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An evolutionary game to describe doping dynamics

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ABSTRACT

The objective of this research is to delve deeply into the complex dynamics of doping in various sports disciplines, seeking to uncover the underlying mechanisms that contribute to its proliferation. Our approach involves employing ecological and biological models in conjunction with evolutionary game theory, an original aspect of this study. These models will be instrumental in simulating and understanding the intricate interactions and competitive strategies that drive athletes toward doping.

The study seeks to propose effective and efficient policies and measures that can be implemented to combat the spread of doping and to identify innovative approaches that could be more effective at deterring athletes from doping, thereby ensuring a fair and level playing field in competitive sports.

1. Introduction

The practice of using prohibited substances to outperform one's own physical capabilities in sporting activities has origins that trace to antiquity. Already during the most ancient Olympic Games, athletes made use of substances that stimulated them to be more competitive, just as Greek and Roman wrestlers made use of particular mushrooms or meats of various kinds in the belief that they would help them in the fight. For this reason, the study of doping in sports presents a range of open issues that are complex and multifaceted. A deeper understanding of the motivations and attitudes driving athletes towards doping is needed, which includes examining factors such as high-performance pressures, the desire for physical enhancement, and social and cultural influences. There's a gap in research on the effects of doping across different athlete groups, spanning various competitive levels, ages, genders, and sports disciplines. More studies are required on the long-term health effects of doping, encompassing not only physical impacts but also psychological and social consequences. The effectiveness of educational interventions and anti-doping policies also requires thorough research to understand which strategies are most effective in preventing doping use. It's also crucial to study how the perception of doping by both the public and athletes influences their behaviour. A definitive evidence confirming a higher incidence of doping in specific sports is missing [1]. While it is commonly perceived that certain sports, particularly those demanding exceptional strength, endurance, or physical performance, might have a higher prevalence of doping [2], concrete empirical data or theoretical models to unequivocally support this claim are limited [3]. The difficulty in obtaining reliable data on doping incidence stems from several factors. First, the clandestine nature of doping makes it challenging to accurately measure its prevalence. Athletes, teams, and their support networks often go to great lengths to conceal doping, making detection difficult. Second, the methods and rigor of drug testing vary significantly across sports, countries, and governing bodies. This variability can lead to inconsistencies in the detection of doping, thereby affecting the accuracy of prevalence estimates. Third, the reliance on self-reporting in surveys can skew the data. Athletes may

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be reluctant to admit to doping due to fear of repercussions, leading to underreporting. Conversely, the perception of widespread doping in a sport may lead to overreporting, as athletes assume doping is more common than it might be. Lastly, high-profile doping cases and media coverage can disproportionately influence public opinion and skew the understanding of the extent of doping in certain sports. This can lead to assumptions about the prevalence of doping based on a few prominent cases rather than systematic evidence. This study attempts to resolve, using a theoretical perspective, the open question concerning the greater incidence of doping in certain sports disciplines by studying, through a model based on evolutionary games, which characteristics determine the greater or lesser incidence of doping and consequently what corrective measures can be put in place. Theoretically defining these factors in quantitative terms is an appreciable and original result that can improve our knowledge of the phenomenon.

In section two, we will describe some aspects about the phenomenon by identifying the motivations that lead to doping, with a review of the literature. Paragraph three will present some empirical data on doping and, in particular, the main doping substances and the state of controls. Sections four and five and six will be devoted to describing the model and its solution in dynamic terms with simulations of the model, section seven provides a discussion of the results, and section eight some conclusions.

2. Background

The studies and literature outlined below show that there is a different degree of spread of doping between the different sporting disciplines, which can be linked to three aspects.

A) The mental and incentivizing factors behind doping are obviously different among players and are randomly distributed across all disciplines.
B) To the benefits in terms of increased performance that doping guarantees
C) To the expected cost in terms of sanctions.

The mental and incentivizing factors behind doping, we can identify six types of factors.

1) Intense ambition to succeed regardless of the methods employed2) Strong reliance on others' opinions (lack of personal conviction)
3) Inappropriate conduct outside of athletic settings
4) Excessive consideration of one's own persona
5) Reliance on a culture that values only victory
6) Compromised ethical standards.

Gleaves et al. [1] study on the widespread occurrence of doping in sports presents a comprehensive and complex picture. It encompasses research spanning from 1975 to 2019, involving over 100,000 athletes from more than 35 countries. The prevalence rates for doping varied widely, ranging from 0 to 73 %, with most studies indicating a prevalence below 5 %. This variation is attributed to diverse methodologies used across studies, including self-reported surveys and sample analyses. The broad range in prevalence also reflects the challenges in defining and detecting doping, as well as the reluctance of athletes to admit to doping due to its punitive and stigmatized nature. Petróczi et al. [4] introduce the Performance Enhancement Attitude Scale (PEAS) to captures a spectrum of attitudes toward doping among athletes [3]. Offer a detailed critique of the existing research landscape on attitudes and beliefs about doping in sports. A key observation of this review is that most existing studies are descriptive and lack a robust theoretical foundation. There is a significant gap in empirically validating doping models and theories in sports. This suggests a need for more refined, theoretically informed research strategies that can effectively inform policy and prevention practices. They. also emphasize the importance of understanding the varying motivations and socio-cultural contexts of athletes. This understanding is critical for developing effective anti-doping interventions. The current research landscape, predominantly quantitative and focused more on explaining doping phenomena, falls short in providing a comprehensive understanding of doping through qualitative measures. This entails developing interventions and policies that are aligned with contemporary societal needs and that leverage current scientific understanding. The call is for an approach that not only addresses the act of doping but also delves into the underlying motivations and contexts that drive athletes towards such decisions, thereby offering a more holistic perspective on doping prevention in sports. Boardley et al. [5], explore seven mechanisms of Moral Disengagement (MD) and three additional themes within the context of athletes' use of performance-enhancing drugs (PEDs). These mechanisms and themes provide valuable insights into the psychosocial processes athletes employ to rationalize and normalize their PED use. Sukys et al. [6] examine how moral identity and perceptions of fair play influence athletes' attitudes towards doping; Ntoumanis et al. [7] explain with meta-analysis personal and psychosocial factors influencing doping use; Ring et al. [8] investigate how basic values predict unethical behaviors in sports, like athletes' likelihood of doping.

The aspects related to points B and C determine the concentration of doping-prone athletes in a particular discipline. Loland [2] with his vulnerability theory in sports proposes that sports emphasizing record-keeping are susceptible to undesirable manipulations that contradict the true spirit of sport, creating a disparity in doping tendencies across different sports. Kavussanu et al. [9] demonstrate how moral intervention can reduce the likelihood of doping among British and Greek athletes; Stamm et al. [10] compare perceptions of doping among top-level athletes, performance-driven leisure athletes, and the general population; The difference in terms of sporting performance that can be achieved through the use of doping is certainly an important aspect, as is the perception of the expected value of the cost in terms of sanctions, where the term expected value indicates the level of sanctions multiplied by the probability of being caught depending on the effectiveness of anti-doping policies. Shelley et al. [11] explore qualitatively the experiences and views of clean British elite long-distance runners on doping and anti-doping,; García-Grimau et al. [12] study examines attitudes and susceptibility to doping among Spanish elite and national-level track and field athletes. The effectiveness of anti-doping policies is, therefore, linked to their ability to affect these determinants. It is therefore useful to conduct a brief survey of the specific literature dealing with the relationship between doping levels and anti-doping policies, and in this light, it is important to first consider the relationships between testing, penalties and doping described in the literature. Rivkin D. [13] proved that, under relatively general assumptions, the minimum amount of random testing necessary to deter doping increases as the number of participants increases. The presence of even a minimal penalty, besides being banned from the contest, makes random testing more effective, especially in large competitions. Music K [14]. highlights potential issues with incentives that could emerge during strategic interactions between anti-doping organizations. Meanning [15] identifies an economic solution to doping that consists of increasing the expected costs of doping by agreeing on sufficiently high financial penalties. Meaning et al. [16] develop a dynamic model to analyze the effects of anticipating changes in testing policy and sanctions and find that such changes affect athletes' behaviour. Mohan V. et al. [17] examine the impact of regulation on the doping decisions of athletes in a Tullock contest. They found that when legal efforts and illegal drugs are substituted, an increase in anti-doping regulation may, counterintuitively, increase the level of doping activity by athletes. Hirschman [18] showed that under certain circumstances, an increase in doping sanctions can decrease the number of participants in a competition. Berentsen [19] examines with a game a strategic scenario where two participants, possessing unequal chances of triumphing in the contest, simultaneously and secretly decide to use performance-enhancing drugs before they compete and studies the incentive system that can lead to a nondoping equilibrium. Dimant et al. [20] noted that corruption and doping are ubiquitous in amateur and professional sports and have assumed the character of a systematic threat. By creating unfair advantages, doping distorts the level of playing in sports competitions. This paper provides a comprehensive overview of the literature on the individual drivers of doping, concomitant harmful effects and respective countermeasures. Goetsch et al. [21] construct a conceptual framework for after-the-fact doping inspections and evaluate how it influences an athlete's decision to dope. Within this framework, the anti-doping agency has the capability to preserve doping samples for future re-examination. The findings indicate that an optimal combination of sample storage and retesting exists for the agency, which can decrease the intensity of doping among athletes. Using game theory, Buechel B. et al. [22] argue that consumer boycotts after scandals can increase fraud rather than reduce it.

As this review shows, doping is certainly a complex and multifaceted phenomenon that tends to grow as a result of an increasingly competitive view of sports and society, and strategies to reduce the level of doping are also quite complex and must take into account side effects and complementarities. Designing policies that are effective at reducing their impact is a priority both as a matter of legality and to be as adherent as possible to the spirit of sporting activity. Investigating the facts related to doping is complex because, because doping is an illegal activity, people who use it take great care to keep everything hidden. What emerges with the controls is only a small part of the phenomenon, also because one must take into account all those medical procedures that, even if they do not technically fall under the definition of doping, still have as their objective the artificial enhancement of athletes' performance. It is well known that many sports make much use of pharmacological tools that, although they do not fall under the notion of doping, at least in the way they

Table 1
Main anabolic substances detected in athlete samples (Taken from: WADA 2022).

stanozolol	235	17 %
the GC/C/IRMS result is consistent with an exogenous origin	175	12 %
drostanolone	138	10 %
metandienone	131	9 %
nandrolone (in 26 cases, the result is consistent with an exogenous origin based on GC/C/IRMS analysis)	150	11 %
dehydrochloromethyl-testosterone	108	8 %
trenbolone	79	6 %
oxandrolone	78	6 %
metenolone	73	5 %
boldenone (in 34 cases, the result is consistent with an exogenous origin based on GC/C/IRMS analysis)	87	6 %
mesterolone	51	4 %
clostebol	25	2 %
methasterone	17	1,0 %
methyltestosterone	13	1,0 %
fluoxymesterone	11	1,0 %
1-androstenedione	8	0,6 %
1-testosterone	7	0,5 %
boldione (androsta-1,4-diene-3,17-dione)	7	0,5 %
desoxymethyltestosterone	6	0,4 %
methylstenbolone	5	0,4 %
oxymetholone	5	0,4 %
methyl-1-testosterone	2	0,1 %
mestanolone	1	0,1 %
oxymesterone	1	0,1 %
danazol	1	0,1 %
stenbolone	1	0,1 %
testosterone	1	0,1 %
gestrinone	1	0,1 %
Totale	1417	

are used, constitute medical support procedures that are not difficult to define with a euphemism borderline. Doping not only violates the fundamental principle of sport, which is fairness but also ultimately constitutes a danger to the health of athletes. However, it is also well understood that as long as the risk of being caught remains low, especially in those sports where results can be drugged with substances, the phenomenon will continue to spread. Increasing controls and increasing sanctions may be important; however, resources must be invested in educating people about clean sports, where winning at any cost is not the best option. However, the level of fierce competition that already exists in everyday life and the enormous economic interest in some sports frustrate this pure idea of sport.

3. Empirical data on doping

Data from the Anti-Doping Testing Figures Report - WADA 2022 are a very important source of doping data. First, it is necessary to define which of the main anabolic substances are used for doping purposes. Tables 1–3 give a fairly complete picture, although the list is constantly being updated.

As shown in the tables, which list only the main substances used for doping, there is an endless list of substances considered illegal in sporting activity that are normally taken by athletes. Sometimes this intake is scientifically programmed during training, so the doping phenomenon is certainly highly pervasive in professional and nonprofessional sporting activities. The following figure can be used to obtain a picture of the time course of tests and positivity:

Taken from: 2022 Anti-Doping Testing Figures Report - Executive Summary - WADA.

Fig. 1 summarises the examinations and positives of the athletes by sport discipline. As shown, the positivity rates, although low, strongly increased in non-Olympic sports from 2008 to 2018 and slightly decreased in Olympic sports, a sign of a strengthening of the control system.

Table 4 (in red, the values that exceed the mean value are highlighted, and in green, the values below the mean value) gives the figures for testing and positive tests per athletic category. Data cover approximately 80 % of the total doping tests. The positivity values, although low overall, show strong variability within the range and provide an a proxy of the different impact of doping in different sports.

4. Evolutionary games and an interpretative model

To show the decision-making process on which it is based the doping phenomenon, we begin by identifying each sport discipline within an ecological system in which two species compete for the same resource. We build a model of a competition series or of a championship as competition between two species for a resource under the condition that the principle of competitive exclusion applies. The competition between two species for a resource can be described with ecological biological models that have been well known in the literature for many years. Models of the 'war of attrition' type can lead to the supremacy of a single species, something known in biology as the Gause principle [23–26]. If the hypothesis that two species can coexist is accepted, it is possible to study the growth and decline of agents within a sector and the dynamics of sectors [27–32].

In particular, three parameters can be identified in the biological selection mechanism:

a) Performance
b) Ability In competition
c) Intensity of controls and sanctions and the risk of being discovered

To adopt an evolutionary perspective, a clear and uniform definition of fitness is essential. Within biological frameworks, fitness is characterized as the probability of survival. In economic modelling, fitness is a synonym for payoff. When it comes to evolutionary game theory, fitness may be described as pertaining to either an individual agent or a collective of agents.

The evolutionary game context has a limited 'behavioural content' of the models. A game theory approach is a very simple description of reality.

Returning to the definition in the previous paragraph, let us consider a particular sport in which an advantageous choice of doping is possible.

Table 2
Peptic hormones and growth factors detected in athlete samples (Taken from: WADA 2022).

erythropoietin (EPO)	60	52 %
darbepoetin (dEPO)	13	11 %
human Chorionic Gonadotrophin (hCG) greater than the Decision Limit of 5.0 IU/L for immunoassay	12	10 %
GHRP-2 (pralmorelin)	11	10 %
GHRP-6	8	7 %
methoxy polyethylene glycol-epoetin beta (CERA)	5	4 %
ibutamoren	3	3 %
growth hormone (GH)	2	2 %
ipamorelin	1	1 %
TOTAL*	115	

Table 3
Stimulant substances detected in athlete samples (Taken from: WADA 2022).

amfetamine	95	16 %
methylphenidate	91	15 %
cocaine	87	14 %
4-methylhexan-2-amine (methylhexaneamine)	72	12 %
1,4-dimethylpentylamine (5-methylhexan-2-amine)	40	7 %
heptaminol	35	6 %
ephedrine	30	5 %
1,3-dimethylbutylamine (4-methylpentan-2-amine)	24	4 %
mephentermine	17	3 %
oxilofrine (methylsynephrine)	16	3 %
D-amfetamine/dextroamfetamine	14	2 %
modafinil	14	2 %
sibutramine	12	2 %
methylenedioxymethamphetamine	9	1 %
phentermine	7	1 %
fonturacetam [4-phenylpiracetam (carphedon)]	7	1 %
methamfetamine (D-)	7	1 %
pseudoephedrine	4	1 %
isometheptene	4	1 %
methylephedrine	4	1 %
tuaminoheptane	3	0,0 %
strychnine	3	0,5 %
cathine	3	0,5 %
hydroxyamfetamine (parahydroxyamfetamine)	2	0,3 %
2-amino-N-ethyl-1-phenylbutane	1	0,2 %
amfepramone	1	0,2 %
octopamine	1	0,2 %
N-methylphenethylamine	1	0,2 %
phenethylamine and its derivatives	1	0,2 %
TOTAL	605	

Players will then be able to choose to behave correctly (by not doping), which we denote as player (nodop), or incorrectly by doping, which we denote as player (dop).

The payoff will be determined by the kind of player that evolves, which in turn will be related to the ratio of dopers to the entire population of players. If k is the ratio of player(dop) to the entire population and (1-k) player(dop) to the entire population, then the expected payoff will be (see equations (1)–(3)):

$$\mathrm{E(nodop)} = \mathrm{k}\Pi^{\mathrm{s}}_{\mathrm{nodop}} + (1 - \mathrm{k})\ \Pi^{\mathrm{c}}_{\mathrm{nodop}} \tag{1}$$

$$\mathrm{E(dop)} = \mathrm{k}\Pi^{\mathrm{c}}_{\mathrm{dop}} + (1 - \mathrm{k})\ \Pi^{\mathrm{s}}_{\mathrm{dop}} \tag{2}$$

where $\Pi^{\mathrm{s}}_{\mathrm{nodop}}$ is the player's payoff when they all choose to use the nodoping strategy, $\Pi^{\mathrm{s}}_{\mathrm{dop}}$ the player's payoff when they all choose to use the doping strategy, $\Pi^{\mathrm{c}}_{\mathrm{dop}}$ is the payoff when the first group plays doping and the second group plays nodoping, $\Pi^{\mathrm{c}}_{\mathrm{nodopig}}$ is the payoff when the first group plays doping, and the second group plays nodoping.

Employing an evolutionary perspective, we are able to characterize the scenario through a specific game known as the "HAWK DOVE" game [1].

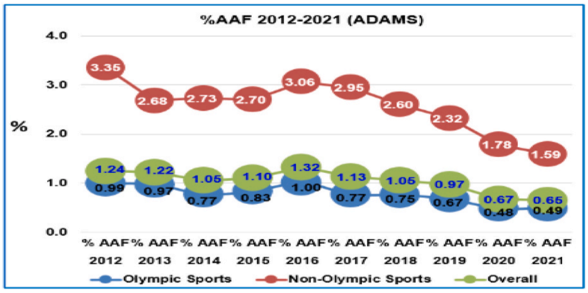

Fig. 1. % of positive[1] tests out of the total number of tests per sport specialities and year.

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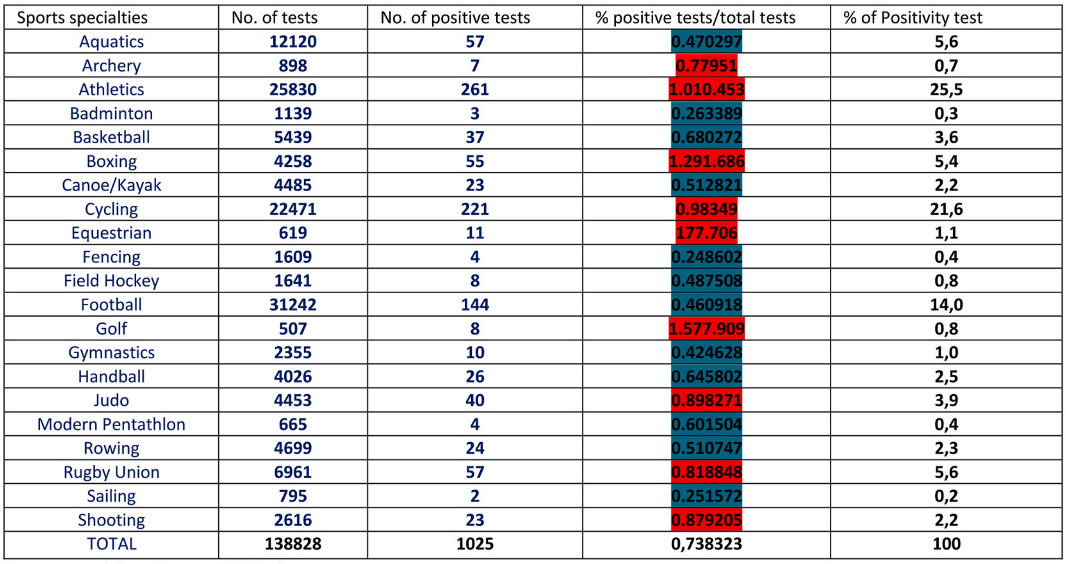
[1] More correctly Adverse Analytical Findings, because a part of this percentage could be related to therapeutic use of the substances and therefore not involve a violation of anti-doping rules.

Table 4
% of positive tests out of the total number of tests per sports specialty.



Sports specialties	No. of tests	No. of positive tests	% positive tests/total tests	% of Positivity test
Aquatics	12120	57	0.470297	5,6
Archery	898	7	0.77951	0,7
Athletics	25830	261	1.010.453	25,5
Badminton	1139	3	0.263389	0,3
Basketball	5439	37	0.680272	3,6
Boxing	4258	55	1.291.686	5,4
Canoe/Kayak	4485	23	0.512821	2,2
Cycling	22471	221	0.98349	21,6
Equestrian	619	11	177.706	1,1
Fencing	1609	4	0.248602	0,4
Field Hockey	1641	8	0.487508	0,8
Football	31242	144	0.460918	14,0
Golf	507	8	1.577.909	0,8
Gymnastics	2355	10	0.424628	1,0
Handball	4026	26	0.645802	2,5
Judo	4453	40	0.898271	3,9
Modern Pentathlon	665	4	0.601504	0,4
Rowing	4699	24	0.510747	2,3
Rugby Union	6961	57	0.818848	5,6
Sailing	795	2	0.251572	0,2
Shooting	2616	23	0.879205	2,2
TOTAL	138828	1025	0,738323	100

Source: our elaborations on WADA data

With the following assumptions:

$$\Pi^{sno}_{dop} > \Pi^{s}_{dop,}\ \Pi^{c}_{dop} > \Pi^{c}_{nodopt} \text{ and } \Pi^{s}_{dop} > \Pi^{c}_{nodop},\Pi^{s}_{nodop} > \Pi^{c}_{dop}, \tag{3}$$

Hence, $\Pi^{s}_{nodop} > \Pi^{s}_{dop} > \Pi^{c}_{dop} > \Pi^{c}_{nodop,}$ which is the classic payoff matrix of the Hawk Dove game. A standard payoff matrix for a Hawk-Dove type game can, therefore, be written in the following form (see Table 5).

An evolutionary game is a description, therefore, of the dynamic evolution within a sport discipline in which there is competition between sport players who use doping and sport palyers who do not use doping. Intuitively, we can identify three possible outcomes.

1) Only athletes who dope manage to achieve important results
2) Only nondoping using athletes succeed in obtaining important results
3) The two groups share victories.

In the first two cases, there will be a specialisation of the discipline in such a way that doping in the first case will be the dominant situation, while the absence of doping will characterize the second type. In the third case, we will have groups of doping players coexisting with groups of nondoping athletes. However, since the payoff depends on the number of players who choose one of the two approaches, the convenience of doping in terms of performance and the presence of a substantial number of athletes who use banned substances constitute incentives that exponentially increase the level of doping in that sport. In the Hawk Dove game and the war for attraction, individuals' strategies can be represented as a combination of choices between doping and nondoping. In these evolutionarily evolutionary games, successful strategies tend to increase their frequency in the population, while unsuccessful strategies tend to decrease their frequency. The ESS (Evolutionarily Stable Strategy) in the game 'Hawk Dove' is achieved when a strategy becomes resistant to invasion by an alternative strategy. In other words, if a population consists mainly of Hawks, then the Hawk strategy becomes the ESS because no Dove can invade the population successfully. However, if the population contains a mixture of both strategies, then there is no single strategy that can be considered the SSE since the equilibrium depends on the frequency distribution of the two strategies. In this case, the SSE can be represented by a population in which the frequency of Hawk and Dove is stable and in equilibrium, with the incidence of the strategies depending on the context-specific conditions. In the Hawk Dove game, Evolutionary Stable Strategies (ESSs) are identified as stable equilibrium points where the strategies of the population do not change since any deviation from these strategies would be disadvantageous.

5. - The dynamic solution

The payoff matrix described in the previous section can be written in synthetic and general form.

Where V is the total payoff of the game and C is the cost in calories resulting from competition when two Hawks meet**.**

To use a model of this type to study the competition between agents who choose to dope and agents who choose not to dope, making it more adherent to reality and therefore, it is necessary to make the Hawk–Dove game more complex by introducing a third type a third strategy, which is that of the Retaliator (i.e., of an agent who plays Hawk when he encounters a Hawk and plays Dove when

Table 5
Payoff matrix nodop dop.

	Π^{s}_{nodop}	Π^{c}_{nodop}
Nodop	Π^{s}_{nodop}	Π^{c}_{dop}
Dop	Π^{c}_{dop}	Π^{s}_{dop}
	Π^{c}_{nodop}	Π^{s}_{dop}

he encounters a Dove), and to introduce a further cost K linked to the Hawk strategy, linked to the expected value of the penalty (level of the penalty per probability of being discovered) in the case of being discovered.

The payoff matrix in this case becomes.

A well-known result in evolutionary game theory is the Hofbauer theorem [33](see equations (4)–(6)). This theorem proves the equivalence of the replicator equation of an evolutionary game with n pure strategies with a system of n-1 differential equations of diffusive type.

In the formula, the replicator equation with n strategies is given by:

$$\dot{x}_i = x_i\left(\sum_{j=1}^{n} a_j x_j - \varphi\right) i = 1, \cdots, n \tag{4}$$

where x_i is the frequency of strategy i, $A = |a_{ij}|$ is the payoff matrix and φ is the average fitness of the population (the growth rate of strategy i $\frac{\dot{x}_i}{x_i}$ is given by the difference between the fitness of i and the average fitness of the entire population). This is equivalent to:

$$\dot{y}_i = y_i\left(r_i + \sum_{i=0}^{n-1} b_{ij}\, y_j\right) \text{ con } i = 1\ldots, n-1 \tag{5}$$

which is a classical Lotka–Volterra equation with n-1 species, where y_i is the number of specimens of species i, r_i is the intrinsic growth rate and b_{ij} describes the interaction between species i and j.

If n are the strategies, n-1 will be the degrees of freedom of the system since one of the frequencies xi will be expressible as a function of the remaining ones since $\sum_1^n x_i = 1$.

Therefore, if we apply these equations to the HDR game, we find:

$$dx / dt = - x(-a_1 + bx + a_{12}y) \tag{6}$$

$$dy / dt = - y(-a_2 + c\, y_+ a_{21}\, x)$$

The model can be constructed as a system of two nonlinear differential equations and can be thought of as a war between two populations trying to conquer a territory. These equations are known in the literature as the Lotka-Volterra equations; therefore, the 'Hawk-Dove-Retaliator Game' can be thought of as the competition of two populations over the same resource. Each exchange of calories can be measured as a payoff in a 'Hawk-Dove-Retaliator Game' (see equations 7- 11).

The model can be developed with simple steps in the following form:

$$\frac{dx}{dt} = a_1 x - a_{12} xy - bx^2 \quad \frac{dy}{dt} = a_2 x - a_{21} xy - cy^2 \tag{7}$$

The parameters a_{12}, a_{21}, b, and c are important because they indicate an inhibitory effect if negative, growth if positive, and indifference if zero, respectively, and are the parameters that determine the dynamic characteristics of equilibrium. The parameters a_1 and a_2 instead refer to the growth characteristics of the individual population in the absence of the other. The terms x^2, xy and y^2 can be considered to be caused by random encounters.

If parameters b and c are equal to zero, the system becomes:

$$\frac{dx}{dt} = a_1 x - a_{12} xy \tag{8}$$

$$\frac{dy}{dt} = a_2 x - a_{21} xy$$

The model describes the process of two populations spreading together. If one population becomes extinct, then the other grows exponentially. The parameters a and c are related to the growth rate and the extinction rate [34,35].

The solutions will then be:

$$x = x_0\, e_1^{a\,t}$$

$$y = y_0\, e_2^{a\,t} \tag{9}$$

In the presence of a quadratic term, exponential growth does not occur, but growth is limited by available resources (logistic function).

If we denote the subjects who play the three strategies by x1, x2, and x3,

$$x_1 = 1/1 + x + y \tag{10}$$

$$x_2 = x/1 + x + y$$

$$x_3 = y/1 + x + y$$

Then, by Hofbauer's theorem (Hofbauer, 1981 [33], Hofbauer et al., 1988 [36], 1990 [37], 1998 [38]), equation (7) is equivalent to equation (4) with a matrix

$$A = \begin{vmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{a_1} & \boldsymbol{b} & \boldsymbol{a_{12}} \\ \boldsymbol{a_2} & \boldsymbol{a_{21}} & \boldsymbol{c} \end{vmatrix} \tag{11}$$

The model expressed by equation (7) can lead to three different equilibria.

(a) stable equilibrium;
(b) unstable equilibrium;
(c) survival of a single population.

For simplicity, assume the following (see equations (12)–(16)):

$$a_1 = a_2 = 1 \tag{12}$$

the conditions for stable equilibrium are as follows:

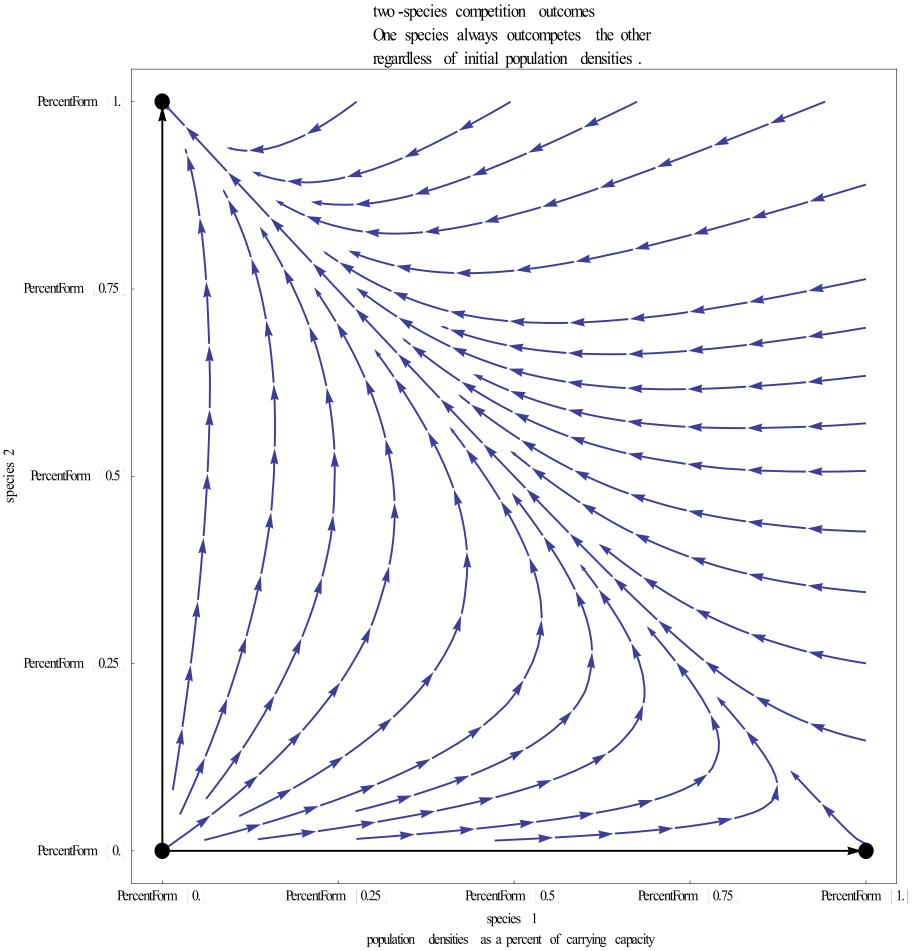


Fig. 2. Species 2 specialisation.

$$b > a_{21}; c > a_{12} \tag{13}$$

the conditions for unstable equilibrium are as follows:

$$b > a_{21}; c < a_{12} \tag{14}$$

and, finally, the conditions for supremacy are, for the first population:

$$a > a_{12}; a_{21} > c \tag{15}$$

while for the second are:

$$b > a_{21}; c < a_{12} \tag{16}$$

The dynamics can then either lead to the coexistence of the two populations or to the supremacy of one population. The coefficients a_{21} and a_{12}represent the competition coefficients, b and c are the reciprocal of the carrying capacity of the system under hypothesis (15), and a_1 and a_2 are the intrinsic growth rates, i.e., the growth rate of population i, when population j has become extinct.

6. Results of the simulations

To have concrete indications in terms of policies, it is necessary to simulate the behaviour of athlete populations to determine how the dynamics of competition between doping and nondoping using athletes evolve. To do this, we simulated the behaviour of the system with the numerical solution of the Lotka Volterra equation obtained with the program Mathematics.

We identified 4 cases corresponding to different values of the parameters of the Lotka-Volterra equation that were described above.

Case 1 (Fig. 2, Fig. 3)

Intrinsic growth rate: equal between two species.

Carrying capacity: Same between two species.

Competition coefficient: 1.48 species 1, 0.53 species 2.

Initial conditions: (0,4, 0,4)

In this case, we see that, in the end, only species 1 survives regardless of the initial population density. We have an equilibrium of specialisation. Either everyone will use doping, or no one will use doping.

Case 2 (Fig. 4, Fig. 5)

Intrinsic growth rate: equal between two species.

Carrying Capacity: same between two species.

Competition coefficient: 0.03 species 1, 1.16 species 2.

Initial conditions (0,4, 0,4)

In this case, we have a solution that mirrors the previous one, with species 2 taking the place of species 1.

Case 3 (Fig. 6, Fig. 7)

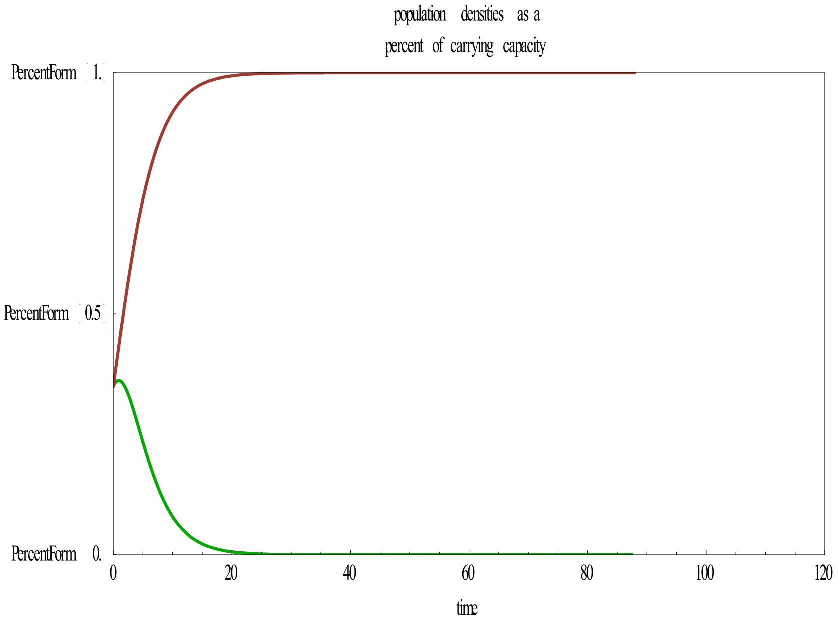


Fig. 3. Two competing population densities in species 2 specialisation.

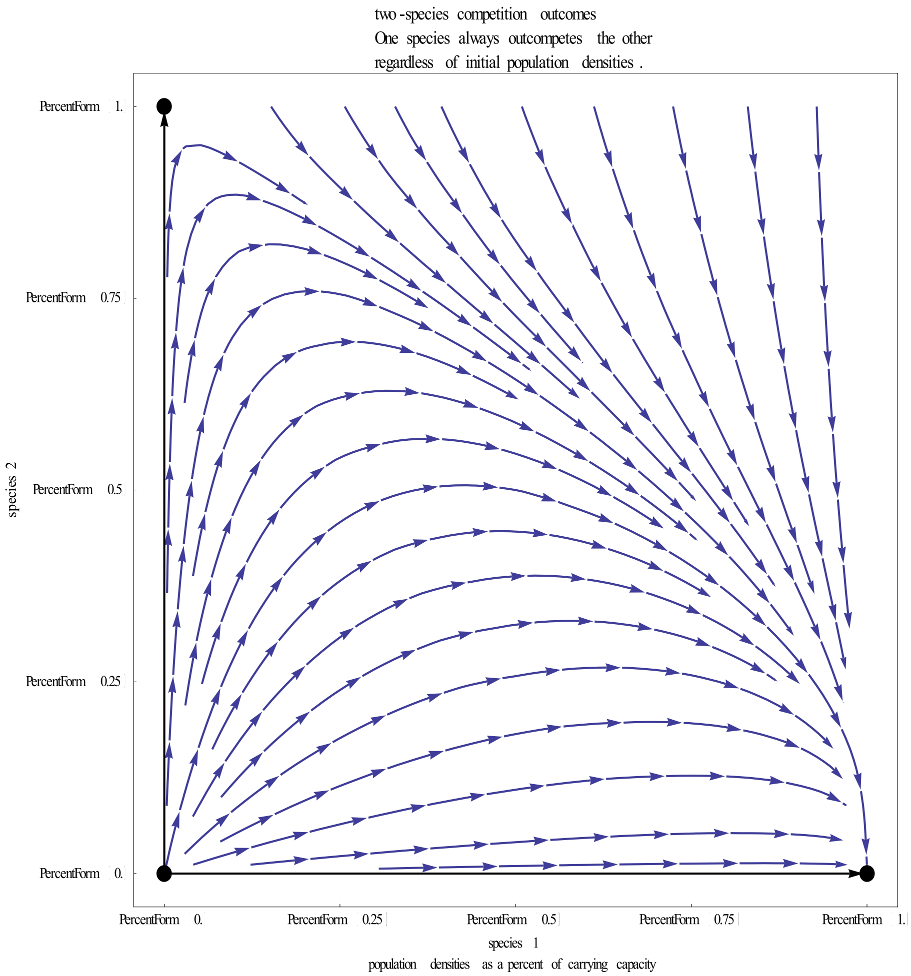


Fig. 4. Species 1 specialisation.

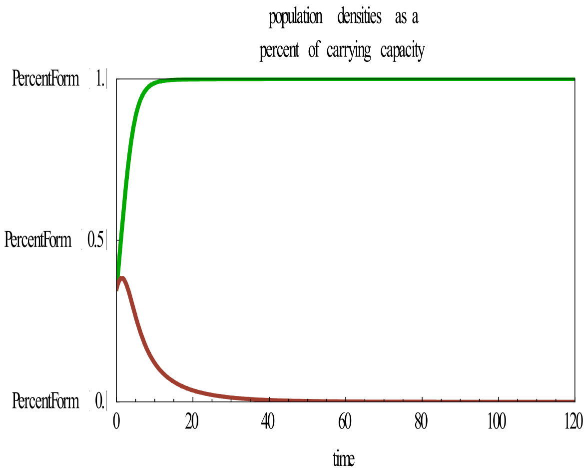


Fig. 5. Two competing population densities in species 1 specialisation.

Intrinsic growth rate: equal between two species.
Carrying Capacity: same between two species.
Competition coefficient: 0.63 species 1, 0.53 species 2.
Initial conditions (0,4, 0,4)

Case 3. is interesting because it describes a market in which there is a coexistence of doping and nondoping using athletes, and this can continue to be valid if players can change their choice at each step of the game.

Case 4 (Fig. 8)
Intrinsic growth rate: equal between two species.
Carrying Capacity: same between two species.
Competition coefficient: 1.57 species 1, 1.16 species 2.
Initial conditions (0,4, 0,4)

In this case, the basins of attraction are highlighted. This is a situation in which the system can evolve into one of the previous conditions as conditions change. With these two colours, the basins of attraction are highlighted, and the point of stable coexistence is also highlighted. The basin of attraction of an SSE is the set of all initial points of populations that converge to the SSE in question, i.e., all populations that, from a given initial frequency of strategies, converge to the frequency of the SSE in the long run. In an evolutionary game such as Hawk and Dove, the attraction basins of SSEs can be described as regions in the space of strategy frequencies where populations starting from these regions converge to the corresponding SSE. These catchment areas depend on context-specific conditions, such as the costs and benefits of different strategies and the relative frequency of these strategies in the population. In general, SSE catchment areas can be quite complex and may include areas where strategy frequencies fluctuate around the SSE. For example, in a context where the Hawk strategy is expensive but leads to high gain, the Hawk SSE catchment area could include populations with a relatively high initial frequency of Hawks but with some variability over time due to fluctuations in strategy frequencies. In any case, SSE catchment areas are important because they indicate the conditions under which strategies evolve and remain stable over time.

6.1. Model with penalty K

To make the model more realistic, it can be advantageous to consider the parameter K not as a constant and study the dynamics as a function of K. In fact, the most recent literature contains models in which strategies co-evolve with the multiplicative factor of cooperation groups (Shag, Y et al., 2019, [39,40]). If we imagine that the value of a_{12} for the doping using athletes is a decreasing function of K (K is the parameter that we have incorporated into the payoffs of the game (see Tables 6 and 7), with K being the cost of doping understood as the expected value of the penalty resulting from being discovered, then as K varies, we can find ourselves in one of the previous situations, and by making K sufficiently high, we can bring the system to equilibrium in which there are only nondoping using athletes.

To analyze in detail how the parameter K affects the coexistence of two competing species through the modification of interspecies competition coefficients a_{12} and a_{21} we write the relationship between a_{12} and K in these terms: $a_{12} = a*_{12}\exp(-K)$ where $a*_{12}$ is the value of the parameter for $K = 0$. If we write the Lotka-Volterra equations in normalized form (see equations (17) and (18)):

$$\frac{dx}{dt} = r_x x\left(1 - \frac{x}{R_x} - \frac{a_{12}}{R_x}y\right) \tag{17}$$

$$\frac{dy}{dt} = r_y y\left(1 - \frac{y}{R_y} - \frac{a_{21}}{R_y}x\right)$$

where:

x and y are the population sizes of species 1 and 2, respectively, r_x and r_y are the natural growth rates of species 1 and 2, respectively,a_{12} is the competition coefficient indicating how strongly species y affects the growth of species x, and a_{21} for the reverse effect.

The normalized form of the Lotka-Volterra equations for interspecies competition directly incorporates the effects of interspecies competition on the growth capacity of each species. The term $\frac{x}{R_x}$ x represents the proportion of the carrying capacity achieved by species x, and the term $\frac{a_{12}}{R_x}$ represents the impact of competition with species y on the ability of species x to achieve its carrying capacity. The same applies for species y. This formalism highlights how competition for limited resources affects the population dynamics of competing species in an ecosystem.

By including the factor K, the equations become:

$$\frac{dx}{dt} = r_x x\left(1 - \frac{x}{R_x} - \frac{a*_{12}e^{-K}}{R_x}y\right) \tag{18}$$

$$\frac{dy}{dt} = r_y y\left(1 - \frac{y}{R_y} - \frac{a_{21}}{R_y}x\right)$$

Considering the Jacobian matrix (see equation (19)):

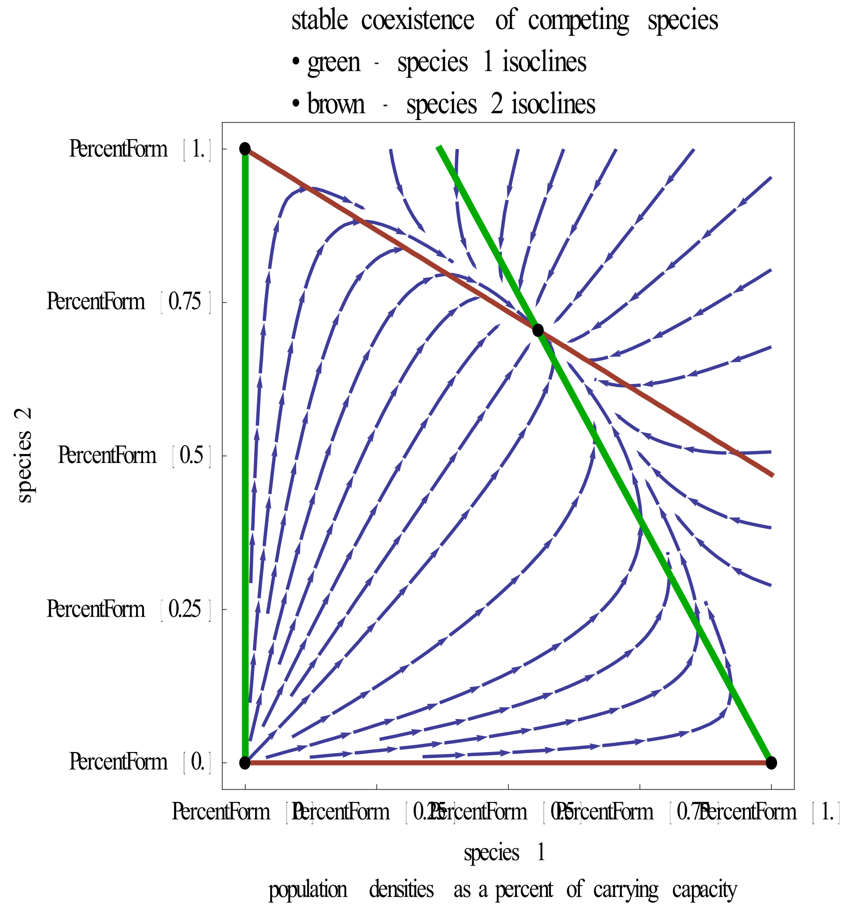


Fig. 6. Isoclines of two competing species in coexistence.

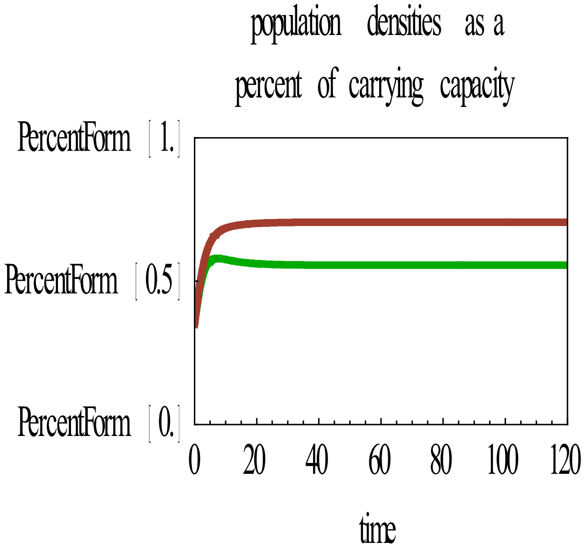
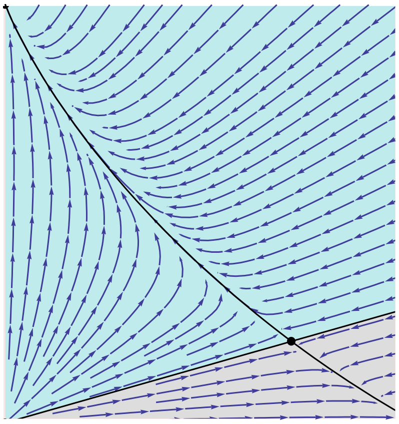


Fig. 7. Two competing population densities in coexistence.



Light grey attraction basin species 1

Pale blue attraction basin species 2

Fig. 8. Basins of attraction.
Light grey attraction basin species 1.
Pale blue attraction basin species 2.

Table 6
Game HD payoff.

	Hawk (dop)	Dove (nodop)
Hawk dop)	(V–C)/2, (V–C)/2	V, 0
Dove (no dop)	0, V	V/2, V/2

Table 7
Game HDR payoff.

	Hawk (dop)	Dove (nodop)	Retaliator
Hawk dop)	(V–C–K)/2, (V–C–K)/2	V–K, 0	(V–C–K)/2, (V–C–K)/2
Dove (no dop)	0, V–K	V/2, V/2	V/2, V/2
Retaliator	(V–C–K)/2, (V–C–K)/2	V/2, V/2	V/2, V/2

$$\begin{matrix} \frac{\partial}{\partial x}\left(\frac{dx}{dt}\right) & \frac{\partial}{\partial y}\left(\frac{dx}{dt}\right) \\ \frac{\partial}{\partial x}\left(\frac{dy}{dt}\right) & \frac{\partial}{\partial y}\left(\frac{dy}{dt}\right) \end{matrix} \tag{19}$$

and studying the stability analysis as a function of K through the study of the eigenvalues given by the solution of the characteristic equation (equation (20)):

$$\det(\mathrm{J} - \lambda \mathrm{I}) = 0 \tag{20}$$

where I is the identity matrix, one can observe how variations in K influence the stability of the equilibrium. This determinant, in fact, reveals the relationship between the parameters of the system (including K) and the stability of the coexistence equilibrium. A positive determinant is one of the necessary conditions for equilibrium stability: it indicates that the Jacobian matrix has eigenvalues with negative real part, a necessary condition for local stability of the coexistence equilibrium. The parameter K appears in the expression in the form of exp(-K), directly influencing the stability of the equilibrium in relation to the costs associated with the rate of interspecific competition, modulated by the effect of K.

Coexistence equilibria can thus occur as a function of K, with both species surviving in a stable equilibrium. The condition for a stable coexistence equilibrium occurs if the determinant of the Jacobian matrix is positive and the trace is negative, indicating that the eigenvalues have negative real parts. Specialisation equilibria occur when one of the two species reaches a population density that excludes the other species. This occurs when the value of one of the two species tends to zero, while the other species approaches its capacity For low values of K, the specialisation equilibrium is favored in which doping prevails or the rate of agents using doping is high. For high values of K, the stable coexistence of species is favored, or the specialisation equilibrium in which there are no doping agents or their number is very low.

By numerically solving the Lotka-Volterra equation for different values of K, these dynamics can be shown. The simulations is carried out considering $a_{12} = 1.51$ and $a_{21} = 2.51$, with an initial value for the two populations of 10. As can be seen for low values of K the species y (non-doping) disappears, while for increasing values of K stable equilibria of coexistence are found (see Fig. 9).

7. - Discussion

The analysis of the literature on made in the previous paragraph, made us realise the complexity of the problem. The question concerning the greater incidence of doping in certain sports disciplines remains open, and also open is the question concerning the factors which determine the greater or lesser incidence of doping. And more, while it is true that an increase in controls and sanctions may in principle lead to a reduction in doping, this higher cost of the decision to dope may be underestimated by athletes or may even be considered irrelevant compared to the greater benefit derived from victories. In addition, according to the literature, this assessment of cost and risk is always linked and continent with the perception of doping within that sport discipline. Often, doping may be the only way to aspire to victory in a discipline where the advantage received from doping is significant. Indeed, as highlighted above, an increase in controls may lead to an increase rather than a decrease in doping [17].

To better grasp and understand this complexity, we have attempted to use evolutionary games as an analysis tool. With this method, we aimed to identify which sports are more likely to face doping risks and understand the transmission process that causes doping to become more prevalent in certain athletic fields. To explain and discuss the competitive economic mechanism behind doping, it is necessary to rewrite Eq. (7) in another way to clarify the situation: a match/race is simply a competition between two species (athletes) and is the caloric cost resulting from competition for a resource when two Hawks meet, provided that the principle of competitive exclusion is present. Obviously, in this form, the biological selection mechanism can be described by only two parameters (the sanction at this stage is not relevant).

a) Individual Performance
b) Competitive capacity

Employing the Lotka-Volterra model for competition over a single resource, where x and y represent two competing species (athletes), and r and s signify the personal performance and competitive ability (interpreted as reduced caloric requirements), the mathematical formulation is:

$$\begin{aligned} \frac{dx}{dt} &= [r_x - s_x(ax + by)]x \\ \frac{dx}{dt} &= [r_y - s_y(ax + by)]y \end{aligned} \tag{21}$$

If we define the ratio of r to s as efficiency, one species will prevail if the ratio

$$r_x / s_x > r_y / s_y \tag{22}$$

The competitive exclusion principle, which comes into play when multiple species vie for a shared scarce resource, dictates that only the species most adept at exploiting this resource will ultimately survive, effectively outcompeting the rest. Translated to the realm of sports, this implies that the superior athlete will emerge victorious.

So, doping is simply the disruptive element that changes the system's asymptotic behaviour and can lead to the least efficient species or individuals winning.

The above reasoning provides us with an answer to the first question. The sign of the inequality can change either because there is an increase in the numerator r or a decrease in the denominator. The numerator is related to individual performance that can be improved through training within a certain limit, of course. The denominator, on the other hand, is linked to individual characteristics that allow a better result for the same amount of effort. In sports, we can distinguish between two different types of competition—those in which individual performance is predominant and those in which individual performance is important but not decisive—complemented by other factors. Team sports and sports such as tennis, which require a high level of mental concentration, fall into this second type.

To change the results of a contest, enhancing personal performance or improving efficiency for the same level of exertion is crucial. Doping influences the former, with little to no impact on the latter. Consequently, sports that heavily rely on individual prowess are more susceptible to doping practices. The presence of doping in other sports, albeit to a lesser degree, is justified by the fact that a decrease in the denominator is much more difficult to influence and that, in any case, regardless of the sport, increased performance always gives a greater probability of victory.

We can therefore draw the following taxonomy (Table 8):

The results of the models are in line with Loland's [2] vulnerability thesis and may constitute, not only the proof, but also the micro-foundation of this theory. These results, also, allow to provide policy indications to put in place effective counteracting actions. We have seen that K, the generalised expected cost of doping, is the parameter that can make the system evolve in different configurations. Obviously, the term generalised cost of doping must be well interpreted because this cost is certainly linked to the efficiency

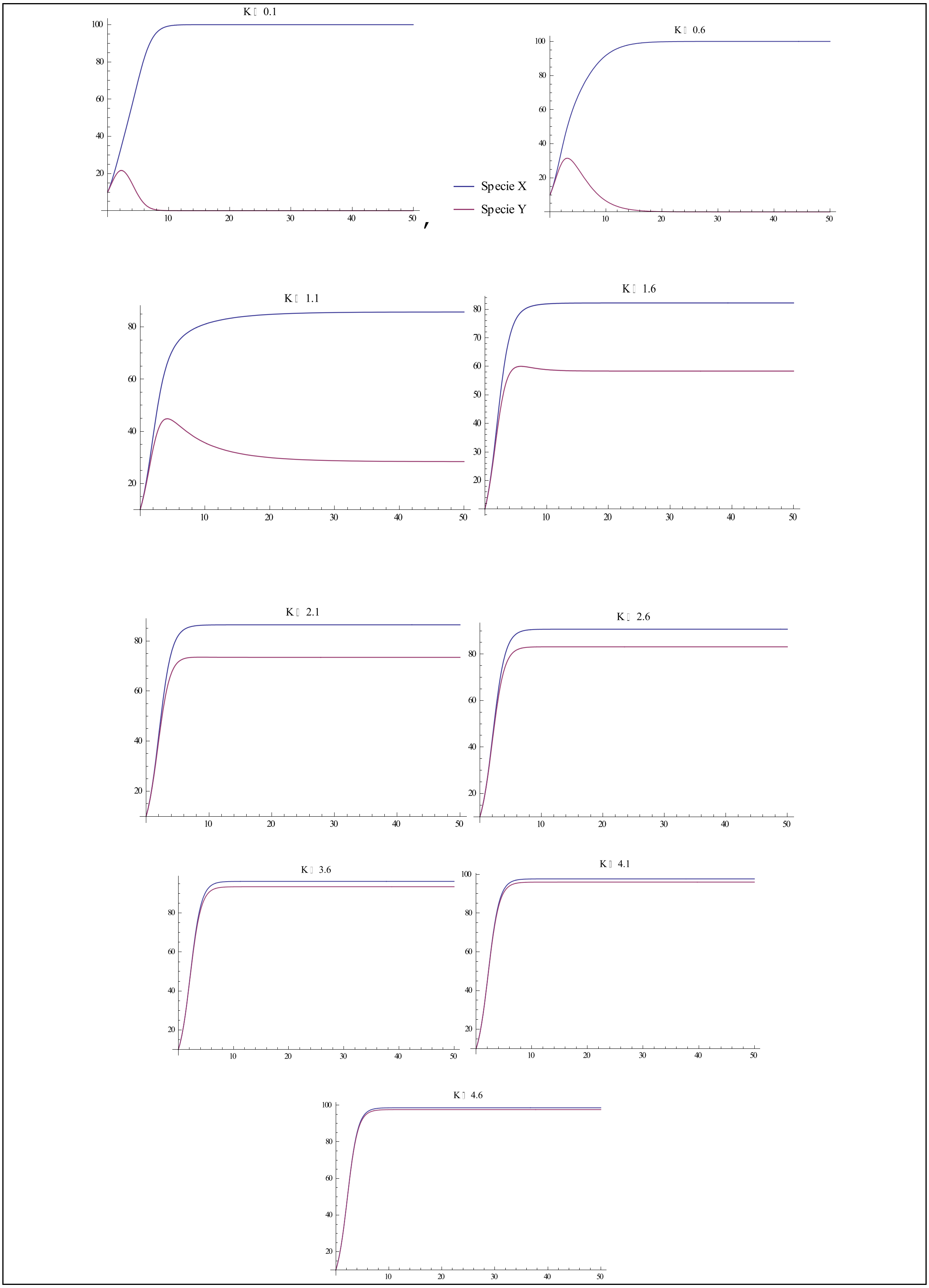


Fig. 9. Population densities for different K value.

Table 8
Taxonomy of the dopant.

$\Delta r > \Delta s$ doping very high $\Delta s > \Delta r$ doping low	$r_x \gg r_y$ doping low $s_x \gg s_y$ doping low
$r_x > r_y$ $s_x < s_y$ doping high	$r_x < r_y$ $s_x > s_y$ doping very low

of anti-doping policies that increase the probability of being discovered in the event of doping; it is certainly linked to the severity of the administrative sanction that is sprayed in the event of anti-doping violations, but it is also linked to the cost in terms of loss of reputation both towards other athletes and towards the public. In fact, the hawk's strategy is only an EES if the value of the asset is greater than the cost of the conflict. This is the most important part of the cost because it undermines the very reason for competing sportingly. A discredited athlete will be expelled from the sporting circuit, not only by the administrative penalty but also by the ostracism of the public and other athletes who will boycott him. This mechanism of social disapproval is very low today. For this reason, especially in some sports where doping is common among athletes, repressive policies based on administrative sanctions alone are insufficient because athletes themselves do not perceive the disvalue of doping and, therefore, are inclined to accept nonclear behaviour. Alongside disqualifications and fines, it is necessary to instil a strong social disapproval mechanism that makes doping a behaviour that is also psychologically costly and discourages its use. Having calibrated all anti-doping actions only on administrative sanctions is also the cause of the substantial failure of these policies. If doping is not recognised as having a social disvalue, anti-doping measures will be ineffective, especially in those contexts where doping is most prevalent.

8. Conclusions

Doping is certainly a complex and multiform phenomenon that tends to grow as a result of an increasingly competitive idea of sport and society. The research presented in this paper offers a analytical insight into the dynamics of doping in relation to individual performance in sports. The key finding is that doping predominantly influences the enhancement of individual performance, a critical factor in sports where performance is directly correlated with success. This understanding stems from a analysis that differentiates between the two primary components impacting an athlete's competitiveness: individual performance and the ability to exert the same level of effort more effectively. Doping's primary impact is on the former - the direct enhancement of an athlete's performance capabilities. This is particularly pertinent in sports where individual performance is the principal determinant of success. In such disciplines, the marginal gains achieved through doping can be the difference between victory and defeat, making them more susceptible to doping practices. On the other hand, sports that rely less on sheer performance and more on strategic or skill-based components are less affected by doping, though not entirely immune. This is because doping does little to enhance these other crucial aspects of competition, such as mental acuity, strategy, or teamwork. Additionally, an attempt has been made in this paper to explain some of its features that can be very important for imagining intervention policies. Convenience of doping and number of players who use it constitute the fundamental fact for understanding how far doping can take root in a discipline and how far it can spread and become a common practice. The battle against doping needs to start with strict measures and deeper inspections, yet it also demands a cultural shift towards valuing the essence of sport. The intense pursuit of success, encompassing victories on the field, economic benefits, and public image, serves as a driving force behind the heightened interest in doping. The research extends beyond the direct implications of doping on sport competition, delving into the broader consequences for the ethos of sportsmanship and the public perception of athletes. It propose for a comprehensive and multi-faceted anti-doping strategy that goes beyond the realm of sanctions. This strategy should include mechanisms aimed at cultivating a culture of social disapproval towards doping, addressing its psychological and societal dimensions. Understanding the evolutionary and 'contagious' dynamics of doping is certainly the first step toward combating doping with appropriate weapons. Complex problems need complex solutions that take into account the many facets that arise. This work constitutes a first research step that will have to be further developed with the attempt to assess the effect of particular incentive schemes or particular counterstrategies based on the efficient use of tests on the basis of evolutionary games.

CRediT authorship contribution statement

Domenico Marino: Writing – review & editing, Writing – original draft, Supervision, Methodology, Investigation, Formal analysis, Conceptualization. **Pietro Stilo:** Writing – review & editing, Writing – original draft, Investigation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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