Closure to "Applying hypothesis of self-similarity for flow-resistance law in Calabrian gravel-bed rivers"

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Closure to "Applying Hypothesis of Self-Similarity for Flow-Resistance Law in Calabrian Gravel-Bed Rivers" by Vito Ferro and Paolo Porto

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First, the writers wish both to thank the discusser for his interest on the original paper and to underline that the aim of the original paper is to establish a theoretically-based flow resistance equation using the Buckingham's Theorem of dimensional analysis and the incomplete self-similarity (ISS) theory (Barenblatt, 1979; 1987; Ferro, 1997). Two different approaches were applied which use the ISS hypothesis for both flow-resistance law (named "first model" by the Discusser) and flow-velocity profile ("second model"), respectively. On the contrary, the aim of the discussion is establishing the best calibration strategy fitting the available measurements to some empirical models which are a slight modification of the theoretically-based equations proposed by the writers.

In the Appendix of the original paper, the writers derived the Π_1 dimensionless group (Eq.26) which is related (Π_1^2 = Z) to the dimensionless variable Z proposed by the Discusser. Following this choice, the functional relationship in Eq.(6) can be rewritten as follows:

$$\Psi\left(\Pi_1^2, \Pi_{1,2,4}, \Pi_{2,1}, \Pi_{4,3,2}\right) = 0 \tag{1}$$

in which Π_i = dimensionless group.

Substituting into Eq.(1) Eqs. (26), (30), (31) and (32) of the Appendix yields to the theoretical deduction of the model proposed by the Discusser:

$$R_* = \phi\left(\mathrm{Re}, F, Z\right) \tag{2}$$

in which R_* = shear Reynolds number, Re = flow Reynolds number and F = flow Froude number.

The deduction of the dimensionless group Π_1 reported in Appendix (Eq.31) clearly shows that the variable Z was used to deduce the flow Froude number F. In other words, the originally involved dimensionless group ($\Pi_1^2 = Z$) can be changed as Barenblatt (1987) explicitly stated "In some cases, it turns out to be convenient to choose new similarity parameters – product of powers of the similarity parameters obtained in the previous step". Furthermore, the choice of the Discusser, which is to substitute Eq.(8) of the paper with Eq.(2), was not fruitful since the exponent of Z is close to zero.

For the second model, the Discusser proposed to recalibrate Eq.(5) and his calibration procedure varied the values of some coefficients. The procedure adopted by the Discusser is merely empirical and does not take into account that Eq.(14) of the paper is a theoretically based flow resistance equation. **Figure 1** shows the comparison between the measured R_* values and those calculated by Eq.(5) (**Fig.1a**), which is the original flow resistance equation, and Eq.(6) recalibrated by the discusser (**Fig.1b**).

This comparison highlights that the scattering of the data pairs corresponding to Eq.(6) is greater than that of Eq.(5). In fact, the flow resistance Eq.(5) allows to calculate R_* values which are characterized by errors, E , which are for 79.8% and 95.2% of the measured R_* values in the ranges of $\pm$5% and $\pm$10%, respectively, while for Eq.(6) the errors are $\leq$ 5% for 65.0% of the measured values and are $\leq$ 10% for 77.9% of the cases. The measurements were only used to determine a function (Eq.20) for estimating Γ_v.

The equation proposed in the original paper has the following mathematical shape:

$$\Gamma_v = a\,\frac{F^b}{J^c} \tag{3}$$

in which a = 0.3043, b = 1.013 and c = 0.5419 were estimated by the measurements carried out in 104 stream reaches. Eq.(3) is applicable for a wide range of bed slope ($0.11 \leq J \leq 6.2$), depth-sediment ratio h/d_{84} ($0.95 \leq h/d_{84} \leq 6.83$), being d_{84} the bed diameter for which 84% of the bed particles are finer, and flow Froude number ($0.18 \leq F \leq 1.25$).

Ferro (2018) theoretically demonstrated that the power velocity distribution can be written as follows:

$$\frac{v}{u_*} = \Gamma\left(\frac{u_* h}{\nu_k}, \frac{h}{d}, J, F\right)\left(\frac{u_*\, y}{\nu_k}\right)^{\delta} \tag{4}$$

in which u_* = shear velocity, h = water depth, ν_k = kinematic viscosity, d = characteristic diameter of the bed particles, y = distance from the bottom and δ = exponent. Ferro (2018) also demonstrated that the flow Froude number F takes into account both the depth-sediment ratio h/d and the shear Reynolds number R_* therefore Eq.(12) can be rewritten as follows:

$$\frac{v}{u_*} = \Gamma(J, F)\left(\frac{u_*\, y}{\nu_k}\right)^{\delta} \tag{5}$$

which states that Γ depends on bed channel slope and F. Ferro (2018), using the measurements by Ferro and Giordano (1991) corresponding to 416 laboratory experimental runs carried out by a quarry rubble bed with different concentrations of coarser elements (boulders), determined the following equation for estimating Γ_v:

$$\Gamma_v = 0.3388\,\frac{F^{1.1108}}{J^{0.5891}} \tag{6}$$

The coefficients a = 0.3388, b = 1.1108 and c = 0.5851 estimated by the measurements of Ferro and Giordano (1991) show a small difference respect to the values estimated by Ferro (2017) and the original paper.

For verifying the applicability of Eq.(6), whose coefficients a, b and c were estimated by flume measurements (Ferro and Giordano, 1991), the fields measurements carried out by Reid and Hickin (2008) and the writers can be used. This testing was carried out for taking into account that data used for calibrating Eq.(6) were obtained in controlled flume conditions, in which the boulder concentration was varied, while for the river data by Reid and Hickin (2008) and the writers the information about bolder concentration is not available and the river measurements, which can be less accurate that in a flume, are carried out in extremely variable natural bed configurations.

The 753 values of F and J corresponding to river conditions were used for calculating the Γ_v value, named $\Gamma_{v,c}$, by Eq.(6) to compare with the measured values $\Gamma_{v,m}$ obtained by Eq.(19) of the paper and the river measurements of water depth h, mean flow velocity V, channel slope J and hydraulic radius R.

The comparison between measured $\Gamma_{v,m}$ values obtained by Reid and Hickin (2008) and data from the original paper and those $\Gamma_{v,c}$ calculated by Eq. (6), which is plotted in **Fig.2**, demonstrates that the following relationship can be established between the $\Gamma_{v,m}$ values corresponding to river measurements and those obtained by Eq.(6):

$$\Gamma_{v,m} = 0.7611\ \Gamma_v = 0.7611 \left(\frac{0.3388\ F^{1.1108}}{J^{0.5891}} \right) \tag{7}$$

Therefore, according to Eq.(7), a scale factor (equal to 0.7611) is established to convert Γ_v values obtained by flume measurements into $\Gamma_{v,m}$ values corresponding to gravel bed rivers. This scale factor highlights that, for given hydraulic conditions (*h*, *i*, *V*), the Γ values obtained by flume data to be upscaled to river conditions have to be reduced; i.e., in the same hydraulic conditions a river bed is characterized by a Darcy-Weisbach friction factor which is higher than that obtained by flume measurements. According to Ferro (2018), this scale factor determines that, for a given hydraulic condition, the Darcy-Weisbach friction factor of a river is 1.6 times the corresponding flume.

In other words, the constant 0.7611 is the scale factor which establishes a similitude ratio between the Γ values obtained by river (prototype) and flume (model) measurements. The Darcy Weisbach friction factor values obtained in the model (flume) have to be upscaled of a factor 1.6 to obtain the *f* values of the real river (prototype).

For the second model, the Discusser proposed to assume the ratio *R/h*, in which *R* = hydraulic radius, as a function of *F* and the flow Reynolds number R to deduce the empirical Eq.(8). This choice is not theoretically or experimentally supported by the Discusser.

For testing the hypothesis of the Discusser the 104 measurements by the writers (Tables 1, 2, and 3) were used. This analysis established that the following power equation can be fitted (**Fig.3**):

$$\frac{R}{h} = 3.5779 \ \frac{F^{0.2137}}{\mathrm{Re}^{0.1}} \tag{8}$$

which is characterized by a low coefficient of determination equal to 0.496.

The scattering shown in **Fig.3** allows to conclude that the power relationship among *R/h, F* and R, hypothesized by the Discusser, is not supported by the experimental data.

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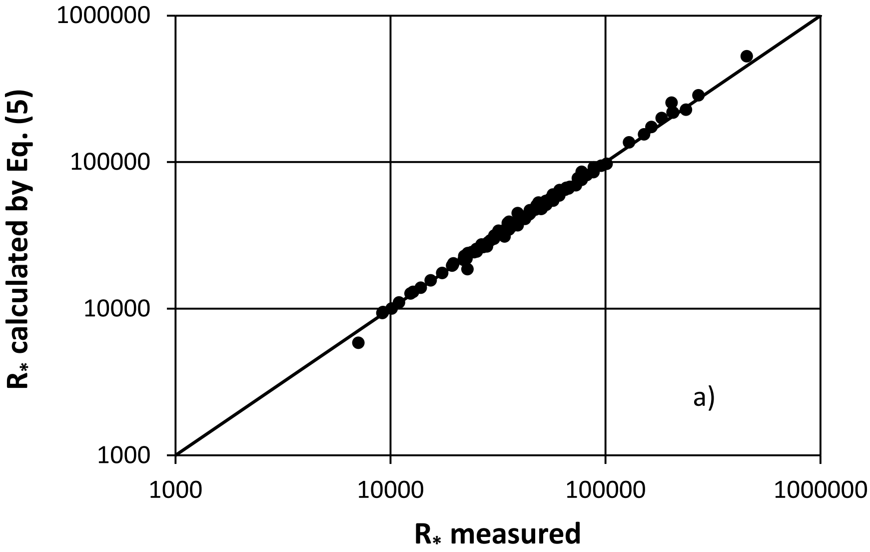
Figure Captions

Fig.1 Comparison between the measured Re* values and those calculated by (a) Eq.(5) and (b) Eq.(6)

Fig.2 Comparison between Γ_v measured values obtained by Reid and Hickin (2008) and data from the original paper and those calculated by Eq. (6)

Fig.3 Comparison between measured *R/h* values and those calculated by Eq.(7)

Fig.1


1000000
100000
10000
1000
R* calculated by Eq. (5)
a)
1000
10000
100000
1000000
R* measured

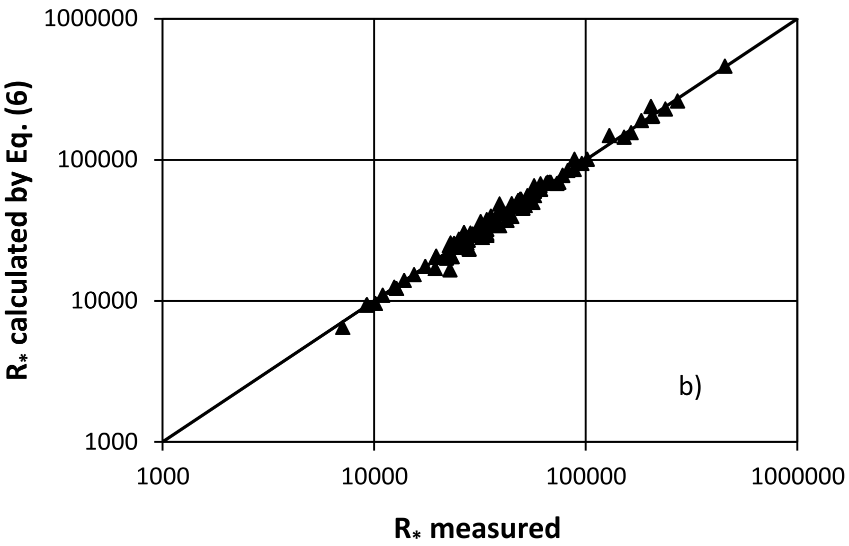
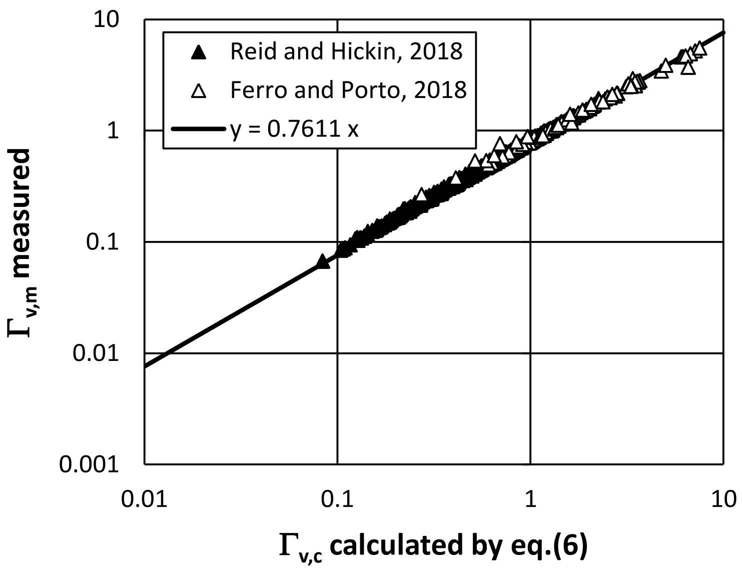

1000000
100000
10000
1000
R* calculated by Eq. (6)
b)
1000
10000
100000
1000000
R* measured

Fig.2



10
1
0.1
0.01
0.001
0.01
0.1
1
10
Reid and Hickin, 2018
Ferro and Porto, 2018
y = 0.7611 x
Γv,m measured
Γv,c calculated by eq.(6)

Fig. 3