retrofitting of frame structures consists of adopting the so-called dissipative exoskeleton. It comprises external rigid walls or steel braces, equipped with energy dissipation devices, which are connected to adjacent storeys of the structure. Compared to traditional solutions, the dissipative exoskeleton offers significant advantages for building retrofit: ease of inspection and replacement, and low maintenance costs (see e.g., Refs. [3–7]). Furthermore, possible interference with the structural and non-structural components inside the existing building is avoided.

Several types of energy dissipation devices, which use different mechanisms to dissipate energy, have been proposed in the literature: viscous dampers, viscoelastic dampers, friction dampers, yielding

dampers, and so on (see, e.g., Refs. [2,8–11]). This study considers a frame structure connected to a much more rigid wall (the exoskeleton) by means of a set of linear viscoelastic energy dissipation devices. A fractional model of the viscoelastic devices is assumed. Indeed, after numerous experimental tests, it has been widely recognized that only the fractional model is able to capture the actual constitutive laws of viscoelastic materials [12–16].

It is well-known that *sensitivity analysis* (SA) plays a crucial role in the design of structures subjected to stochastic excitations. Indeed, it can be used to reveal how uncertainty in structural parameters of a finite element model affects a specific response quantity of interest (see e.g., Refs. [17,18]). This analysis is particularly useful for complex structural systems, where assessing the impact of parameter variations on dynamic properties is challenging. Benfratello et al. [19] proposed a time-domain approach for evaluating response sensitivities of discrete structural systems. Operating in the frequency domain, Bhattacharyya and Chakraborty [20] evaluated the stochastic sensitivity of structures with uncertain structural parameters subjected to stationary random

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earthquake loading. Liu [21] proposed three analytical methods to calculate the sensitivity and Hessian matrix of the *Power Spectral Density (PSD)* functions of stochastic seismic responses. The most significant among these methods is based on direct differentiation once the *Pseudo-Excitation Method* is applied. Johnson and Wojtkiewicz [22] proposed evaluating the sensitivities of the transfer functions, *PSDs*, and mean-square responses of a structural system by changing design parameters that are spatially localized within some small portions of the computational model. Relying on the use of the so-called Rational Series Expansion, Muscolino et al. [23] proposed a semi-analytical approach for the sensitivity analysis of the response of linear discretized structures subjected to stationary multi-correlated Gaussian random excitations. This approach was extended to the evaluation of reliability sensitivities of linear structures with interval parameters within the framework of the classical *first-passage* problem, under the Poisson assumption of independent up-crossings of a specified threshold [24]. Ding et al. [25] presented two numerical methods to determine the sensitivity and Hessian matrix of the *PSD* function for non-classically damped systems subjected to stationary stochastic excitations. Valdebenito et al. [26] evaluated the sensitivity associated with first excursion probabilities by performing the partial derivative of the probability with respect to parameters that affect the structural response.

In the framework of *SA*, the *design sensitivity analysis (DSA)* requires the determination of the rate of change of performance measures with respect to a set of design variables. This is essential to define an optimal design, even if a systematic design optimization method is not used. *DSA* results may assist engineers in determining the direction and extent of design changes required to improve performance measures [27,28]. Several approaches based on the use of *DSA* have been proposed in the literature. In particular, Nalecz and Wicher [29] performed *DSA* of mechanical systems in the frequency domain. In the paper by Van Keulen et al. [30], four different options for *DSA* calculation are summarized in terms of accuracy, computational costs, and implementation complexity. Applying the finite difference, direct differentiation, and adjoint variable methods, Li et al. [31] performed the *DSA* of the deterministic dynamic response of linear systems with non-viscous damping forces, which take into account the past history of motion via convolution integrals over some kernel functions. Yun and Youn [32] adopted the discretize-then-differentiate approach for deriving discrete adjoint equations. Ding et al. [33] evaluated the first- and second-order derivatives of the transient response using a modified precise integration method in conjunction with the direct differentiation method. The *DSA* of eigenvalues and eigenvectors for structural systems with viscoelastic dampers rheologically modelled with fractional derivatives was performed by Lewandowski and Łasecka-Plura [34] using the direct differentiation and adjoint variables methods.

The main purpose of this paper is to develop an analytical approach for estimating the sensitivities of the first excursion probabilities of linear frame structures controlled by external fractional viscoelastic devices, subjected to seismic excitation. The constitutive behaviour of each viscoelastic damper is described by a fractional differential equation involving two parameters i.e., the coefficient and the order of the fractional derivative, which are assumed as design parameters. Ground motion acceleration is modelled as a mono-correlated zero-mean stationary Gaussian random process, well representative of the *strong motion* phase of natural accelerograms. Under this assumption, the motion of the controlled structure is governed by a set of linear fractional differential equations forced by a stationary stochastic process. The solution of these equations should be performed by using stochastic fractional analysis tools. In this paper, stochastic analysis of the controlled frame is performed in the frequency domain by means of the approach recently proposed by Sofi et al. [35]. Then, structural safety assessment is addressed within the framework of the *first-passage* theory by evaluating the *reliability function* for the *extreme value* response process of interest. By applying Vanmarcke's failure criterion [36,37], the *reliability function* is approximated by an explicit function of the first

three spectral moments of the response process, which depend on the parameters of the fractional viscoelastic dampers. Relying on this approximation, the sensitivities of the *reliability function* to the main parameters of the external dampers are derived in explicit form by direct differentiation. Notably, the evaluation of reliability sensitivities involves the sensitivities of the first three spectral moments of the response process, which can be derived by direct differentiation as well. To demonstrate the effectiveness of the proposed formulation, numerical results concerning a five-story shear-type frame controlled by external fractional viscoelastic dampers are presented.

The rest of the paper is organized as follows: in Section 2, the problem formulation for frame structures controlled by external fractional viscoelastic dampers under seismic excitation is outlined; in Section 3, the formulation of local sensitivity analysis of the stochastic response is presented; in Section 4, closed-form expressions of reliability sensitivities to the parameters of the fractional viscoelastic dampers are derived; in Section 5, numerical results are presented and discussed; in Section 6, some conclusions are drawn.

2. Problem formulation

2.1. Frame structure controlled by external fractional viscoelastic dampers

In this paper, an effective vibration control strategy for an n -degree-of-freedom (n -DOF) linear frame structure subjected to seismic ground motion acceleration is implemented. The frame is connected to a much more rigid wall through a set of external energy dissipation devices characterized by fractional linear viscoelastic behaviour, as illustrated in Fig. 1 [35]. The configuration of these devices may be adjusted for design purposes.

Since the beginning of the twentieth century, it has been shown that experimental creep and relaxation tests on viscoelastic materials can be very well fitted by a time-decaying power-law for deformation and stress, respectively [12–15]. According to the *principle of intermediacy*, introduced by Scott Blair [13], the force-displacement relationship of the j -th energy dissipation viscoelastic device is described by the following fractional differential equation:

$$F_j(t) = C_{\beta_j} \mathcal{D}_{0^+}^{\beta_j} \langle U_j(t) \rangle \quad (1)$$

where $U_j(t)$ and $F_j(t)$ denote the j -th horizontal displacement of the

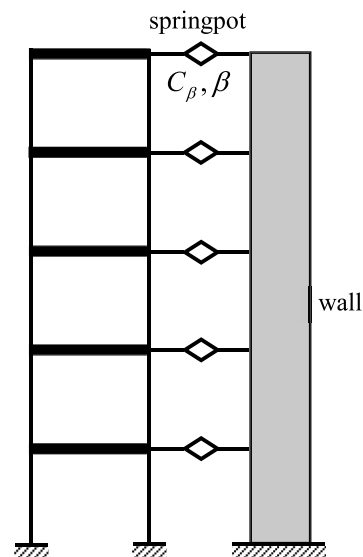


Fig. 1. Sketch of a frame structure controlled by external fractional viscoelastic dampers.

frame and the associated force at time t ; ${}^C_0 \mathcal{D}_t^{\beta_j} \langle \bullet \rangle$ indicates *Caputo's* fractional operator defined as [38,39]:

$${}^C_0 \mathcal{D}_t^{\beta_j} \langle x(t) \rangle = \frac{1}{\Gamma(1-\beta_j)} \int_0^t \frac{1}{(t-\tau)^{\beta_j}} \dot{x}(\tau) d\tau, \quad 0 \leq \beta_j < 1 \quad (2)$$

where $x(t)$ is a differentiable function; $\Gamma(\bullet)$ is Euler's Gamma function; β_j is the fractional derivative order; and the dot over a variable denotes differentiation with respect to time. *Caputo's* fractional operator is particularly suitable for the dynamic analysis of structures with viscoelastic materials, quiescent at time $t = 0$. This fractional derivative operator possesses the following property in the Fourier transform domain [15]:

$$\begin{aligned} \mathcal{F} \left\langle {}^C_{-\infty} \mathcal{D}_t^{\beta_j} \langle x(t) \rangle \right\rangle &= (i\omega)^{\beta_j} X(\omega) \\ &\equiv |\omega|^{\beta_j} \left(\cos \frac{\beta_j \pi}{2} + i \operatorname{sgn}(\omega) \sin \frac{\beta_j \pi}{2} \right) X(\omega) \end{aligned} \quad (3)$$

where $i = \sqrt{-1}$ is the imaginary unit; $|\bullet|$ indicates absolute value; $\mathcal{F}(\bullet)$ denotes the Fourier transform operator; and $X(\omega)$ represents the Fourier transform of the function $x(t)$, defined as follows:

$$X(\omega) \equiv \mathcal{F} \langle x(t) \rangle = \int_{-\infty}^{+\infty} \exp(-i\omega t) x(t) dt. \quad (4)$$

As a consequence of the *principle of intermediacy* [13], a rheological element called springpot is introduced, providing a mechanical model of the fractional viscoelastic devices, as sketched in Fig. 1. The constitutive behaviour of the springpot is ruled by Eq. (1), which involves two parameters only, i.e., the coefficient C_{β_j} and the order of the fractional derivative β_j . Equation (1) generalizes Hooke's and Newton's laws, which can be obtained as particular cases by setting $\beta_j = 0$ and $\beta_j = 1$, respectively.

The analysis of the springpot response in the frequency domain provides information on the energy dissipation capacity of a viscoelastic material. The Fourier transform of the fractional differential equation in Eq. (1) yields an algebraic equation involving the so-called *complex* or *dynamic modulus* [40]

$$\begin{aligned} G_{\beta_j}(\omega) &= (i\omega)^{\beta_j} C_{\beta_j} = \left(\cos \frac{\beta_j \pi}{2} + i \operatorname{sgn}(\omega) \sin \frac{\beta_j \pi}{2} \right) |\omega|^{\beta_j} C_{\beta_j} \\ &= G'_{\beta_j}(\omega) + i G''_{\beta_j}(\omega). \end{aligned} \quad (5)$$

In the positive frequency domain, the real part, $G'_{\beta_j}(\omega)$, is the *storage modulus*, which represents the elastic portion and measures the maximum energy "stored" in the device; the imaginary part, $G''_{\beta_j}(\omega)$, is the *loss modulus*, which represents the viscous portion and measures the dissipated energy. The dimensionless ratio of the *loss modulus* to *storage modulus* in a viscoelastic material, $\operatorname{Im}\{(i\omega)^{\beta_j}\} / \operatorname{Re}\{(i\omega)^{\beta_j}\} = \tan(\beta_j \pi / 2)$, is the so-called *loss factor*, which provides a measure of damping in the material. Notice that, by adopting the fractional model of viscoelasticity, this factor is almost constant, in accordance with the *principle of frequency independence of loss factor* [41]. For a material with $\tan(\beta_j \pi / 2)$ greater than 1 (β_j greater than 1/2), the energy dissipation viscous component of the *complex modulus* prevails. Notice also that, if $\beta_j = 0$, the device exhibits a purely elastic behaviour, while, if $\beta_j = 1$, the behaviour is purely viscous.

2.2. Equations of motion

To perform *design sensitivity analysis* (DSA), the coefficient C_{β_j} and the order of the fractional derivative β_j in Eq. (1) can be expressed as follows:

$$C_{\beta_j}(\alpha_{C_{\beta_j}}) = C_{\beta_{j0}}(1 + \alpha_{C_{\beta_j}}) \quad (6)$$

and

$$\beta_j(\alpha_{\beta_j}) = \beta_{j0}(1 + \alpha_{\beta_j}) \quad (7)$$

where $j = 1, 2, \dots, n$ if a device is installed at each floor; $C_{\beta_{j0}}$ and β_{j0} denote the nominal values; $\alpha_{C_{\beta_j}}$ and α_{β_j} are dimensionless variables representing small variations around the nominal values, which are assumed as design parameters and are collected into the following $2n$ -vector:

$$\boldsymbol{\alpha} = \left[\boldsymbol{\alpha}_{C_{\beta}}^T \quad \boldsymbol{\alpha}_{\beta}^T \right]^T = \left[\alpha_{C_{\beta 1}} \quad \alpha_{C_{\beta 2}} \quad \dots \quad \alpha_{C_{\beta n}} \quad \alpha_{\beta 1} \quad \alpha_{\beta 2} \quad \dots \quad \alpha_{\beta n} \right]^T \quad (8)$$

where the apex T denotes the transpose operator. Based on Eqs. (6) and (7), the nominal values of the parameters C_{β_j} and β_j are obtained setting $\boldsymbol{\alpha} = \mathbf{0}$.

Under these assumptions, the equations of motion of the linear frame with energy dissipation viscoelastic devices, quiescent at initial time $t = 0$, can be written in the following form:

$$\begin{aligned} \mathbf{M} \ddot{\mathbf{U}}(t, \boldsymbol{\alpha}) + \mathbf{C} \dot{\mathbf{U}}(t, \boldsymbol{\alpha}) + \mathbf{K} \mathbf{U}(t, \boldsymbol{\alpha}) + \sum_{j=1}^n \mathbf{C}_{\beta_j}(\alpha_{C_{\beta_j}}) {}^C_0 \mathcal{D}_t^{\beta_j(\alpha_{\beta_j})} \langle \mathbf{U}(t, \boldsymbol{\alpha}) \rangle \\ = -\mathbf{M} \boldsymbol{\tau} \ddot{\mathbf{U}}_g(t) \end{aligned} \quad (9)$$

where \mathbf{M} , \mathbf{C} , and \mathbf{K} are the $(n \times n)$ symmetric and positive-definite mass, damping, and stiffness matrices of the structure; $\mathbf{U}(t, \boldsymbol{\alpha})$ is the n -dimensional vector of floor horizontal displacements relative to the ground; $\boldsymbol{\tau}$ is the influence coefficient vector; and $\ddot{\mathbf{U}}_g(t)$ denotes seismic ground motion acceleration; in view of Eq. (1) and taking into account that the frame is connected to a much more rigid wall, $\mathbf{C}_{\beta_j}(\alpha_{C_{\beta_j}})$ is an $(n \times n)$ matrix with all zero entries except for the j -th diagonal element, which is equal to the coefficient $C_{\beta_j}(\alpha_{C_{\beta_j}})$ of the fractional derivative operator (see Eq. (6)), depending on the thickness and area of the viscoelastic material adopted for the j -th device; $\beta_j(\alpha_{\beta_j})$ is the order of the fractional derivative (see Eq. (7)), which depends only on the viscoelastic material selected for the j -th device. In Eq. (9), the dependence of the matrix $\mathbf{C}_{\beta_j}(\alpha_{C_{\beta_j}})$, the fractional derivative order $\beta_j(\alpha_{\beta_j})$, and the response vector $\mathbf{U}(t, \boldsymbol{\alpha})$ on the design variables, collected into the vector $\boldsymbol{\alpha}$ (Eq. (8)), is highlighted.

2.3. Power Spectral Density function of the stochastic response

Let us assume that the seismic excitation, $\ddot{\mathbf{U}}_g(t)$, is modelled as a zero-mean stationary mono-correlated Gaussian random process completely characterized by its one-sided *Power Spectral Density* (PSD) function $G_{\ddot{\mathbf{U}}_g}(\omega)$. Under this assumption, the solution of Eq. (9), $\mathbf{U}(t, \boldsymbol{\alpha})$, is a zero-mean stationary Gaussian stochastic vector process, fully characterized by its one-sided PSD function matrix $\mathbf{G}_{\mathbf{U}\mathbf{U}}(\omega, \boldsymbol{\alpha})$, which can be evaluated relying on the Fourier transform of Eq. (9), obtaining:

$$\mathbf{G}_{\mathbf{U}\mathbf{U}}(\omega, \boldsymbol{\alpha}) = \mathbf{H}^*(\omega, \boldsymbol{\alpha}) \mathbf{p} \mathbf{p}^T \mathbf{H}^T(\omega, \boldsymbol{\alpha}) G_{\ddot{\mathbf{U}}_g}(\omega) \quad (10)$$

where $\mathbf{p} = -\mathbf{M}\boldsymbol{\tau}$ is the participation coefficient vector, and the asterisk means complex conjugate; $\mathbf{H}(\omega, \boldsymbol{\alpha})$ represents the *transfer function* matrix, also known as *frequency response function* (FRF) matrix. Considering Eq. (3), $\mathbf{H}(\omega, \boldsymbol{\alpha})$ can be evaluated in the positive frequency domain as:

$$\mathbf{H}(\omega, \boldsymbol{\alpha}) = \left[-\omega^2 \mathbf{M} + i\omega \mathbf{C} + \mathbf{K} + \sum_{j=1}^n (i\omega)^{\beta_j(\alpha_{\beta_j})} \mathbf{C}_{\beta_j}(\alpha_{C_{\beta_j}}) \right]^{-1}. \quad (11)$$

3. Local sensitivity of stochastic response

Local sensitivity analysis (LSA) consists of evaluating the change in structural response due to parameter variations around assigned values, called “nominal parameters”. In practice, *local sensitivities* are defined as the partial derivatives of the output with respect to the input parameters at the nominal values.

Since the response of linear structural systems subjected to zero-mean stationary Gaussian stochastic excitation is fully characterized by the relevant *PSD* function, the local sensitivities of response statistics can be evaluated in the frequency domain. To this aim, one can perform direct differentiation of the one-sided *PSD* function matrix $\mathbf{G}_{\text{UU}}(\omega, \boldsymbol{\alpha})$ of the displacement vector process given in Eq. (10), which depends on the parameters characterizing the fractional viscoelastic devices, and then set $\boldsymbol{\alpha} = \mathbf{0}$, i.e.:

$$\begin{aligned} \mathbf{S}_{\mathbf{G}_{\text{UU}},k}(\omega) &= \frac{\partial}{\partial \alpha_k} \mathbf{G}_{\text{UU}}(\omega, \boldsymbol{\alpha}) \Big|_{\boldsymbol{\alpha}=\mathbf{0}} \\ &= \mathbf{G}_{\tilde{U}_k}(\omega) [\mathbf{T}_k^*(\omega) \mathbf{p} \mathbf{p}^T \mathbf{H}_0^*(\omega) + \mathbf{H}_0^*(\omega) \mathbf{p} \mathbf{p}^T \mathbf{T}_k^*(\omega)] \end{aligned} \quad (12)$$

where

$$\begin{aligned} \mathbf{T}_k(\omega) &= \frac{\partial}{\partial \alpha_k} \mathbf{H}(\omega, \boldsymbol{\alpha}) \Big|_{\boldsymbol{\alpha}=\mathbf{0}} \\ &= \begin{cases} -(i\omega)^{\beta_{k0}} \mathbf{H}_0(\omega) \mathbf{C}_{\beta_{k0}} \mathbf{H}_0(\omega), & \text{if } \alpha_k \in \boldsymbol{\alpha}_{C_{\beta}} \\ -\beta_{k0} (i\omega)^{\beta_{k0}} \log(i\omega) \mathbf{H}_0(\omega) \mathbf{C}_{\beta_{k0}} \mathbf{H}_0(\omega), & \text{if } \alpha_k \in \boldsymbol{\alpha}_{\beta} \end{cases} \end{aligned} \quad (13)$$

with $\mathbf{H}_0(\omega) = \mathbf{H}(\omega, \boldsymbol{\alpha}) \Big|_{\boldsymbol{\alpha}=\mathbf{0}}$ and $\mathbf{C}_{\beta_{k0}} = \mathbf{C}_{\beta_k}(\alpha_{C_{\beta k}}) \Big|_{\boldsymbol{\alpha}=\mathbf{0}}$.

The matrix collecting the first three spectral moments of the displacement vector is defined as:

$$\Lambda_{j,\text{UU}}(\boldsymbol{\alpha}) = \int_0^{\infty} \omega^j \mathbf{G}_{\text{UU}}(\omega, \boldsymbol{\alpha}) d\omega, \quad j = 0, 1, 2. \quad (14)$$

The sensitivity matrix of the spectral moments with respect to the k -th parameter can be evaluated as:

$$\mathbf{S}_{\Lambda_{j,\text{UU}},k}(\omega) = \frac{\partial}{\partial \alpha_k} \Lambda_{j,\text{UU}}(\boldsymbol{\alpha}) \Big|_{\boldsymbol{\alpha}=\mathbf{0}} = \int_0^{\infty} \omega^j \mathbf{S}_{\mathbf{G}_{\text{UU}},k}(\omega) d\omega, \quad j = 0, 1, 2 \quad (15)$$

where $\mathbf{S}_{\mathbf{G}_{\text{UU}},k}(\omega)$ is given by Eq. (12).

Relying on the knowledge of the sensitivities of the spectral moments of the response process of interest, explicit expressions of reliability sensitivities to the design parameters will be derived in the next section, within the framework of the *first-passage* theory.

4. Reliability sensitivity analysis

4.1. Definition of the reliability function

Failure of a structural system is herein identified with the *first-passage failure*, which occurs when the *extreme value* random process for some response measure (e.g., displacement, strain or stress) first exceeds a prescribed safe domain within a specified time interval $[0, T]$. Specifically, it is assumed that the structure fails in a *first-passage* sense when the *extreme value* random process of the generic structural zero-mean stationary Gaussian random response process $Y_h(t, \boldsymbol{\alpha})$, defined as:

$$Y_{h,\text{max}}(T, \boldsymbol{\alpha}) = \max_{0 \leq t \leq T} |Y_h(t, \boldsymbol{\alpha})| \quad (16)$$

first exceeds the critical level b within the time interval $[0, T]$. In the

previous equation, the symbol $|\bullet|$ denotes absolute value, and $Y_h(t, \boldsymbol{\alpha})$ is defined as follows:

$$Y_h(t, \boldsymbol{\alpha}) = \mathbf{q}_h^T \mathbf{U}(t, \boldsymbol{\alpha}) \quad (17)$$

where \mathbf{q}_h is a vector collecting the combination coefficients relating $Y_h(t, \boldsymbol{\alpha})$ to the displacement vector $\mathbf{U}(t, \boldsymbol{\alpha})$.

In the context of the *first-passage* theory, the aim of reliability analysis is the evaluation of the *cumulative distribution function (CDF)*, also called *reliability function*, defined as the probability that the *extreme value* random process $Y_{h,\text{max}}(T, \boldsymbol{\alpha})$ does not exceed the barrier level b in the time interval $[0, T]$ [42]:

$$L_{Y_{h,\text{max}}}(b, T, \boldsymbol{\alpha}) = \Pr[Y_{h,\text{max}}(T, \boldsymbol{\alpha}) \leq b | 0 \leq t \leq T]. \quad (18)$$

Unfortunately, the exact probability distribution of the *extreme value* random process is not available, and several approximations have been proposed in the literature (see e.g., Ref. [42]). In this paper, the criterion proposed by Vanmarcke [36,37] is adopted. According to this criterion, the *CDF* of the *extreme value* random process $Y_{h,\text{max}}(T, \boldsymbol{\alpha})$, for unit initial probability ($\Pr[Y_{h,\text{max}}(0, \boldsymbol{\alpha}) \leq b] = 1$), can be approximated as:

$$L_{Y_{h,\text{max}}}(b, T, \boldsymbol{\alpha}) \cong \exp[-T \eta_{Y_h}(b, \boldsymbol{\alpha})] \quad (19)$$

where $\eta_{Y_h}(b, \boldsymbol{\alpha})$ is the so-called *hazard function* (dimensionally equal to the inverse of time, $[s^{-1}]$), defined as:

$$\eta_{Y_h}(b, \boldsymbol{\alpha}) = 2\nu_{Y_h}^+(\boldsymbol{\alpha}) \left[\frac{1 - \exp[-\sqrt{\pi} \delta_{Y_h}^{1,2}(\boldsymbol{\alpha}) r_{Y_h}(b, \boldsymbol{\alpha})]}{\exp[r_{Y_h}^2(b, \boldsymbol{\alpha})] - 1} \right]. \quad (20)$$

In the previous equation, the *zero-level up-crossing frequency*, $\nu_{Y_h}^+(\boldsymbol{\alpha})$, the *bandwidth parameter*, $\delta_{Y_h}(\boldsymbol{\alpha})$, and the *dimensionless barrier*, $r_{Y_h}(b, \boldsymbol{\alpha})$, are given, respectively, by:

$$\begin{aligned} \nu_{Y_h}^+(\boldsymbol{\alpha}) &= \frac{1}{2\pi} \sqrt{\frac{\lambda_{2,Y_h}(\boldsymbol{\alpha})}{\lambda_{0,Y_h}(\boldsymbol{\alpha})}}; \\ \delta_{Y_h}(\boldsymbol{\alpha}) &= \sqrt{1 - \frac{\text{Re}\{\lambda_{1,Y_h}(\boldsymbol{\alpha})\}^2}{\lambda_{0,Y_h}(\boldsymbol{\alpha})\lambda_{2,Y_h}(\boldsymbol{\alpha})}}; \\ r_{Y_h}(b, \boldsymbol{\alpha}) &= \frac{b}{\sqrt{2\lambda_{0,Y_h}(\boldsymbol{\alpha})}} \end{aligned} \quad (21a-c)$$

where $\lambda_{j,Y_h}(\boldsymbol{\alpha})$ is the j -th spectral moment of the response process $Y_h(t, \boldsymbol{\alpha})$ [36,37]:

$$\lambda_{j,Y_h}(\boldsymbol{\alpha}) = \mathbf{q}_h^T \Lambda_{j,\text{UU}}(\boldsymbol{\alpha}) \mathbf{q}_h = \mathbf{q}_h^T \left[\int_0^{\infty} \omega^j \mathbf{G}_{\text{UU}}(\omega, \boldsymbol{\alpha}) d\omega \right] \mathbf{q}_h, \quad j = 0, 1, 2. \quad (22)$$

4.2. Evaluation of the reliability sensitivity function

The evaluation of the sensitivity of the *reliability function* is a crucial step to perform *DSA* of linear structural systems with viscoelastic dampers, rheologically modelled using fractional derivatives. In this section, reliability sensitivities to the parameters of the fractional viscoelastic dampers are derived in explicit form by direct differentiation of the approximate analytical expression in Eq. (19).

The sensitivity of the *reliability function* (Eq. (19)) with respect to the k -th parameter reads:

$$S_{L_{Y_{h,\text{max}}},k}(b, T) = \frac{\partial L_{Y_{h,\text{max}}}(b, T, \boldsymbol{\alpha})}{\partial \alpha_k} \Big|_{\boldsymbol{\alpha}=\mathbf{0}} = -TL_{Y_{h,\text{max}}}^{(0)}(b, T) S_{\eta_{Y_h},k}(b) \quad (23)$$

where $S_{\eta_{Y_h,k}}(b)$ is the sensitivity of the *hazard function* (Eq. (20)) and $L_{Y_h,\max}^{(0)}(b, T)$ is the nominal value of the *reliability function*, defined respectively as:

$$S_{\eta_{Y_h,k}}(b) = \left. \frac{\partial \eta_{Y_h}(b, \alpha)}{\partial \alpha_k} \right|_{\alpha=0}; \quad (24a,b)$$

$$L_{Y_h,\max}^{(0)}(b, T) = L_{Y_h,\max}(b, T, \alpha)|_{\alpha=0} = \exp[-T\eta_{Y_h}^{(0)}(b)]$$

with $\eta_{Y_h}^{(0)}(b) = \eta_{Y_h}(b, \alpha)|_{\alpha=0}$ denoting the nominal value of the *hazard function*.

The evaluation of the sensitivity of the *hazard function* with respect to the k -th parameter requires the knowledge of the sensitivity of the *zero-level up-crossing frequency*, $S_{\nu_{Y_h,k}^+}$, *bandwidth parameter*, $S_{\delta_{Y_h,k}}$, and *dimensionless barrier*, $S_{r_{Y_h,k}}(b)$, defined, respectively, as follows:

$$S_{\nu_{Y_h,k}^+} = \left. \frac{\partial \nu_{Y_h}^+(\alpha)}{\partial \alpha_k} \right|_{\alpha=0} = \frac{1}{2} \nu_{Y_h}^{+(0)} \left[\frac{1}{\lambda_{2,Y_h}^{(0)}} S_{\lambda_{2,Y_h,k}} - \frac{1}{\lambda_{0,Y_h}^{(0)}} S_{\lambda_{0,Y_h,k}} \right];$$

$$S_{\delta_{Y_h,k}} = \left. \frac{\partial \delta_{Y_h}(\alpha)}{\partial \alpha_k} \right|_{\alpha=0}$$

$$= -\frac{1}{2\delta_{Y_h}^{(0)}} \left(\frac{\lambda_{1,Y_h}^{(0)}}{\lambda_{0,Y_h}^{(0)} \lambda_{2,Y_h}^{(0)}} \right)^2 \left[\frac{2S_{\lambda_{1,Y_h,k}}}{\lambda_{1,Y_h}^{(0)}} - \frac{S_{\lambda_{0,Y_h,k}}}{\lambda_{0,Y_h}^{(0)}} - \frac{S_{\lambda_{2,Y_h,k}}}{\lambda_{2,Y_h}^{(0)}} \right]; \quad (25a-c)$$

$$S_{r_{Y_h,k}}(b) = \left. \frac{\partial r_{Y_h}(b, \alpha)}{\partial \alpha_k} \right|_{\alpha=0} = -\frac{r_{Y_h}^{(0)}(b)}{2\lambda_{0,Y_h}^{(0)}} S_{\lambda_{0,Y_h,k}}$$

where $S_{\lambda_{j,Y_h,k}}$ is the sensitivity of the j -th spectral moment, and

$$\lambda_{j,Y_h}^{(0)} = \lambda_{j,Y_h}(\alpha)|_{\alpha=0}; \quad \nu_{Y_h}^{+(0)} = \nu_{Y_h}^+(\alpha)|_{\alpha=0}; \quad (26a-d)$$

$$\delta_{Y_h}^{(0)} = \delta_{Y_h}(\alpha)|_{\alpha=0}; \quad r_{Y_h}^{(0)}(b) = r_{Y_h}(b, \alpha)|_{\alpha=0}.$$

By inspection of the previous equations, it appears that evaluating the sensitivities of the *hazard function* and the associated *reliability function* requires knowledge of the sensitivities of the first three spectral moments of the selected structural response process. According to Eqs. (15) and (22), these sensitivities can be determined as follows:

$$S_{\lambda_{j,Y_h,k}} = \left. \frac{\partial \lambda_{j,Y_h}(\alpha)}{\partial \alpha_k} \right|_{\alpha=0} = \mathbf{q}_h^T \frac{\partial}{\partial \alpha_k} \Lambda_{j,UU}(\alpha) \Big|_{\alpha=0} \mathbf{q}_h$$

$$= \mathbf{q}_h^T \left[\int_0^\infty \omega^j \mathbf{S}_{GUU,k}(\omega) d\omega \right] \mathbf{q}_h, j = 0, 1, 2 \quad (27)$$

where $\mathbf{S}_{GUU,k}(\omega)$ is given by Eq. (12).

By performing direct differentiation of Eq. (20) and taking into account Eqs. (25)–(27), the sensitivity of the *hazard function* to the k -th parameter can be expressed as follows:

$$S_{\eta_{Y_h,k}}(b) = 2S_{\nu_{Y_h,k}^+} \frac{A_{Y_h}^{(0)}(b)}{B_{Y_h}^{(0)}(b)} + 2\nu_{Y_h}^{+(0)} \left[\frac{S_{A_{Y_h,k}}(b)}{B_{Y_h}^{(0)}(b)} - \frac{A_{Y_h}^{(0)}(b) S_{B_{Y_h,k}}(b)}{(B_{Y_h}^{(0)}(b))^2} \right] \quad (28)$$

where

$$S_{A_{Y_h,k}}(b) = \sqrt{\pi} \left[1 - A_{Y_h}^{(0)}(b) \right] \left(\delta_{Y_h}^{(0)} \right)^{0.2}$$

$$\times \left[1.2 r_{Y_h}^{(0)}(b) S_{\delta_{Y_h,k}} + \delta_{Y_h}^{(0)} S_{r_{Y_h,k}}(b) \right]; \quad (29a,b)$$

$$S_{B_{Y_h,k}}(b) = 2 \left[B_{Y_h}^{(0)}(b) + 1 \right] r_{Y_h}^{(0)}(b) S_{r_{Y_h,k}}(b)$$

and

$$A_{Y_h}^{(0)}(b) = 1 - \exp \left[-\sqrt{\pi} \left(\delta_{Y_h}^{(0)} \right)^{1.2} r_{Y_h}^{(0)}(b) \right]; \quad (30a,b)$$

$$B_{Y_h}^{(0)}(b) = \exp \left[\left(r_{Y_h}^{(0)}(b) \right)^2 \right] - 1$$

are the sensitivities and nominal values of the following functions

$$A_{Y_h}(b, \alpha) = 1 - \exp \left[-\sqrt{\pi} \delta_{Y_h}^{1.2}(\alpha) r_{Y_h}(b, \alpha) \right]; \quad (31a,b)$$

$$B_{Y_h}(b, \alpha) = \exp \left[r_{Y_h}^2(b, \alpha) \right] - 1.$$

Substituting Eq. (28) into Eq. (23) and taking into account Eq. (24b), an explicit expression of the sensitivity of the *reliability function* with respect to the k -th parameter is obtained.

4.3. Design sensitivity analysis

DSA is an effective tool for quantifying the influence of various parameters on the structural response of interest. It first requires the definition of a *performance measure function*, $\varphi(t, \alpha)$, which generally depends on the design variable vector α and time t . Then, DSA involves evaluating the so-called *function of sensitivity*, namely a dimensionless percentage measure of the influence of the generic parameter on the selected performance measure. For the k -th parameter, such a function can be defined as follows:

$$\varepsilon_{\varphi,k}(t)(\%) = \frac{S_{\varphi,k}(t)}{\varphi^{(0)}(t)} \alpha_k \times 100 \quad (32)$$

where $S_{\varphi,k}(t) = \partial \varphi(t, \alpha) / \partial \alpha_k |_{\alpha=0}$ and $\varphi^{(0)}(t) = \varphi(t, \alpha) |_{\alpha=0}$ are the sensitivity with respect to the k -th parameter and the nominal value of the selected *performance measure function*, $\varphi(t, \alpha)$; α_k denotes a small dimensionless fluctuation of the k -th parameter around the nominal value ($\alpha_k = 0$). If the performance measure is a function of the design variables only, i.e., $\varphi(\alpha)$, Eq. (32) reduces to the so-called *coefficient of sensitivity*.

In the present study, DSA is applied to identify the parameters of the external fractional viscoelastic dampers that have the most significant influence on the *reliability function* of the response quantity of interest. To this aim, within the context of the *first-passage* formulation outlined in Section 4.1, the first three spectral moments of the response process, $\lambda_{j,Y_h}(\alpha)$, $j = 0, 1, 2$, and the *reliability function*, $L_{Y_h,\max}(b, T, \alpha)$, are selected as *performance measure functions*.

By applying Eq. (32), the *coefficient of sensitivity* of the spectral moments of the selected response process with respect to the k -th parameter is obtained:

$$\varepsilon_{\lambda_{j,Y_h,k}}(\%) = \frac{S_{\lambda_{j,Y_h,k}}}{\lambda_{j,Y_h}^{(0)}} \alpha_k \times 100, \quad j = 0, 1, 2 \quad (33)$$

which can be evaluated based on the knowledge of the nominal value $\lambda_{j,Y_h}^{(0)}$ and k -th sensitivity $S_{\lambda_{j,Y_h,k}}$ of $\lambda_{j,Y_h}(\alpha)$ given by Eqs. (26a) and (27), respectively.

Similarly, Eq. (32) yields the *function of sensitivity* of the *reliability function* $L_{Y_h,\max}(b, T, \alpha)$ with respect to the k -th parameter in the following form:

$$\varepsilon_{L_{Y_h,\max,k}}(b, T)(\%) = \frac{S_{L_{Y_h,\max,k}}(b, T)}{L_{Y_h,\max}^{(0)}(b, T)} \alpha_k \times 100 = -TS_{\eta_{Y_h,k}}(b) \alpha_k \times 100 \quad (34)$$

which can be evaluated once the functions $S_{L_{Y_h,\max,k}}(b, T)$ and $L_{Y_h,\max}^{(0)}(b, T)$, defined in Eqs. (23) and (24b), respectively, are determined.

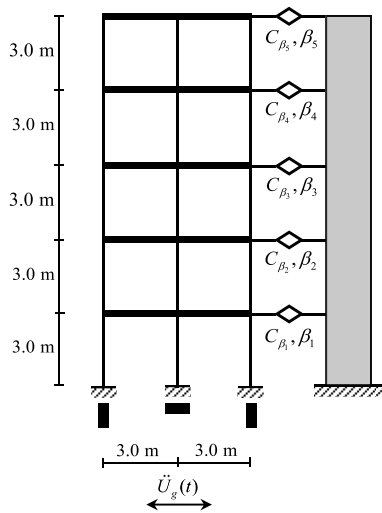


Fig. 2. Five-story shear-type frame with external fractional viscoelastic dampers subjected to seismic excitation.

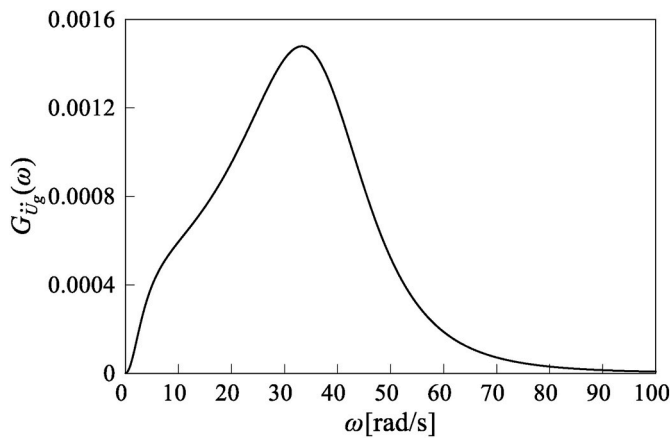


Fig. 3. PSD function of ground motion acceleration.

5. Numerical application

The presented formulation for DSA is applied to a two-bay five-story shear-type frame controlled by external fractional viscoelastic dampers (see Fig. 2). The frame is characterized by the following geometrical and mechanical properties [35]: mass lumped at each floor $m = 60000$ kg; Young's modulus of the material $E = 25$ GPa; modal damping ratio $\zeta_0 = 0.05$ equal for all vibration modes; rectangular cross-section of columns with width $b = 0.3$ m and thickness $h = 0.5$ m (see Fig. 2). The fundamental period of the frame is $T_0 = 0.699$ s.

The nominal values of the coefficient C_{β_j} and order β_j of the fractional derivative, $C_{\rho 0} = 180995$ Ns $^\beta$ /m and $\beta_0 = 0.461$, assumed equal for all the devices, are defined relying on the results of experimental tests carried out by Montgomery and Christopoulos [43] to characterize

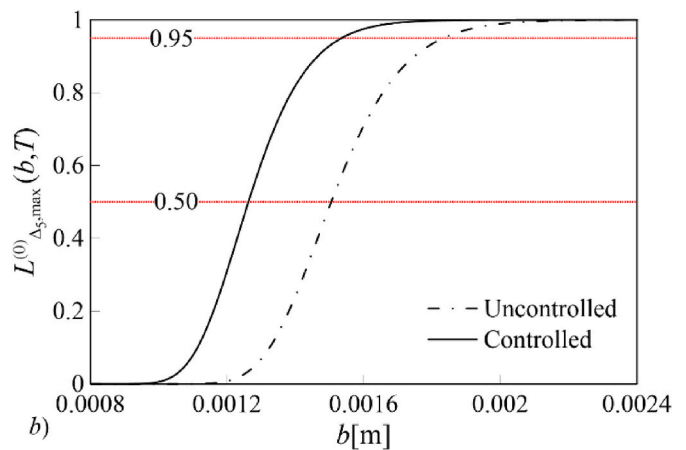
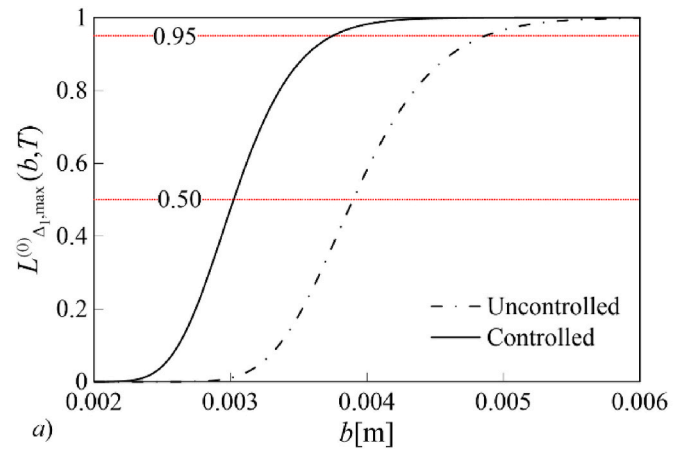


Fig. 4. Nominal CDF of the extreme value random processes a) $\Delta_{1,max}(T, \alpha)$ and b) $\Delta_{5,max}(T, \alpha)$ for the controlled and uncontrolled structure ($T = 30$ s).

viscoelastic materials (see also [35]).

The shear-type frame is subjected to ground motion acceleration modelled as a zero-mean stationary Gaussian random process fully characterized by the following unimodal one-sided PSD function [44]:

$$G_{\ddot{U}_g}(\omega) = \sigma_{\ddot{U}_g}^2 \gamma_0 \left(\frac{\omega^2}{\omega^2 + \omega_H^2} \right) \left(\frac{\omega_L^4}{\omega^4 + \omega_L^4} \right) G_0^{(CP)}(\omega) \quad (35)$$

where $G_0^{(CP)}(\omega)$ is the Conte and Peng unimodal one-sided PSD function, defined as [45]:

$$G_0^{(CP)}(\omega) = \frac{\rho_0}{\pi} \left[\frac{1}{\rho_0^2 + (\omega + \Omega_0)^2} + \frac{1}{\rho_0^2 + (\omega - \Omega_0)^2} \right] \quad (36)$$

In the previous equations, $\sigma_{\ddot{U}_g}^2 = E\langle \ddot{U}_g^2(t) \rangle$, with $E\langle \cdot \rangle$ denoting the stochastic average operator, is the variance of the stochastic excitation process; ρ_0 and Ω_0 are the circular frequency bandwidth and the predominant circular frequency of the filtered stationary process, respectively [45]; $\omega_H = 0.1\Omega_0$ and $\omega_L = \Omega_0 + 0.8\rho_0$ are the control frequencies of the second-order low-pass and first-order high-pass Butterworth

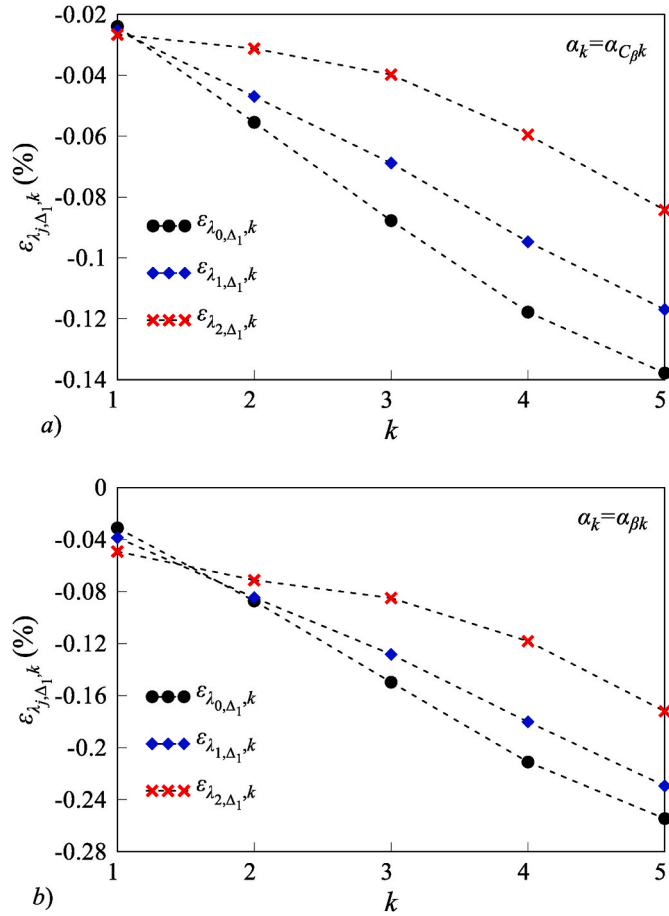


Fig. 5. Coefficients of sensitivity of the first three spectral moments of the first-floor drift with respect to the a) coefficients and b) orders of the fractional derivatives of the viscoelastic dampers.

filters [44]; the coefficient γ_0 (denoted with β_0 in Ref. [44]) is evaluated in such a way that the process $\ddot{U}_g(t)$ possesses variance $\sigma_{\ddot{U}_g}^2$. Relying on the statistical analysis [35] of a set of accelerograms, with $PGA > 0.05g$, recorded by stations located on rock subsoil in regions with high and moderate seismicity i.e., with *surface-wave magnitude* $M_s > 5.5$ [46], the three main parameters characterizing the PSD function in Eq. (35) are selected as follows: $\sigma_{\ddot{U}_g}^2 = 0.051 \text{ m/s}^2$, $\Omega_0 = 36.279 \text{ rad/s}$, and $\rho_0 = 16.022 \text{ rad/s}$. The relevant plot of the PSD function is displayed in Fig. 3.

For design purposes, it is important to identify the parameters of the external viscoelastic dampers which have the most significant impact on the seismic performance of the controlled frame. To this aim, the first-floor drift, $\Delta_1(t, \alpha)$, and top-floor drift, $\Delta_5(t, \alpha)$, are selected as performance measures. Sensitivities of the spectral moments and *reliability function* of the selected drift processes to the fractional derivative order and coefficient, $\beta_k(\alpha_{\beta k})$ and $C_{\beta k}(\alpha_{C\beta k})$, $k = 1, 2, \dots, 5$, (see Eqs. (6) and (7)), of the five external viscoelastic dampers are evaluated by applying the presented analytical expressions.

Fig. 4 shows the nominal CDF of the *extreme value* random processes (Eq. (24b)), $\Delta_{1,\max}(T, \alpha)$ and $\Delta_{5,\max}(T, \alpha)$, for the controlled and uncontrolled structure ($T = 30\text{s}$). It can be observed that the external

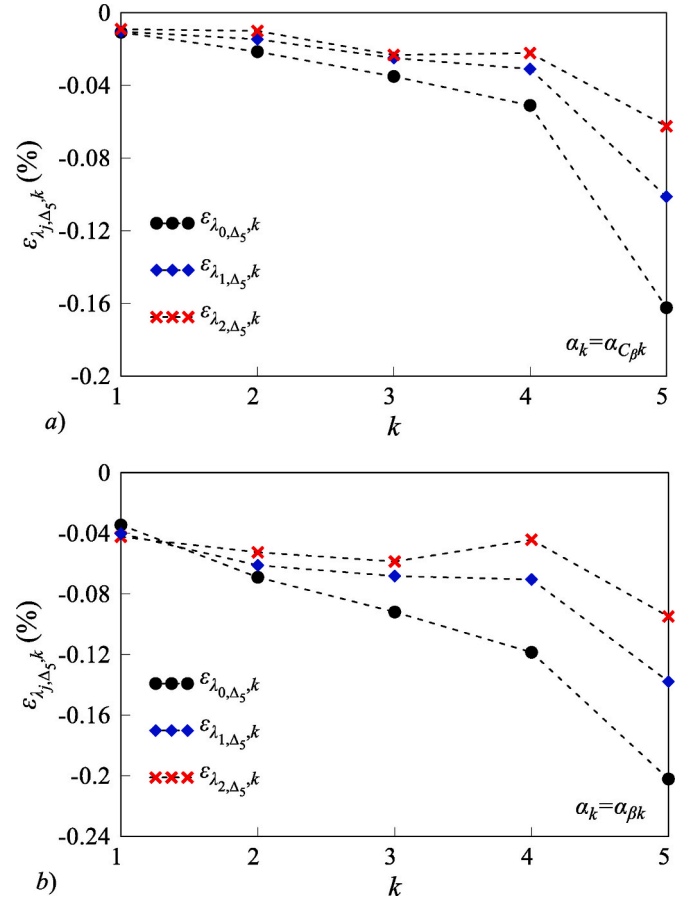


Fig. 6. Coefficients of sensitivity of the first three spectral moments of the top-floor drift with respect to the a) coefficients and b) orders of the fractional derivatives of the viscoelastic dampers.

control system significantly increases the safety level of the frame structure [35]. However, as highlighted in Ref. [35], the viscoelastic dampers on the various floors have different influences on the selected seismic performance measures.

To assess the impact of the damper parameters on the seismic behaviour of the controlled structure, a sensitivity analysis of the first three spectral moments of the response quantities of interest is carried out as a first step. Fig. 5 shows the *coefficients of sensitivity* $\varepsilon_{\lambda_{j,\Delta_1},k}$ (%) of the spectral moments (see Eq. (33)) $\lambda_{j,\Delta_1}(\alpha)$, $j = 0, 1, 2$, of the first-floor drift process $\Delta_1(t, \alpha)$ with respect to the coefficients and orders of the fractional derivative of the viscoelastic dampers, $C_{\beta k}(\alpha_{C\beta k})$ and $\beta_k(\alpha_{\beta k})$, $k = 1, 2, \dots, 5$, for $\alpha_k = \alpha_{C\beta k} = \alpha_{\beta k} = 0.01$. It can be observed that the *coefficients of sensitivity* are always negative, implying that a small increase of the considered parameters results in a decrease in the spectral moments. Furthermore, the parameters of the fractional viscoelastic damper on the first and fifth floors appear to have the least and greatest impact on the spectral moments, respectively. In particular, the zero-order spectral moment is generally the most affected by changes in the damper parameters. It is also noted that the spectral moments are

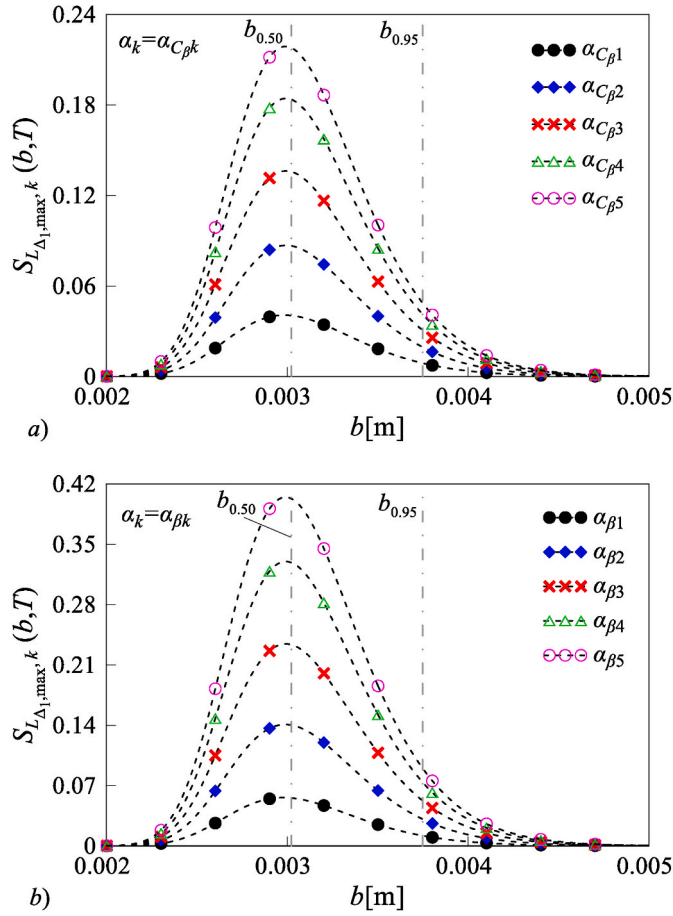


Fig. 7. Sensitivities of the CDF of the extreme value random process $\Delta_{1,\max}(T, \alpha)$ with respect to the a) coefficients and b) orders of the fractional derivatives of the viscoelastic dampers ($T = 30$ s).

more influenced by the fractional derivative order β_k ($\alpha_{\beta k}$) than by the coefficient $C_{\beta k}$ ($\alpha_{C_{\beta k}}$) of the viscoelastic dampers. Fig. 6 shows similar results for the coefficients of sensitivity $\varepsilon_{\lambda_{j,\Delta_5},k}(\%)$ of the spectral moments of the top-floor drift $\lambda_{j,\Delta_5}(\alpha)$, $j = 0, 1, 2$. As expected, in this case, the parameters of the fractional viscoelastic damper on the fifth floor have a much more significant impact than the others.

In Fig. 7, the sensitivities of the CDF of the extreme value random process $\Delta_{1,\max}(T, \alpha)$ to the damper parameters (Eq. (23)) versus the barrier b are plotted ($T = 30$ s). The values of the barrier $b_{0.50}$ and $b_{0.95}$ corresponding to the fractiles of order 0.50 and 0.95 of the nominal CDF $L_{\Delta_{1,\max}}^{(0)}(b, T)$ are also highlighted. Consistently with the results of the sensitivity analysis of the spectral moments (see Fig. 5), it is observed that the impact of the fractional derivative order and coefficient on the safety level increases when moving from the viscoelastic damper on the first floor to the fifth-floor device. Furthermore, as expected, sensitivities are always positive, implying that a small increase in the parameters leads to an increase in reliability for any barrier level b . Notably, sensitivities of the CDF achieve the maximum value when the barrier b is

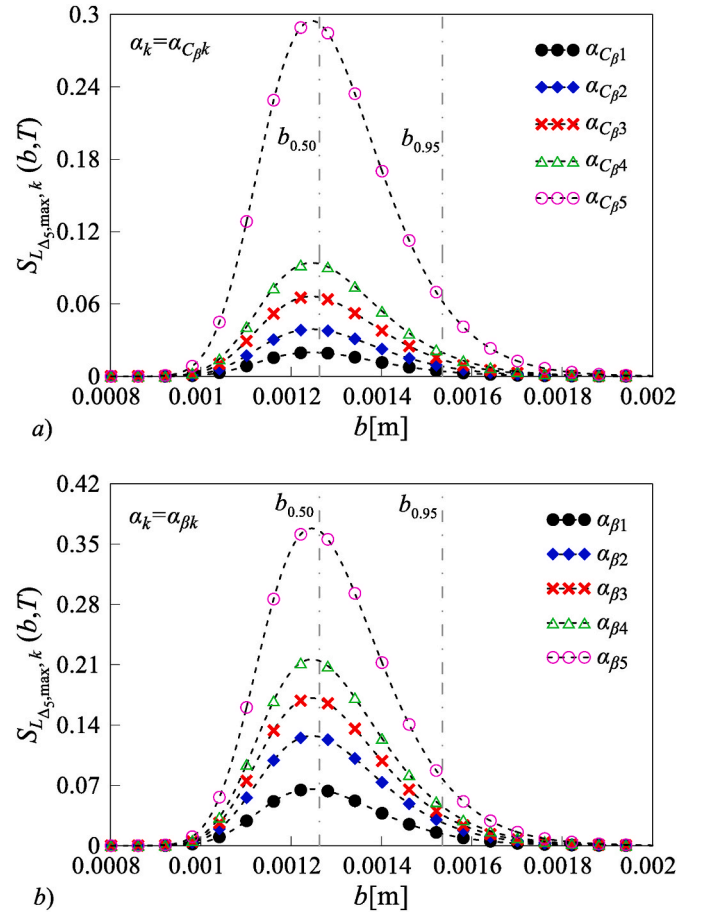


Fig. 8. Sensitivities of the CDF of the extreme value random process $\Delta_{5,\max}(T, \alpha)$ with respect to the a) coefficients and b) orders of the fractional derivatives of the viscoelastic dampers ($T = 30$ s).

close to $b_{0.50}$. Fig. 8 shows similar results for the sensitivities of the CDF of the extreme value random process $\Delta_{5,\max}(T, \alpha)$. As expected, small changes in the parameters of the viscoelastic damper on the fifth floor have a significantly greater influence on the CDF than those on the other floors. In this case, as well, the sensitivities reach their maximum value when the barrier b is close to $b_{0.50}$.

Figs. 9 and 10 show the functions of sensitivity $\varepsilon_{L_{\Delta_{h,\max}}^{(0)}}(b, T)(\%)$, $h = 1, 5$, of the CDF (Eq. (34)) of the extreme value random processes $\Delta_{1,\max}(T, \alpha)$ and $\Delta_{5,\max}(T, \alpha)$ for a range of the barrier level b enclosed by $b_{0.50}$ and $b_{0.95}$ ($\alpha_k = \alpha_{C_{\beta k}} = \alpha_{\beta k} = 0.01$; $T = 30$ s). In agreement with the information provided by the sensitivities of the CDF (see Figs. 7 and 8), it is observed that the functions of sensitivity decrease as the barrier level moves from $b_{0.50}$ to $b_{0.95}$, implying that small changes in the damper parameters have a higher impact on the median of the CDF. Moreover, the percentage influence of the damper parameters on the CDF increases when moving from the damper on the first floor to the fifth-floor device, even when the first-floor drift is considered as the performance measure. As expected, the function of sensitivity of the CDF of the extreme value

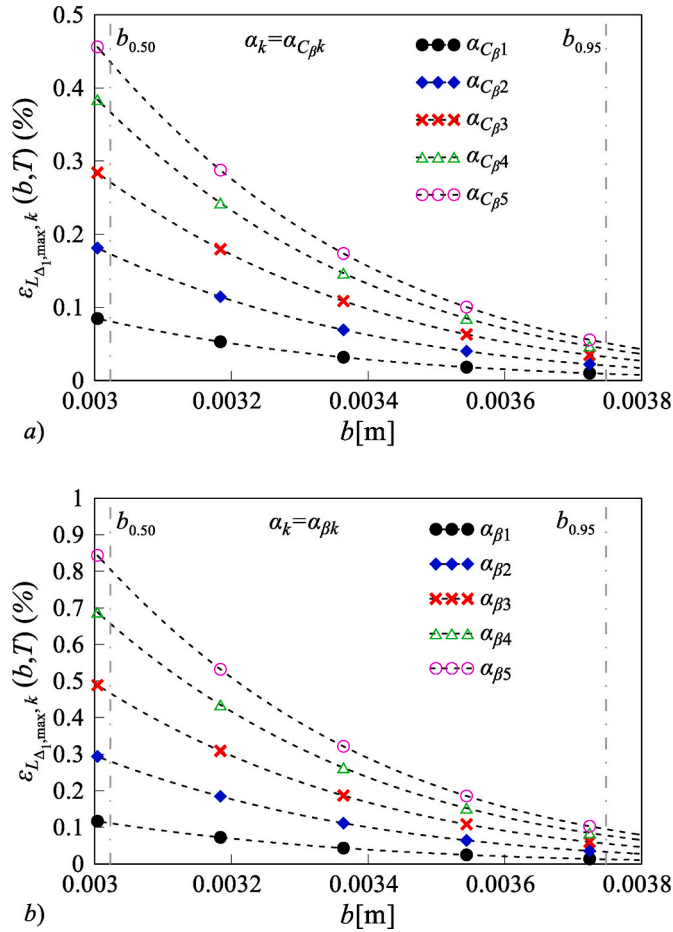


Fig. 9. Functions of sensitivity of the CDF of the extreme value random process $\Delta_{1, \max}(T, \alpha)$ with respect to the a) coefficients and b) orders of the fractional derivatives of the viscoelastic dampers ($T = 30$ s).

random process $\Delta_{5, \max}(T, \alpha)$ with respect to the parameters of the damper on the fifth floor takes much higher values than those corresponding to the dampers on the other floors. Furthermore, reliability appears to be more affected by the fractional derivative orders β_k ($\alpha_{\beta k}$) than by the coefficients $C_{\beta k}$ ($\alpha_{C_{\beta k}}$) for both the drift processes selected as performance measures.

The same conclusions can be drawn from Figs. 11 and 12 where the coefficients of sensitivity $\varepsilon_{L_{\Delta_h, \max, k}}(b, T)$ (%), $h = 1, 5$, of the CDF of the extreme value random processes $\Delta_{1, \max}(T, \alpha)$ and $\Delta_{5, \max}(T, \alpha)$ for assigned values of the barrier, $b_{0.50}$ and $b_{0.95}$, are plotted ($\alpha_k = \alpha_{C_{\beta k}} = \alpha_{\beta k} = 0.01$; $T = 30$ s).

The results provided by DSA may offer valuable guidance for the optimal design of the external retrofit system. For instance, viscoelastic dampers should preferably be placed on upper floors, given their greater influence on structural seismic performance compared to those on lower floors.

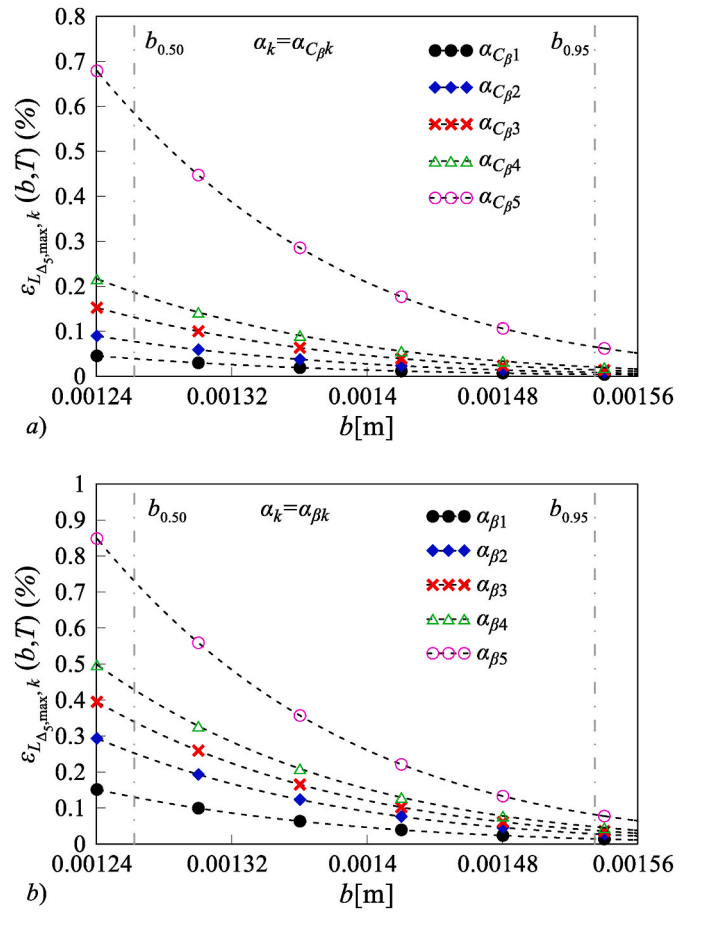


Fig. 10. Functions of sensitivity of the CDF of the extreme value random process $\Delta_{5, \max}(T, \alpha)$ with respect to the a) coefficients and b) orders of the fractional derivatives of the viscoelastic dampers ($T = 30$ s).

6. Conclusions

An analytical approach for estimating the sensitivities of the first excursion probability of linear frame structures controlled by external viscoelastic dampers subjected to seismic excitation has been presented. Ground motion acceleration has been modelled as a zero-mean stationary Gaussian random process within the *strong motion* duration. The constitutive behaviour of the energy dissipation devices has been described by a fractional differential equation involving two parameters i.e., the coefficient and the order of the fractional derivative, which have been assumed as design variables. Relying on Vanmarcke's approximation, explicit analytical expressions of the sensitivities of the *reliability function* to the main parameters of the fractional viscoelastic devices have been derived by direct differentiation. These expressions involve the sensitivities of the spectral moments of the response process of interest, which can be evaluated by direct differentiation as well. Analytical expressions of reliability sensitivities may be very helpful in performing *design sensitivity analysis* (DSA) of controlled structures as they allow a straightforward identification of the most influential

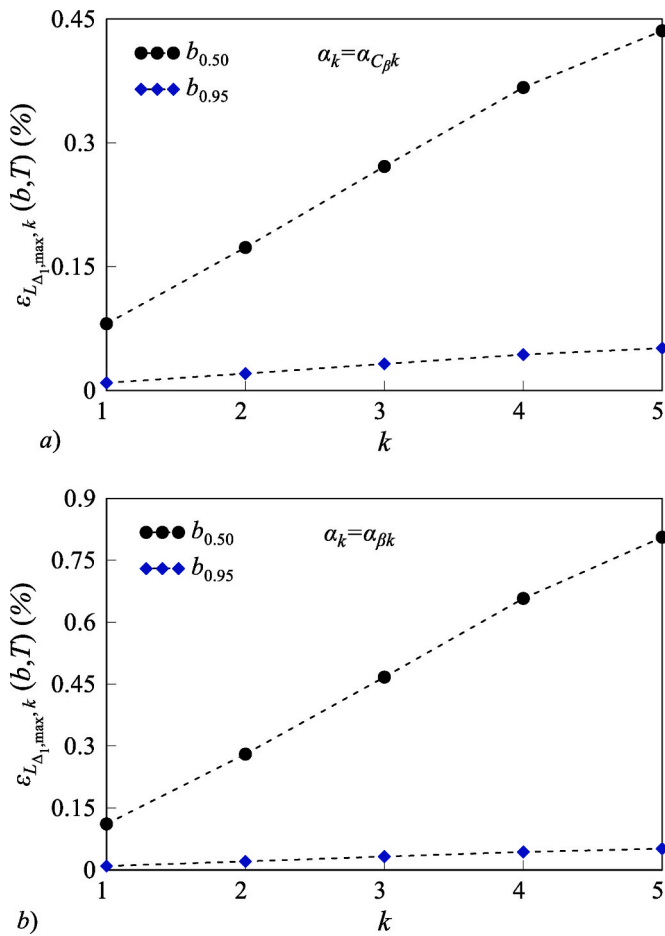


Fig. 11. Coefficients of sensitivity of the CDF of the extreme value random process $\Delta_{1, \max}(T, \alpha)$ with respect to the a) coefficients and b) orders of the fractional derivatives of the viscoelastic dampers for assigned values of the barrier, $b_{0.50}$ and $b_{0.95}$ ($T = 30$ s).

parameters on the selected performance measure.

Numerical results have highlighted that the percentage influence of the parameters of the fractional viscoelastic dampers on the seismic performance of the controlled structure increases when moving from the device on the first floor to the one on the top floor, even when the first-floor drift process is considered as the performance measure. In this context, DSA may assist designers in the optimal placement of dampers and calibration of the relevant parameters. Moreover, the proposed analytical approach may be useful to perform reliability-based design optimization of controlled structures subjected to seismic excitation.

Notably, the derived analytical expressions of reliability sensitivities can be readily particularized to different design variables, such as the material or geometrical properties of the controlled structure, enabling a comprehensive and efficient DSA.

CRedit authorship contribution statement

Alba Sofi: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Conceptualization. **Giuseppe Muscolino:** Writing – review & editing, Writing – original draft, Validation, Methodology, Investigation, Conceptualization.

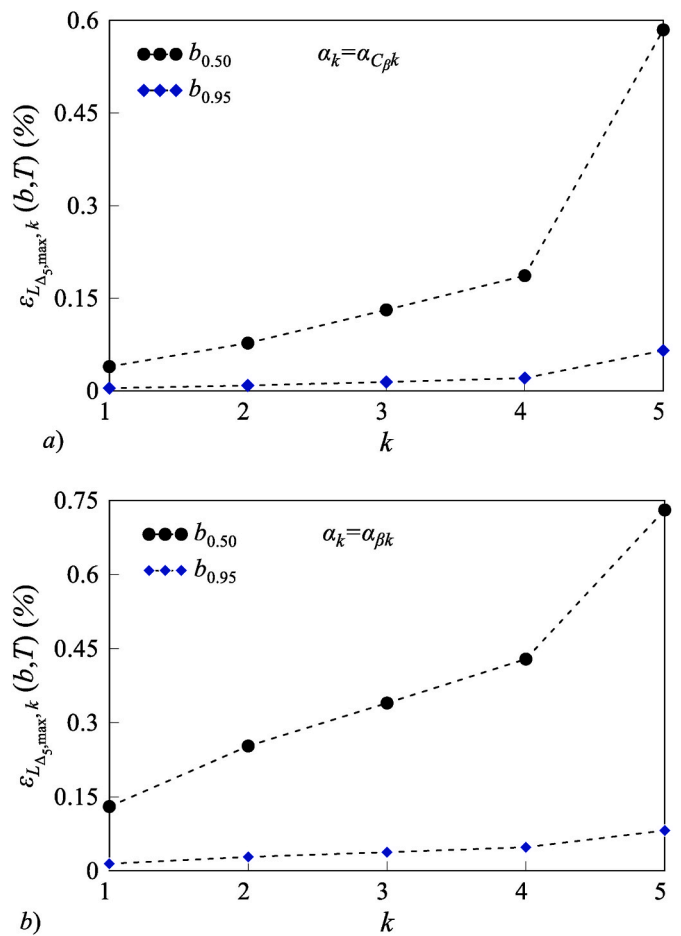


Fig. 12. Coefficients of sensitivity of the CDF of the extreme value random process $\Delta_{5, \max}(T, \alpha)$ with respect to the a) coefficients and b) orders of the fractional derivatives of the viscoelastic dampers for assigned values of the barrier, $b_{0.50}$ and $b_{0.95}$ ($T = 30$ s).

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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