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Beamforming-based algorithms for recovering information from fetal electrocardiographic sensors

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ABSTRACT
We deal with the extraction of the fetal electrocardiography (ECG) signal from the raw ECG signals of the mother by the beamforming-based algorithms. The foetal ECG sensors bring out signals containing information from the pregnant mother and the infant. Detailed and separate signals are already provided by the foetal ECG instruments; but for some specific studies related to the infant conditions, it is necessary to improve the quality of the signal with a dedicated processing. In this paper, four techniques, with some enhancements, are proposed to perform the processing; we have applied the following techniques: Least Mean Square (LMS) with adaptive noise cancellation technique, Discrete Wavelet Transform (DWT)-based technique, Empirical Wavelet Transform (EWT) technique, and Multiple Signal Classification (MUSIC). The LMS and the MUSIC pertain to beamforming approach. The techniques were used to decompose and identify the different elements constituting the source signal (mother's signal) and noise cancellation by Multivariate Empirical Mode Decomposition (MEMD) technique. The signal was adaptively decomposed by LMS, DWT and MUSIC according to optimised parameters to extract some hidden components of the source signal, such as the foetal features, QRS, heartbeat etc. The results have showed that LMS, with enhancements, is more effective in identifying and removing useless noise. The techniques were applied to the ECG signal of a 30-year-old healthy pregnant woman, which allowed to verify their applicability.
The present research leads to the below main contributions among others: separation of the ECG signal of the foetus from the mother, highlighting the functional state of the foetal heart rhythm (heart rate and heartbeat,) and this can show us if the foetal ECG has malfunctions.

Section: RESEARCH PAPER

Keywords: Beamforming; array of sensors; ECG; fetal signals; adaptive filtering; MUSIC technique; apnea; applied mathematics

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1. INTRODUCTION

Eminent researchers in the scientific world are in the process of or have studied the separation or extraction of signals contained in an original envelope. The medicine applied in the field of paediatrics, more precisely in the branch of cardiology, encounters certain difficulties, to extract the fetal component of the electrocardiography (ECG) signal from the mother's ECG signal to acknowledge problems of congenital diseases of the fetal/infant; in order to allow doctors to treat this as soon as possible, and to prevent the infant from being born with deformities or other problems that may complicate his/her life in the period of growth. The objective of this paper is to separate, by extracting from the raw ECG of the mother, the ECG of the fetal by preserving information content using Beamforming [1], [2], [3], [4], [5]. Currently, there are many methods for processing ECG signals, among which the transient electromagnetic method, short-time Fourier transform, de-shape short-time Fourier transform, and nonlocal median become the preferred method of researchers in ECG signal processing due to its fast signal extraction and decomposition, high efficiency, and great depth of exploration [6], [7], [8], [9]. The authors used the support vector machine to classify the electroencephalography signals of focal and non-focal electroencephalography using entropies [10]. Experimental results showed that the proposed approach distinguishes between seizure and non-seizure electroencephalography signals using performance measures such as sensitivity, accuracy, specificity, score, and Matthew Correlation Coefficient (MCC) to study the single-channel ECG to reconstruct the waveform of the respiratory signal, using the Discrete Wavelet Transform (DWT) [11]. The author applied Multivariate Empirical Mode Decomposition (MEMD) technique to separate maternal ECG from the abdomen ECG, and wavelet-based technique is employed to find fetal R-peaks. The performance was analysed by calculating cross correlation between the true and detected fetal heart rate signals [12]. The author applied an adaptive noise cancellation technique followed by the Stationary Wavelet Transform (SWT) in extracting fetal ECG from composite signal using Identification of Systems (DaISy) and PhysioNet database. Also, the fetal heart rate and Heart Rate Variability have been determined to identify fetal suffering [13]. In relation to all the techniques previously used, we have applied Beamforming with a combination of its two techniques (LMS and MUSIC) to perform decomposition to accurately extract the fetal electrocardiogram (fECG) components from the mother's ECG: The techniques of decomposition (MUSIC - Multiple Signal Classification, MEMD - Multivariate Empirical Mode Decomposition, LMS - Least Mean Square, and EWT - Empirical Wavelet Transform) it is obvious that the application of this combination has allowed to obtain one of the best results on the processing of the ECG the result obtained that we had implemented in this study is to have applied some enhanced techniques for the decomposition and extraction of the hidden components in the source signal.

2. ELECTROCARDIOGRAM

The data quantity in the medical field is rapidly increasing, thus obliging scientists to find effective and adequate techniques and means for their rapid treatments to save human lives. From this need a series of methods have emerged, however electrocardiogram (ECG) occupies the best place in the field of measuring cardiac activities. ECG is a test that studies the functioning of the heart by measuring its electrical activity. With each heartbeat, an electrical impulse (or 'wave') passes through the heart. This wave causes the heart muscle to contract so that it expels blood from the heart. An ECG measures and records the electrical activity that passes through the heart. A doctor can determine whether the electrical activity seen is normal or irregular. An ECG may be recommended if one has arrhythmia, chest pain or palpitations. Abnormal ECG results can be used to detect various heart problems [13].

The ECG is used to detect arrhythmias that can lead to blood clots; that is to detect heart problems such as recent or ongoing heart attacks, arrhythmias (irregular heartbeats), blockages in the coronary arteries, damaged areas of the heart muscle (caused by a previous heart attack), enlargement of the heart and inflammation of the heart wall (pericarditis). In the case of a pregnant woman, it is mandatory to monitor the heart rate of the fetal to take appropriate precautions for treatment before and after birth if the baby has heart problems.

2.1. Heartbeat modelling

Heartbeat modelling is an essential step to automatically identify characteristic waves. The aim is to find a mathematical representation, as simple and compact as possible, of the shape of each wave constituting the heartbeat. Thus, the most natural representation of waves is to describe the signal by its amplitude at each instant. Thus, a vector in a space whose dimension would

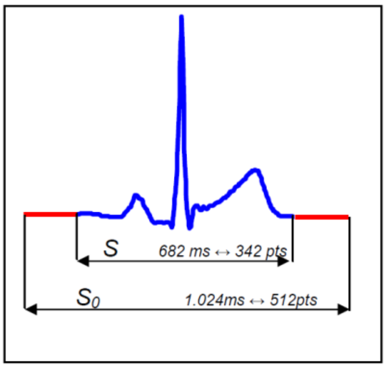

Figure 1. The orthogonal wavelet transforms.

be equal to a few hundred. The identification of the characteristic waves of the cardiac rhythm is carried out in two stages: segmentation and labelling. Segmentation corresponds to the division of the cardiac rhythm into zones likely to contain each cardiac wave. Labelling corresponds to the attribution of a medical label (P, Q, R, S or T) to each of the zones defined during segmentation. The signal to be decomposed is therefore an isolated heartbeat.

For example, if we consider the signal S of a beat to be modelled (Figure 1) sampled at 500 Hz, it is composed of 342 points. The signal S_0, used for the decomposition, is the vector composed of the signal S preceded by 85 zeros and followed by 84 zeros, which brings the dimension of this vector to 512, or $2^9 \cdot S_0$ is therefore also a vector S_0 of the 512-dimensional space whose i-th coordinate in the canonical base is the value of the signal at point i. In all that follows, the vectors of this space are noted in bold type.

The first step in the decomposition is the construction of the wavelet basis. If S_0 is the signal, to be decomposed, of length N_p (the number of points), the basis consists of N_p orthogonal wavelets S^I, all of which are deduced from the "mother" wavelet by translations and dilations. Let φ be the mother wavelet; the basis is constructed as follows:

$$B = \{\varphi(2^m \times \pm n), n \in [1..2^{m-2}], m \in [1..\log_2(N_p)]\}. \quad (1)$$

In (1) it is shown that, m and n are respectively the expansion and position coefficients of each wavelet, and $N_p$the length of the signal to be modelled wavelets, and $N_p$is the length of the signal. The $N_p - p$ basic functions are denoted $\{\varphi_1\}_{i=[1..N_p-1]}$ in the following. Such a library is shown in Figure 2; wavelets

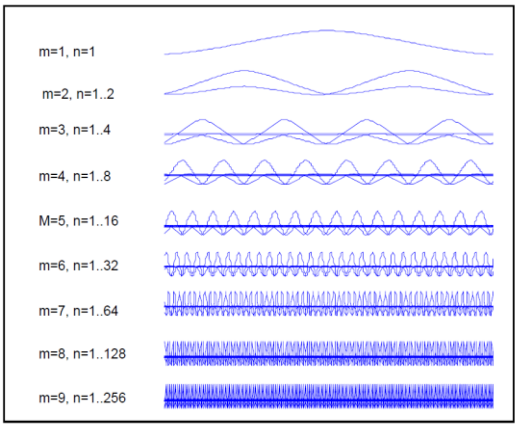

Figure 2. Wavelet family used for the decomposition of the S_0 signal.

(here Coiflets) that have the same expansion (constant m) are represented on the same line. The modelling of the signal is computationally inexpensive due to the orthogonality property of wavelets mentioned above.

Once the basis is constructed, the decomposition of the signal S_0 is to apply to the vector S_0 the matrix of passage from the canonical basis to the in other terms, to calculate the coordinates of the S_0 vector in wavelet basis wavelet basis:

$$S_0 = \sum_{i=1}^{N_p - 1} \langle S_0 | \varphi_i \rangle \varphi_i \,. \qquad (2)$$

Also, in (2) where $\langle S_0 | \varphi_i \rangle$ represents the i-th coordinate of the signal in the wavelet base. Thus, if we decide to choose $N < N_p - 1$ wavelets to model the signal S_0, the best Y model will be obtained with the N wavelets having the largest scalar product in value.

The model Y demonstrated in [15] will be obtained with the wavelets having the largest absolute scalar product with the signal

$$Y(t) = \sum_{i=\{A\}} \langle S | \varphi_i \rangle \varphi_i(t), \qquad (3)$$

where A represents the indices of the N largest absolute scalar products between the φ_i and S_0, the mean square modelling error is then written in (4) as

$$J = \frac{1}{N_p} \sum_{j=1}^{N_p} ((S(i) - Y(i))^2 \,. \qquad (4)$$

The results of the decomposition the example of an $N = 10$ wavelet model of the previous beat is shown below in Figure 3.

3. SIGNAL POST PROCESSING TECHNIQUES

Broadly speaking, we have two procedures for extracting fetal ECG signal (fECG). The first is to attach an electrode directly to the fetal scalp (head), but this makes the mother too uncomfortable. The last and second one, for the one we used, consists in the extraction of the fECG using electrodes placed on the mother (Figure 4) to mean that here we will have in a single envelope carrying the signal both ECGs, which will carry out the objective of our study to carry out the extraction of the fECG from the ECG signal of the mother (MECG). The importance of this method is that it avoids any contact between the fetal and any external energy.

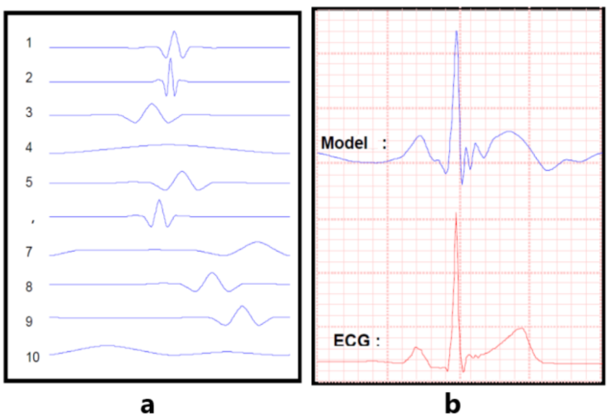

Figure 3. The best Y model with N = 10 Coiflets for the signal S_0 is shown on the left (a). The decomposition is the weighted sum of the 10 wavelets shown on the right (b).

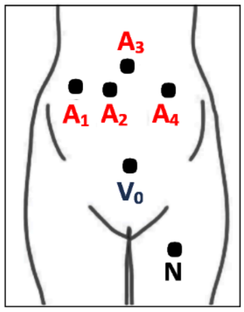

Figure 4. Acquisition of the maternal and fetal ECG signal, A_1 … A_4: abdominal leads, V_0: reference electrode, N: active ground.

Figure 5. ECG signal collection in a pregnant mother.

It is very important to point out that the ECG technique is a painless, non-invasive procedure, which means that nothing is injected into the body. The technique is carried out by means of electrodes (Figure 4 and Figure 5), usually between 12 and 15, which are attached to various parts of the body such as the arm, leg, and chest, in a particular way its electrodes are attached with small suction cups or adhesive patches, which have sensors that detect the electrical activity of the heart. An ECG normally takes between 5 and 10 minutes [14].

For the adaptive filter unit, the two main parts (blocks) are: first digital filter and last the adaptive algorithm; the filtering is performed by the digital filter and the adaptive algorithm performs a weight adjustment; the operation is carried out automatically, see Figure 6, and see also the adaptive filter with explanation: $d(n)$ desired signal, $y(n)$ output of a digital filter driven by input signal $x(n)$, and the error signal $e(n)$ is the difference between $d(n)$ and $y(n)$. A predetermined cost function that is related to $e(n)$ that the adaptive algorithm can minimize by updating the weights of the filters [15].

The adaptive filter is implemented with a different number of structures and realization: the calculation, the complexity and the level of performance are decided by the choice of the structure and the number of iterations. In its adaptive implementation: speed of learning or convergence, computational complexity

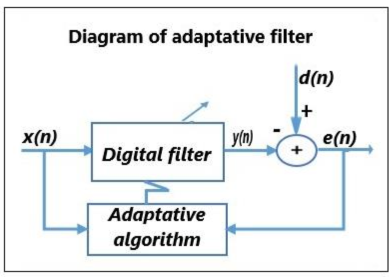

Figure 6. Diagram of adaptative filter.

numerical precision and the stability of the algorithm are of the fundamental questions. LMS is a simple and stable algorithm but, its speed is slow. In the LMS algorithm, the weight update is written in the form below. The relationships of the filter algorithm are given in (5) and (6).

$$y(n) = w^{\mathrm{T}}(n-1)\, u(n) \tag{5}$$

$$e(n) = d(n) - y(n) \tag{6}$$

$$w(n) = \alpha\, w(n-1) + f(u(n), e(n), u). \tag{7}$$

LMS filters allow us to find the filter coefficients, minimizing the difference between the desired signal and the error signal, which formulas are described in (8) to (10)

$$f(u(n), e(n), u) = u(n)\, u^{*}(n) \tag{8}$$

$$w(i-1) + u\, e(i)\, x(i) \tag{9}$$

where $e(i)$ is the error signal, $x(i)$ is the input, $u(n)$ is step size parameter, $u^{*}(n)$ is the estimated filter; interpreted as the estimation of the filter coefficients after n samples, and $w(i)$ is the weight function. The parameter u satisfies the condition

$$0 < u < \frac{1}{\lambda_{\max}}. \tag{10}$$

Figure 7 shows detection and extraction of the fetal heartbeat from the mother's heartbeat (measured signal) by adaptive filtering technique; Figure 8 shows the cancellation and error detection also noise contained in the signal to output reference signal with MEMD.

4. PROPOSED APPROACH

We have proposed four joint approaches, including wavelet transformation technique. The wavelet technique is essential in clinical studies to detect the different signals that make up a raw one; the wavelet transformation is a recent technique in non-invasive electrocardiology. Details of the ECG signal are highlighted in time and frequency resolution using wavelet transformation [16]. During our study, we have opted for the wavelet technique to extract up to the minute information of the abdominal signal and knowing that this algorithm remains complex. The goodness of this set of algorithms is the following: the best representation of the signal missed by one member could necessarily be represented by another, to have the fECG (fetal ECG) signal. We have tried to apply the wavelet (dB10), with the regularity of the wavelet is dB10 (1.25) which is a little higher than that of dB4 (2.90). fECG signal is extracted from the original signal using a two-level wavelet transform. Wavelet transformation technique uses the convolution of the wavelet function $\psi(t)$ with the signal $x(t)$ is the wavelet transform. In this technique, the dyadic orthonormal discrete wavelets will be put together to scaling functions $\varphi(t)$, the coefficient approximation is produced by convolving the scaling function and the signal. The DWT is written in Eq. (11), [17].

To reconstruct the original signal, we have selected a wavelet base $\psi_{m,n}(t)$ of coefficient approximation, at the scale and in the location as represented by the m, n, is described in (11) and (12)

$$S_{m,n} = \int_{-\infty}^{\infty} x(t)\, \varphi_{m,n}(t)\, \mathrm{d}t \tag{11}$$

$$x_0(t) = x_M(t) + \sum_{m=1}^{M} d_m(t). \tag{12}$$

The least mean square is a search algorithm in which a simplification of the gradient vector computation is made possible by appropriately modifying the objective function. The LMS algorithm, as well as others related to it, is widely used in various applications of adaptive filtering due to its computational simplicity. The convergence characteristics of the LMS algorithm are studied in order to establish a range for the convergence factor that will guarantee stability; (difference between the desired and the actual signal), as shown from (13), up to (16).

$$W_k(n+1) = W_k(n) - \mu\, \nabla J\,, \tag{13}$$

where, ∇ is the gradient of MSE J, μ is the step size, $W_k(n)$ is tap value of k^{th} tap n^{th} iteration. The step size can be variable or constant. In LMS algorithm, it is a constant positive number whose value ranges from

$$0 < \mu < \frac{2}{Y_{\max}}, \tag{14}$$

where $Y_{\max}$ is maximum eigen value of *R*. If μ exceeds the limit, then the trajectory of W_k becomes unstable.

In (15) and (16) of steepest descend algorithm [18], [19]. Now LSM algorithm estimates the gradient as:

$$W_k(n+1) = W_k(n) + \mu\, E[e(n)^{*}\, X(n)] \tag{15}$$

$$W_k(n+1) = W_k(n) + u^{*} e_n \times X_n. \tag{16}$$

With this algorithm, calculating the value of the next catch becomes easier. Updating the k^{th} value requires only 1 multiplication and 1 addition. Therefore, for a filter of order P+1, *P*+1 multipliers and adders are needed. An adder is needed to find $e(n)$ and a multiplier for $u^{*}\, e(n)$ [20], [21]. Finally, *P* adders and *P*+1 multipliers are needed to find the output $y(n)$.

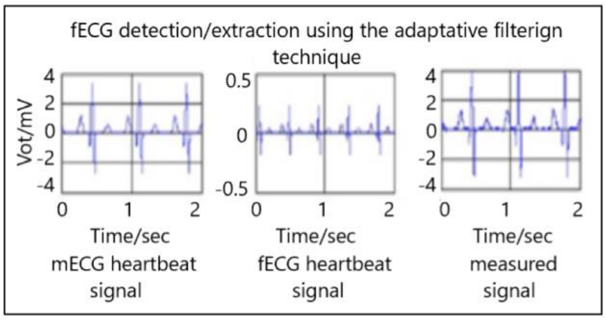

Figure 7. Fetal ECG detection/extraction using the adaptative filtering technique.

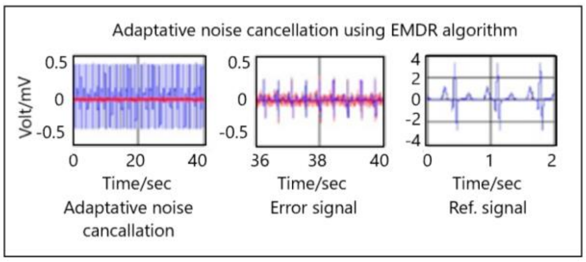

Figure 8. Adaptative noise cancellations.

Thus, a total of 2 P+3 multipliers and 2 P+2 adders are needed. Flowchart of LMS algorithm, the flowchart of is based on the following three equations:

$$y(n) = w(n)^* \, x(n) \tag{17}$$

$$w(n+1) = w(n) + \mu^* \, e(n)^* \cdot x(n). \tag{18}$$

Equation (17) calculates the output of filter by multiplying input with the filter weights, (6) calculates the error between desired signal and output. The equation (18) is of filter weight adaptation. It denotes the LMS algorithm. The weights are calculated by changing the previous weight and the converging factor. μ^* is the complex conjugate of the vector of input samples. We also used the MUSIC technique, the literature on the MUSIC algorithm is described in [20].

Pre-processing

In the field of signal processing, it is obviously necessary to prepare the signal before processing; to improve the quality of the signal and the operation is called pre-processing.

The objective is to remove all kinds of impurities and unwanted elements from the signal as shown in Figure 9. We have used the MEMD technique to pre-process the signal, which has allowed us to detect and remove the noise as shown in Figure 9.

Segmentation

The segmentation of components of the mother's ECG signal allows to point out them , at this stage also the components of our fetal ECG signal (peak, QRS, frequency, heartbeat, apnea peak etc.) will appear automatically, by means of the MUSIC algorithm (Figure 10 and Figure 11).

The use of the algorithms saves time in the interpretation of the ECG signal and especially for a pregnant woman, which brings the fetal signal inside [21].

Post-processing

An ECG is a test that looks at how the heart works by measuring its electrical activity. With each heartbeat, an electrical impulse or wave passes through the heart. This wave causes the heart muscle to contract so that it expels blood from the heart, as discussed in Section II. Here we are not dealing with an ordinary ECG signal, but with an ECG signal from a pregnant woman, so by taking this ECG automatically we have a double ECG in its components (mother and fetus). The objective of this study is to separate all the components of the source signal to highlight the ECG of the fetal as well as its internal components, as shown by the different results. This is of paramount importance to provide information to the cardiologist or paediatrician to diagnose cardiological condition of the mother and much more so of the fetal, and to prepare for the different eventualities if the fetal heart has abnormalities. The algorithms allow to clearly separate the components of the two signals in terms of frequency, wave peaks, power spectrum, decimations, interpolations.

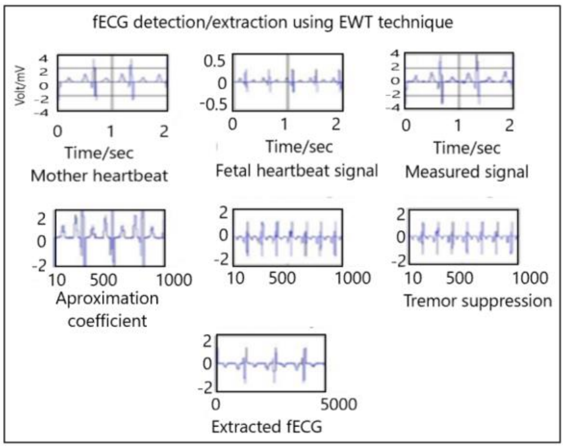

Figure 9. Fetal ECG detection/extraction using EWT technique.

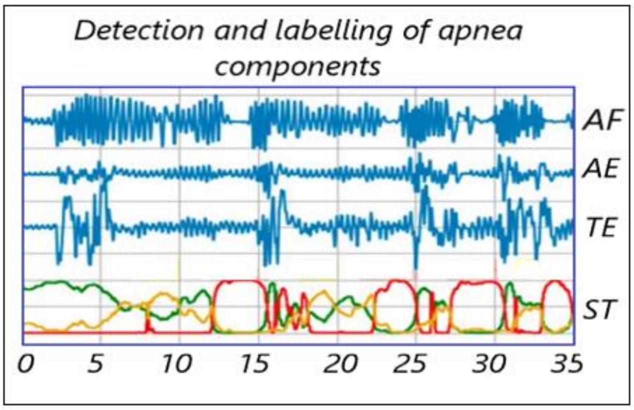

Figure 10. Detection and labelling of apnea components.

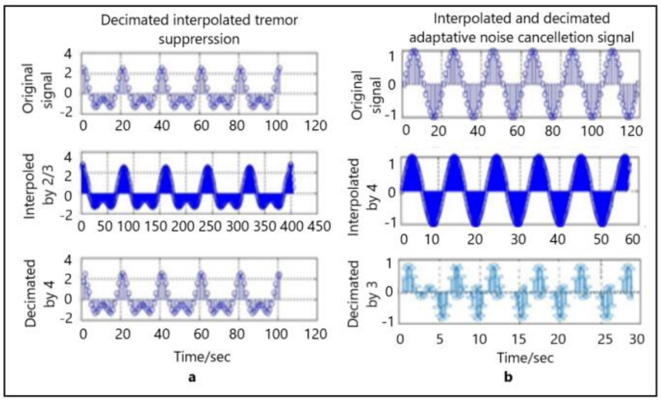

Figure 11. Decimated and interpolated tremor suppression (left), Interpolated and Decimated and noise cancellation signal (right).

5. RESULTS

ECG, by its nature, is a non-invasive technique that provides electrical signals of the heart activity. In this paper, ECG signal processing with appropriate techniques (MUSIC, MEMD, LMS, EWT) have brought a significant improvement in the interpretation of an ECG signal,

Especially in the interpretation of an ECG signal from a pregnant woman. The detection of the different components of the signal, the most innovative improvement is the separation the components of two ECG signals (mother and fetus), which were initially in one ECG signal, the mother's as we have mentioned at the beginning of our work, that is the framework in which we have worked, the results Figure 12–Figure 20 show how the two ECG signals are separated.

In order to help the cardiologist in saving time in the interpretation of the ECG signal of a pregnant woman, which will allow him/her to have precise information on the cardiac activities of the fetus, to see if the heart of the fetus is in good health, on the contrary, to prepare an appropriate therapy immediately, because the algorithms will highlight the problems detected so the cardiologist can give a precise diagnosis.

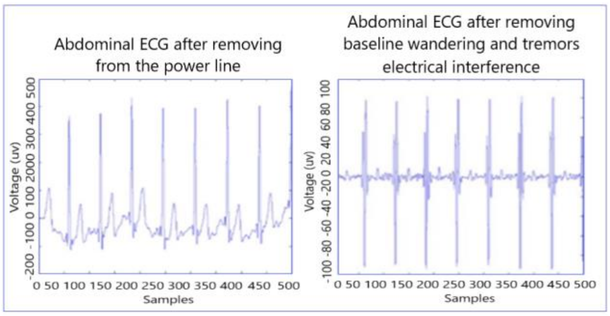

Figure 12. Abdominal ECG after suppressed power line interference noise at 50 Hz and Abdominal ECG after removing interference from the power line, line based on the wandering and trembling.

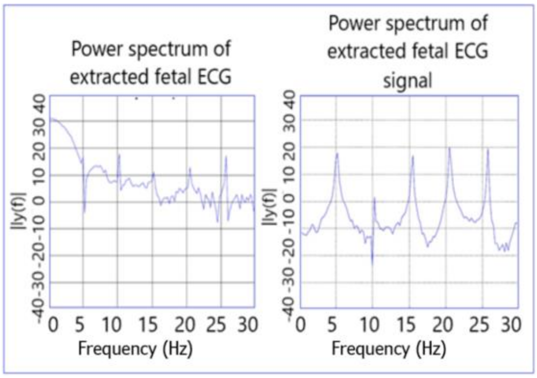

Figure 13. Power spectrum of simulated mother ECG signal and power spectrum extracted fetal ECG signal.

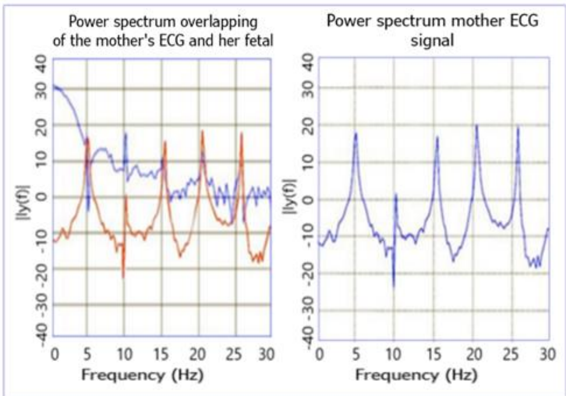

Figure 14. Power Spectrum overlapping of the mother's ECG signal and her fetal component.

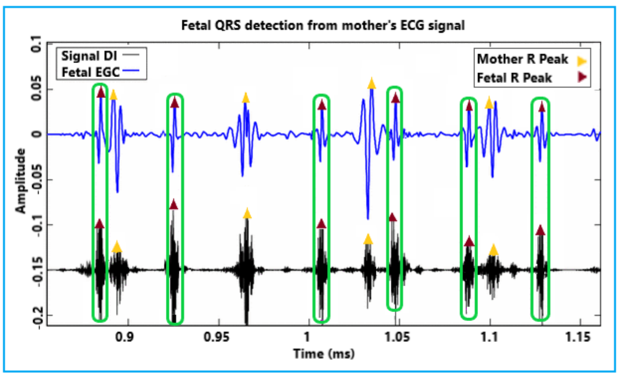

Figure 15. Fetal QRS detection from mother's ECG signal.

The plot of the Figure 13 shows a overlapping of the two signals (Power Spectrum of the ECG signal of (the red) mother and that of the fetal in blue); the plot indicates that the fetal heart is still dependent on that of its mother, since the peaks of the two signals are at the same rate with just a small difference because that of the fetal lives on the blood of its mother. This leads us to confirm that the algorithm has worked.

Figure 15 shows a separation of the signals which were contained in a single envelope of origin or source (maternal ECG signal) with the different *R* peaks of each signal; the present result is obtained thanks to the Multiple Signal Classification algorithm (MUSIC), which allows to classify, to separate and to provide

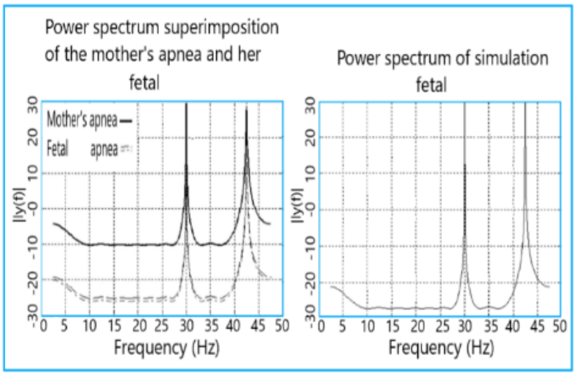

Figure 16. Power spectrum overlapping of the monther's apnea and her fetal signal.

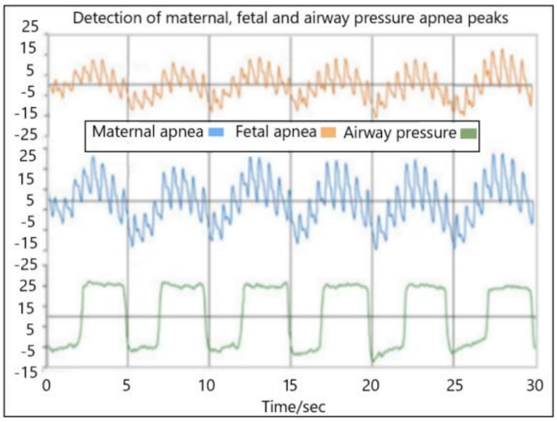

Figure 17. Power spectrum overlapping of the mother's apnea and her fetal signal on the left, Detection of maternal, fetus and airway pressure apnea peaks on the right.

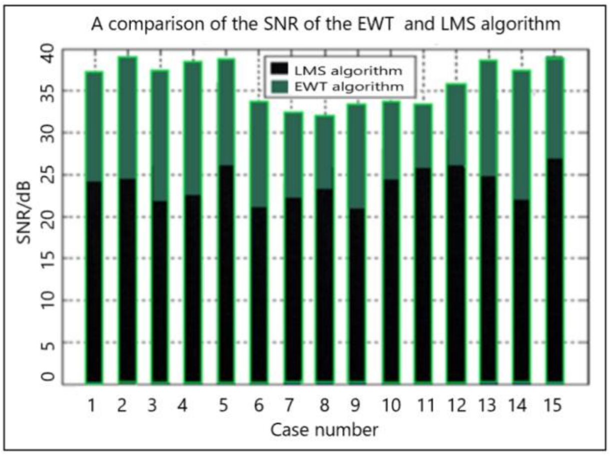

Figure 18. A comparison of the SNR of the EWT and LMS algorithm.

unbiased estimates of the number of muniscule signals and also to discriminate the signals from the noise which was on board during transmission, recording or during its diffusion in different transmission channels and, sometimes with certain details as detected, see Figure 19.

We have decided, given the objectives of the paper to introduce decimation and interpolation for a further interpretation. Decimation and interpolation are two operations that affect the timescale of a signal. For discrete signals, time scaling corresponds to increasing or decreasing the length of the signal. However, we must pay attention to how these operations are carried out. Decimation is the time compression of a signal. This corresponds to speeding up a signal or reducing its sampling rate. Suppose we have a signal $x[n]$ which corresponds to a continuous signal $x(t)$ samples at t_s intervals. The signal $y[n]$ is then equivalent to the compressed signal which is sampled at and contains the samples $x[0], x[2], x[4], \dots$. $y[n]$ can also be obtained directly if the signal $x(t)$ is sampled at intervals of $2\ t_s$. Decimating by a factor of N is equivalent to keeping all N samples of a signal. This can lead to information loss. Figure 10 details the procedure for detecting the components of the apnoea signal and labelling them with the MUSIC algorithm, on a normal class of apnoea including hypopnoea which are shown here in bright colours: red, yellow, and green, as well as other lines labelled in this way, with this legend: AF: Airflow, AE: Abdominal Effort, TE: Thoracic Effort ST: Stage Label.

Another needed operation is the interpolation. It is the stretch over time. This corresponds to slowing down the signal or increasing the sampling rate. Suppose we have a signal $x[n]$ which corresponds to a continuous signal $x(t)$ sampled at intervals t_s. The signal $y[n] = x\left[\frac{n}{2}\right]$ is then equivalent to the signal $x(t)$ which is sampled at intervals of $x(t)$ (or at a rate of

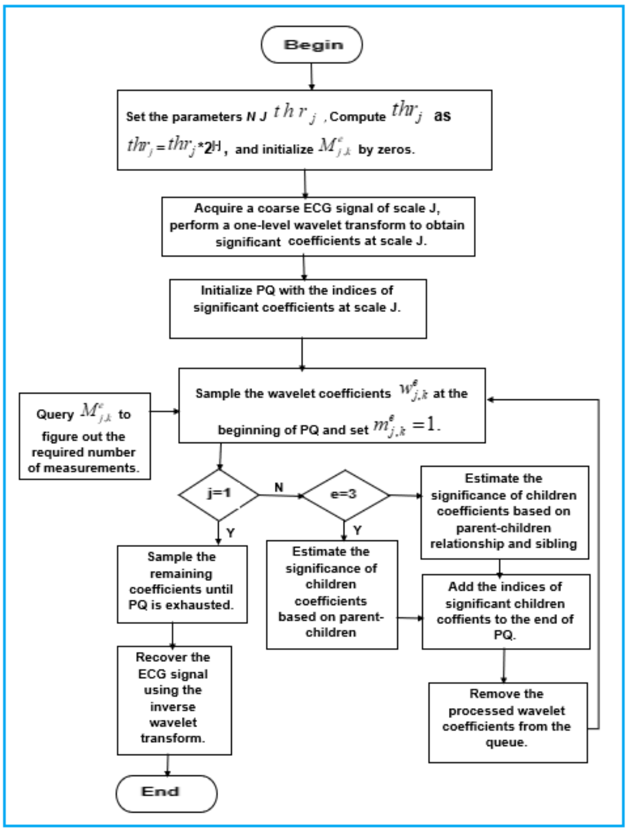

Figure 19. EWT Flowchart.

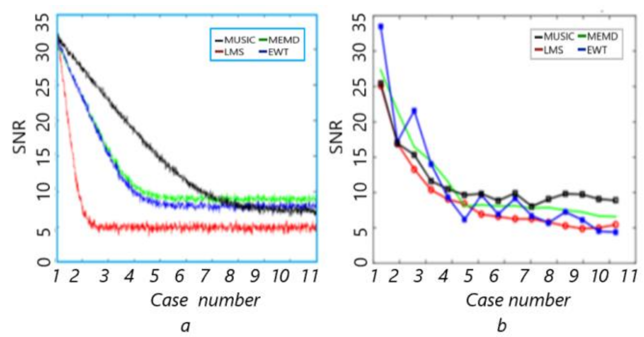

Figure 20. A comparison of the SNR of the MUSIC, MEMD, LMS and EWT algorithms for ECG (a) and apneas (b) of the mother and the fetus.

$S = \frac{2}{t_s}$). We therefore have twice as many samples: we have stretched the signal (8) in both directions. However, how do you calculate the value of the new samples? Three popular techniques exist for doing the interpolation: 1. Zero interpolation: Implies that each new sample is zero, 2. Step interpolation: We take the previous value for the new sample, and 3. Linear interpolation: The values on each side of the new sample are averaged, the figure: interpolation shows an example of three types of interpolation. Figure 11 is the original signal. The samples in red are the new samples. Decimation is the reverse of interpolation, but the inverse is not necessarily true. When we apply the decimation, we lose some information; so we are not able to reproduce the original signal correctly. As an example, we use the sequence. $\{1,2,6,4,8\}$ First, we apply the step interpolation to this sequence, and we get: $\{1,1,2,2,6,6,4,4,8,8\}$

However, if we inverse the order of operations, we will not find the original sequence. First, we apply the decimation on the sequence $x[n]$, and then we apply the step interpolation: $\{1,1,6,6,8,8\}$ which is not the same sequence as the original.

6. CONCLUSIONS

Currently, considerable progress has been made with the advent of the beamforming technique. Beamforming, also called spatial filtering, or channel formation, is a signal processing technique used in antenna and sensor networks for the transmission or directional reception of signals. four approaches are used (LMS, MUSIC, MEMD, EWT) as shown in Figure 20 during different experiments.

The objective of this study is to decompose, visualize and highlight internal components of the ECG signal of a pregnant woman, as we are seeing in Figure 11 – Figure 20, the various techniques made it possible to carry out a normal decomposition and separation of the internal components of the initial signal (mother's ECG). referring to the previous paragraph, the goal is to save time for health professionals (cardiologist) in the interpretation of an ECG signal, especially the ECG of a pregnant woman including that of the fetus.

The approach used in this paper remains mostly filtering to separate components of a signal by their frequencies and on their channels; the beamforming approach (adaptive filtering) is reliable in the processing of ECG signals; each component of the signal source has been filtered by its appropriate frequency.

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