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Optimal Synthesis of Continuous Aperture Sources and their Discretization into Isophoric Sparse Arrays

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Groza VOICU

Amor, ch'a nullo amato amar perdona,
mi prese del costui piacer sì forte,
che, come vedi, ancor non m'abbandona…"

Inferno - Canto V

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Abstract

An innovative deterministic approach to the optimal power synthesis of mask-constrained shaped beams through concentric-ring isophoric sparse arrays is presented and tested. The design procedure exploits at best the state-of-the-art techniques respectively available in the cases of circular-ring isophoric arrays radiating pencil beams and of linear isophoric arrays generating shaped beams. Moreover, it avoids the exploitation of global-optimization algorithms (with the inherent advantages in terms of computational burden) and compares favorably to the (few) available procedures.

The proposed deterministic design procedure starts from the definition of the power mask constraints for the radiation pattern for the overall azimuth cuts. After that, the workflow foresees the definition of the optimal continuous circular aperture (which acts as a reference and benchmark in the following step) able to meet the requirements for the far field. Finally, the arrays synthesis is performed by means of an optimal discretization of the reference source where, by minimizing the difference between the array's and continuous source's cumulative functions, an optimal isophoric sparse array arranged in circular rings is obtained.

The optimal results achieved in the first part of the research activity suggested applying the proposed approach also to the optimal, mask-constrained power synthesis of circular continuous aperture sources able to dynamically reconfigure their radiation behavior by just modifying their phase distribution. The design procedure relies on an effective a-priori exploration of the search space which guarantees the achievement of the globally-optimal solution. The synthesis is cast as a convex programming problem and can handle an arbitrary number of pencil and shaped beams. The achieved solutions are then exploited as reference and benchmark in order to design phase-only reconfigurable isophoric circular-ring sparse arrays. Numerical results concerning new-generation telecommunication systems are provided in support of the given theory.

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Introduction

1.1 New antenna architecture and status of the arrays antenna synthesis

The satellite communications in the last years have been the object of considerable changing in terms of performance and services offered to the customers. The new payloads require flexibility in terms of frequency/bandwidth or power allocation of coverage on the Earth surface, in order to have a product that satisfies the customers and the business needs. Flexible payload products are designed to reprogram a satellite mission when the spacecraft is already in orbit.

A satellite, which can reconfigure its characteristics, frequencies, coverages and/or power allocation, will allow the operator to follow in a very quick manner the evolving market request or access new businesses. Moreover, a flexible satellite opens plenty of opportunities for operators in their fleet, frequency rights and orbital slots management as well as allowing for the progressive deployment of the associated ground segments and gateways. Beam reconfiguration permits satellite to track mobile terminals. For example, in the marine industry, a beam can be re-configured to seamlessly track the progress of a terminal across an ocean, without having to lease multiple beams to cover the relevant regions.

Quantum satellite, manufactured by Airbus, will be the first generation of universal satellites able to serve any region of the world and adjust to new business needs without the user procures and launches a new satellite. Quantum will be able to adjust its coverage and capacity to suit customers' needs as and when they change. In addition, the re-configurability is not only an essential feature in the new telecommunications satellite. Also in the radar sector it is a crucial aspect to achieve a reconfiguration of the radiative properties in order to have a product which can be adapted to several cases.

Regarding the request of flexibility (re-configurability in terms of spot beam shape, steerable antennas and high gain), the DRA (Direct Radiating Array, see Fig. 1.1-1) may represent an attractive alternative to the conventional well-known solution based on single or multi-reflector system. All the reflector solutions based on single feed per beam configuration or multiple feed per beam configuration are not able to meet the requested re-configurability.

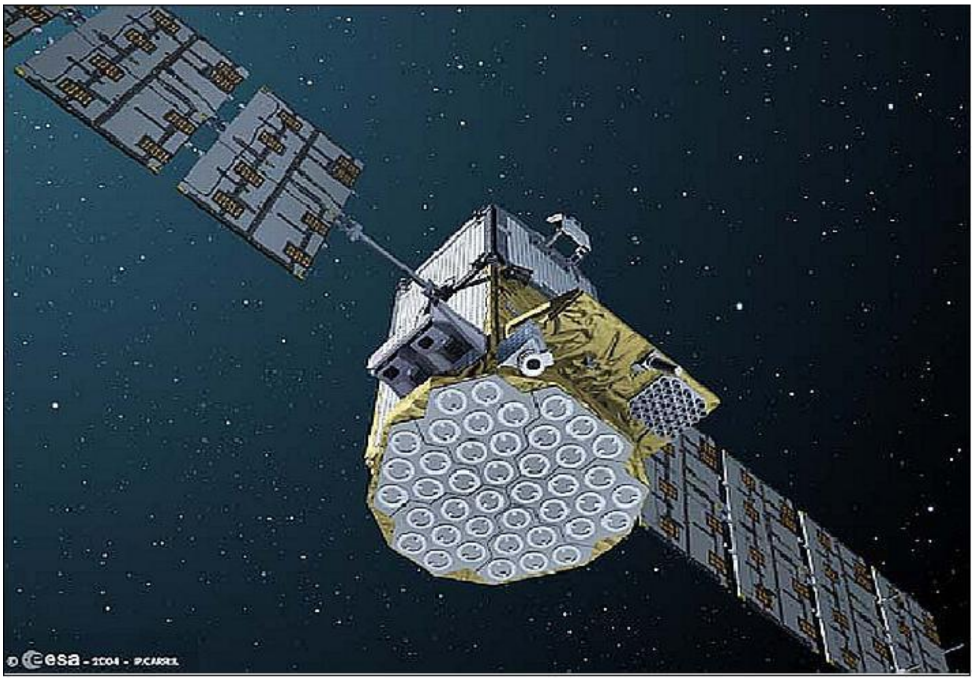


Fig. 1.1-1: Active phased arrays in GIOVE-B satellite

The main limitation for the classical architectures with single feed per beam is the impossibility to reconfigure the antenna during the lifetime of the satellite, in other words, to follow the changes of the user traffic request in terms of coverage footprint (beam pointing/steering), EIRP (beam power), and frequency plan (beam frequency). The second limitation is the fact that, for generating contiguous spot coverage, the satellite should embark more reflectors with large dimensions and consequently it has an impact on the overall mass and volume (critical aspect for the satellite). The alternative solution, multi-feed per beam that represents still the State of the Art, solves the mass and volume issues but it is in any case not completely reconfigurable. In the multi feed per beam antenna more feeds participate for generating a spot beam on the Earth surface (see Fig. 1.1-2)

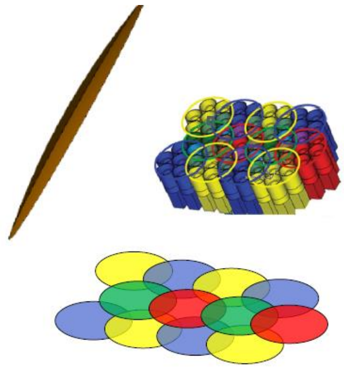


Fig. 1.1-2: Multi-feed for beam concept

The active antenna is the suitable candidate architecture to solve the mentioned problems and to design an antenna with maximum flexibility. Active antennas allow to re-configure the footprint coverage by the pointing capabilities, matching the possible modification of the SATCOM mission scenario due to longitude satellite position modification or to a new coverage shape or position, see Fig. 1.1-3.

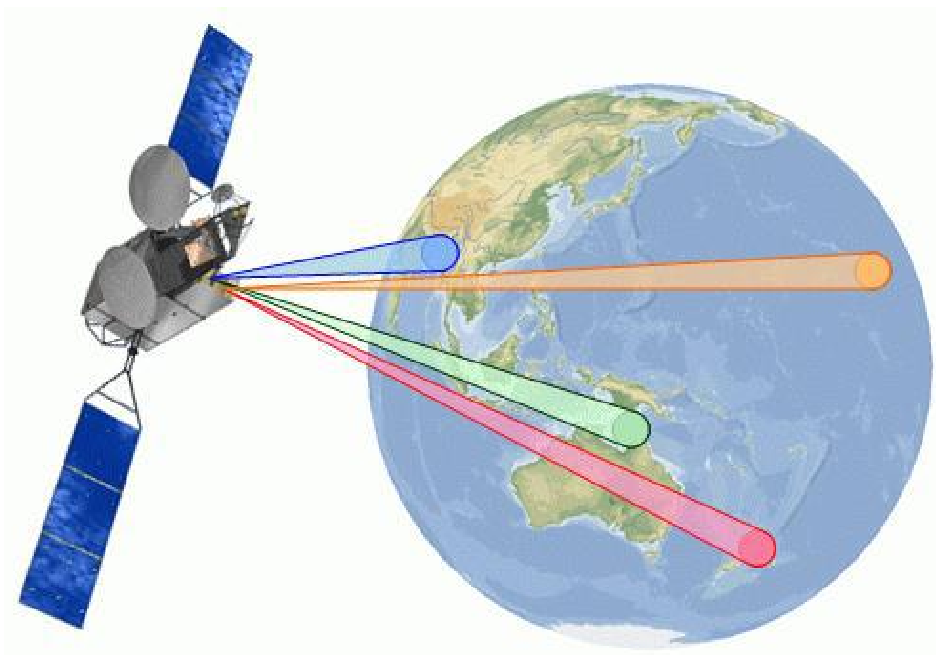


Fig. 1.1-3: Active antenna generating reconfigurable beam coverage

Active antennas allow setting the EIRP (Equivalent Isotropic Radiated Power) performance of each beam matching the user traffic request sharing the power available from the radiating elements amplifiers. Moreover, active antennas allow generating multiple beams coverage with continuous footprint by means of the same aperture minimizing the complexity of the satellite layout.

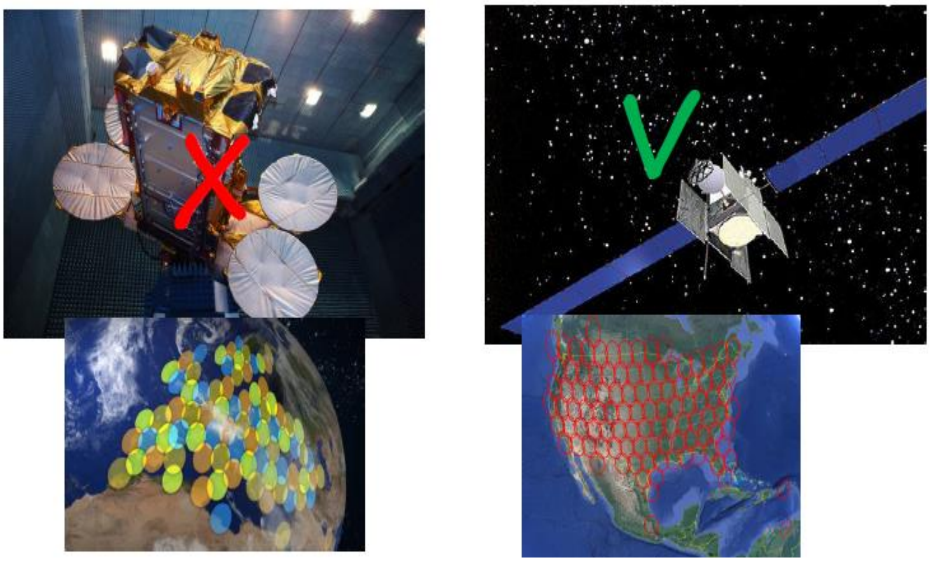


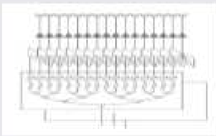
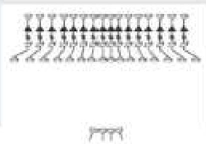
Fig. 1.1-4: Active DRA generating multibeam coverage

The active antenna family is constituted by several antenna architectures as:

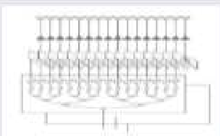
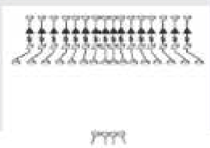
- Arrays Fed Reflector (AFR) – focused or defocused
- Direct Radiating Arrays (DRA)
- Imaging Confocal Arrays (ICA)
- Active Discrete Lens (ADL)

Tab. 1.1-1 and Tab. 1.1-2 resume shortly the main advantages and disadvantage between the main active antennas configuration which are under investigations by ESA in the last ten years.

As it can be noted all these configurations are based on the use of a DRA integrated with a single o multi-reflector system or in stand-alone condition. The techniques for the design of arrays in regular o irregular lattice are shared from all the active antennas.

Concept		Strength
AFR		Reduced flexibility by the BFN Limited power pooling Minimized complexity
DRA		Very high flexibility by the BFN Full power pooling High Coverage reconfigurability High Graceful degradation
ICA		Reduced size of the DRA Beam Gain achieved by magnification: Offset pointing by reflector steering
ADL		No BFN Full power pooling Low Coverage reconfigurability High Graceful degradation

Tab. 1.1-1: Advantages of each active antenna configuration

Concept		Weakness
AFR		Less flexibility than a DRA or ICA More feeds than conventional AFR Less power pooling than a DRA or ICA
DRA		Narrow beams require high number of radiating elements for the larger aperture Complex BFN for high reconfigurability
ICA		Complexity increase due to reduced size of DRA Heat concentrated in reduced area
ADL		Narrow beams require high number of radiating elements Switch matrix and large primary feed cluster for reconfigurability

Tab. 1.1-2: Disadvantages of each active antenna configuration

In the family of the DRA one particular class of antenna is very appealable and promising for the new onboard missions: the isophoric sparse DRA, an arrays with an equal signal amplitude at each radiating element which are not accommodate in a periodic lattice. The isophoric sparse arrays permits to optimize the efficiency of the SSPAs (Solid State Power Amplifiers) allowing them to work at the same working point. In addition, this kind of antenna architecture guarantees the performance (bandwidth and SLL) which are comparable with the full populated periodic arrays by using a smaller number of elements. The reduction of active elements inside the antenna permits to reduce the complexity of the antenna, of the cost and of the overall weight.

The mentioned above requests from the new business market give the opportunity to the scientific community to investigate new arrays antenna architectures and encouraging the development of new synthesis techniques as demonstrated by several publications on the subject.

Considering the kind of radiated pattern which will be designed and the available degrees of freedom a very large number of synthesis problems can be identified. Executing an excursus of the literature on the arrays synthesis problems, a possible classification can be carried out as the one reported in Tab. 1.1-3 where the rows refer to the kind of radiated pattern (pencil beams, difference patterns, shaped patterns, steerable patterns, reconfigurable patterns), while the columns refer to the available degrees of freedom [just excitations, constrained excitations (binary or quantized), just locations, both excitations and locations, phase-only, combinations of the above].

Available Parameters \ Far Field Kind	EXCITATIONS	EXCITATIONS + LOCATIONS	LOCATIONS	PHASE-ONLY	CONSTRAINED EXCITATIONS	CONSTRAINED LOCATIONS	CONSTRAINED LOCATIONS + EXCITATIONS
PENCIL							
DIFFERENCE							
SHAPED							
RECONFIGURABLE							
...							

Tab. 1.1-3: Classification of the arrays antenna synthesis problems

Due to intrinsic simplicity of the problems, a large number of contributions in the literature ensuring the achievement of an optimal solution is available for a fixed geometry and arbitrary excitation (corresponding the first column of Tab. 1.1-3).

In particular, optimal solutions have been designed for the synthesis of pencil beams or difference beams subjected to full arbitrary bounds on the sidelobe level by means of arbitrary geometry arrays (including linear, planar, and conformal arrays, the only constraint being the fact that positions of the elements are fixed in advance).

While previous analytical results exposed were limited to the case of an equal level of the sidelobes [5-7], the approach in [8-11] by Isernia et all suggests that the problems of maximizing the field (or its slope) in a given direction while keeping the field below given values can be conveniently reduced to Convex Programming [8-10] or even to Linear Programming [11] problems and therefore a unique global optimum exists for such a class of problems. In addition, this approach is applied to the maximization of directivity subject to given constraints for the sidelobes. As a consequence, these classes of canonical problems are solved. An important result achieved by these solutions is the absence of global optimization algorithm, characterized often by a very high computational cost.

Optimal solution strategies have been designed for the optimal synthesis of shaped beams with uniformly-spaced one-dimensional arrays [12] (wherein an effective procedure for general arrays is also devised).

Unlike the previous cases, the literature presents much less theoretical results for the classes of arrays synthesis problems where the locations are the unknowns. In fact, in all cases wherein the locations of the radiating elements have to be determined the radiated field depends in a non-linear manner from the unknowns and therefore a more difficult relationship has to be considered in the synthesis.

The interesting theoretical question arises of how to tackle these classes of problems in such a way to achieve a kind of optimal solution to the design problem at hand without exploiting global optimization procedures, wherein by 'optimal' it means

> *"an arrays able to fulfill given design goals by exploiting the minimum number of elements (or the minimum aperture size, or the minimum number of other resources) or, equally, to optimize given performances for a fixed number of elements (or fixed aperture dimensions, or fixed other resources)".*

It is worth noting that the need for using possibly deterministic procedures is due to a number of reasons. The main ones being that of avoiding the computational burden of the global optimization algorithms and the fact that local (or quasi-analytical) solution procedures allow one to better understand and check the expected properties of the different solutions which can be devised. In [17] and in [21] the deterministic procedures for the synthesis of uniform-amplitude linear sparse arrays radiating a pencil beam or a shaped beam are discussed. These approaches include the well know density taper procedure. These methods exploit an analytical formulation of the problem and the convex programming routine achieving the optimal solution without recurring to global optimization.

A more general problem, with an increased number of degrees of freedom, is the case of the synthesis using both locations and excitations of the elements of the array. The goal is generally that of exploiting the additional degrees of freedom deriving from the possibility to choose locations in order to save a number of elements, thus saving costs. The problem, which is of interest in radar applications as well as in many other cases, has attracted the interest of a number of researchers, giving rise to solution procedures ranging from global optimization, hybridization of global and local procedures, to the use of the so-called 'Matrix Pencil Method' [30] and its modifications [31-32], up to the exploitation of the theory of 'compressive sensing' [33]. Hybridizations amongst all these different methods have also been considered.

Another very interesting class of problem of large applicative interest is the one wherein one tries to synthesize two, or more, different patterns by means of excitations having a common amplitude distribution for the different modalities, so that reconfigurability is achieved by simply acting on phase distributions. In the last years several publications ([34-35]) appears for the case of linear arrays but a gap is presented for the case planar arrays able to be reconfigured in terms of radiation patterns (pencil and shaped beam). The few synthesis procedures for the reconfigurable planar are not completely deterministic or they are sub-optimal solution ([36]). In order to fill up this gap, this Ph.D. thesis introduces a deterministic procedure design instead of stochastic optimization to deal with the open problems for the synthesis of isophoric sparse arrays organized in concentric rings which are able to be reconfigurable acting only in the phase distribution.

The next paragraph reports the novelty included in this thesis work and how the chapters are organized.

1.2 Objective, motivation and outline of the thesis

The aim of this thesis is to consider an open point in the array synthesis that concerns the optimal synthesis of reconfigurable isophoric sparse arrays organized in concentric rings. In order to fulfill this need a preliminary step is required in order to solve another open problem in the arrays synthesis that is related to the optimal synthesis of shaped beams radiated by a concentric ring isophoric sparse array architecture. These two array synthesis problems have a common kernel which is represented by the need to use a full deterministic procedure for the discretization of a reference complex source into an isophoric sparse array.

The sparse array antennas, arrays whose elements are located over an aperiodic layout such to fulfill given radiation requirements, represent an important topic for antenna designers. This is shown by a large number of both 'classic' ([2]-[4]) and more recent ([13]-[26]) contributions.

Such a large interest is due to the advantages offered by these systems with respect to equispaced arrays. In fact, for a fixed aperture size, sparse arrays allow decreasing the number of elements without significantly affecting the beamwidth [19]. Such a lowering, in turn, mitigates mutual-coupling issues (due to the increased value of the average inter-element spacing) and it implies a reduced cost, weight, and complexity of the feeding network [2]. Moreover, the aperiodicity of the layout allows reducing grating lobes in the radiation pattern and hence an improvement of performance in terms of both sidelobes level (SLL) and bandwidth [17]. Finally, aperiodic arrays may allow a SLL reduction without resorting to an excitation-amplitude tapering [22]. The last advantages above led over the years to the large diffusion of a particular kind of sparse arrays: the so-called 'Isophoric' Sparse Arrays (ISA) [17]-[26], aperiodic arrays having a constant excitation amplitude over the whole aperture. This feature allows the feeding power amplifiers to operate at their point of maximum efficiency and it greatly simplifies the beam forming network [19],[22].

Amongst all ISAs planar architectures, Concentric Ring Isophoric Sparse Arrays (CRISAs), isophoric sparse arrays whose elements are located onto concentric rings, appear being the most convenient ones due to their capability of uniformly spreading the antenna energy over all azimuth directions [18],[21]-[26]. In fact, CRISAs constitute one of the usual choices to realize the satellite multibeam coverage of Earth [18],[23],[24].

Of course, isophoric sparse arrays configuration has also its disadvantages. The most critical drawback is related to the corresponding synthesis procedures. In fact, since the elements' locations are an unknown of the design problem, the synthesis is unaffordable through Convex Programming (CP) procedures of the kind presented in [27]. Therefore, as done for instance in [13]-[16], the antenna designers often recur to Global Optimization (GO) procedures. However, due to their high computational weight, GO techniques practically result unsuitable for the synthesis of isophoric sparse arrays composed by a very large number of elements. To overcome such difficulties, the following two-steps procedure has been recently devised for the design of Linear Isophoric Sparse Arrays (LISA) and Concentric Ring Isophoric Sparse Arrays (CRISAs) [17]-[24]:

1. Identify a Reference Continuous Aperture Source (RCAS) fulfilling 'at best' the radiation requirements at hand;
2. Derive the arrays layout as a discretization of the RCAS.

This procedure allowed to outperform previous approaches [17]-[24]. In fact, a number of well-assessed methods already exist to perform step 1 ([28], [29]) and, only in the 'pencil beams' case, step 2 ([2], [17], [18], [22]-[24]). Unfortunately, much fewer alternatives to perform step 2 are available in the 'shaped beams' case. The reason of such lack derives from a simple circumstance. The RCASs generating sufficiently narrow pencil beams are real functions [28], in the shaped beams case they result to be complex ones [29]. This issue, which drastically complicates step 2 [23], has been recently solved for the case of LISAs (Linear Isophoric Sparse Arrays) in [21] but still results unsolved for CRISAs. In fact, the unique approach currently available to perform step 2 in the CRISAs case is the 'rough' one in [23], which bypasses the problematic of the RCAS's complexity by:

a) Identifying the elements' locations by applying the technique presented in [18] only to the RCAS's amplitude;

b) Assigning to each arrays element an excitation phase equal to value assumed in its location by RCAS's phase.

This procedure neglects the fact that, as discussed in [21], the arrays elements locations must be a function of both the RCAS's amplitude and phase distributions, and hence its performance is considerably improvable. On the other side, beyond [23], the unique contribution addressing the synthesis of CRISAs in the shaped beams case is [16]. It relies on GO and hence it is exploitable only in case of CRISAs composed by a low number of elements. In the attempt of filling such a gap, this thesis proposes a new approach to the mask-constrained power synthesis of shaped beams through CRISAs. The technique can be seen as the extension of the approach in [21] to the case of ring symmetric arrays layouts and it results fast and effective even in case of arrays composed by a large number of elements.

After the definition of the deterministic procedure for the optimal synthesis of the isophoric sparse concentric ring arrays radiating a shaped beam, the introduced method can be used in the synthesis of only phase reconfigurable isophoric sparse concentric ring array. The only-phase isophoric sparse concentric ring arrays are a particular class of the reconfigurable antennas. In particular, the reconfigurability, i.e., the possibility to change the kind of radiating pattern (pencil beam, shaped beam, cosecant beam, fan beam, etc), is obtained only by changing the phase on the array while keeping the isophoricity requirement in terms of amplitude. This constraint permits to have several advantages at the satellite level. In facts, the reconfigurability permits to change the radiation modality in according to the load on the Earth surface (large beam in low-density traffic zone and small beams in the high-density traffic region). Moreover, the only-phase reconfigurability in active phased arrays can be implemented in very easy way without to increase the number of components and the complexity of the arrays (the same phase shifter of the phased arrays can be reprogrammable changing the phases on the antenna aperture).

To perform the synthesis of an only-phase reconfigurable isophoric sparse concentric ring arrays the following steps are required:

1. Identify a Reference Continuous Aperture Source that is an only-phase reconfigurable continuous aperture.
2. Derive the arrays layout as a discretization of the only-phase reconfigurable continuous aperture exploiting the synthesis procedure implemented for the shaped beam case.

Regarding point 1 a new synthesis technique for continuous source aperture will be derived and presented in this thesis.

The novelty of this thesis work respect to the state of the state-of-the art is the definition of a full deterministic and fast procedure that exploits at the best the characteristic of a planar complex source used as reference for the array synthesis. The introduced method does not have any type of limitation in terms of array dimension and therefore in terms of variable that are to be addressed. The proposed method permits also to define a complete workflow for the synthesis of only-phase reconfigurable source and its optimal discretization to design the isophoric sparse array organized in concentric rings.

In summary, in the thesis the following arguments will be dealt with:

i. State-of-the-art on the power mask constrains synthesis of arrays radiating pencil beam, shaped beam and reconfigurable beams
ii. Optimal synthesis of a continuous source for the synthesis of pencil beam, shaped beam and the introduction of the synthesis procedure of continuous source for the only-phase reconfigurable beams exploiting at the best the optimal synthesis of pencil and shaped beams.
iii. Synthesis of isophoric sparse concentric ring arrays radiating pencil beam, shaped beam and reconfigurable beams. In this chapter the starting point will be the presentation of the results present in the literature for the case of pencil beam (1-D and 2-D problem) and shaped beam (only 1-D was literature) in order to derive the synthesis of shaped beam and only phase reconfigurable beams for the 2-D case (objective of the work).
iv. Conclusions and recommendations for further developments

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2

State-of-the-art on the power mask constraints deterministic synthesis

2.1 Optimal synthesis of linear arrays with fixed geometry radiating pencil beam

In this chapter a survey on the state of the art for the synthesis of linear and planar arrays with fixed geometry able to radiate pencil beam, shaped beam and/or reconfigurable beam is performed. As it just explained, this kind of synthesis problem is simpler than the other ones which were presented in the introduction of this thesis (i.e. where the location of the elements are the unknowns). This survey is carried out in order to acquire the methodologies used for the optimal research of the synthesis problem solutions and to well understand what is needed for the synthesis of isophoric sparse arrays radiating reconfigurable beam.

In the case of linear uniformly spaced arrays with constant sidelobe levels globally optimal solutions to the pencil beam problem are obtained by exploiting the properties of the Chebyshev polynomials ([37], [38]). Once the number of elements, the uniform spacing, and the SLL have been fixed, these arrays exhibit a larger directivity than conventional Chebyshev arrays, provided the number of elements exceeds a minimum value depending on the SLL and element spacing. However, these “modified” Chebyshev polynomials also exhibit a larger beamwidth. The drawback associated to these approaches is that they are unable to deal with non-uniform, asymmetric, sidelobes pattern. In a few of words, they are not able to shape the sidelobes to reduce the arrays radiation on angle range where an undesired interference is present. A second important limitation in the modified Chebyshev approaches is that they can only work with arrays having a uniform spacing (on a line or on a plane) and with identical radiating elements.

In [9] Isernia et all provided an improvement and the problem of determining the excitations of a given set of arbitrarily positioned sources so as to produce a far-field intensity that is maximum in a prescribed direction and subjected to completely arbitrary upper bounds elsewhere was tackled. The formulation includes any kind of fixed geometry arrays and can be naturally applied to considering mutual coupling.

In particular, considering a set of *N* radiating element, each characterized by a proper radiation behavior, $\Psi_i(\underline{r})$, the total radiation pattern of the array can be written as:

$$E(\underline{r}) = \sum_{i=1}^{N} a_i \, \Psi_i(\underline{r}) \qquad (1)$$

where the a_i, $i = 1, \ldots, N$ are the complex excitations of the *N* radiating elements and they represent the unknowns of the problem. An important consideration is required. The expression (1) relates the desired far field with the unknown excitations in linear manner. This relationship permits to consider the synthesis problem with fixed geometry as the simpler one problem.

Therefore, the synthesis problem can be formulated as follows:

"Determine the set of complex excitations $\{a_i, i = 1, \ldots . N\}$ *such that*

$$\left|E\left(\underline{r_0}\right)\right|^2 = \left|\sum_{i=1}^{N} a_i \, \Psi_i(r_0)\right|^2 \qquad (2)$$

is maximum and respects the constraints

$$\left|E(\underline{r})\right|^2 = \left|\sum_{i=1}^{N} a_i \, \Psi_i(r)\right|^2 \leq SLL(\underline{r}) \qquad (3)$$

where $\mathrm{SLL}(\underline{r})$ is the non-negative arbitrary function which defines the sidelobes level in the whole observation space and r_0 is the antenna point direction.

Recurring to the bandlimitedness properties of the far field, considering a sufficiently dense distribution of sampling points over the observation domain *S* and without any loss of degrees of freedom, the problem can be formulated as:

Determine the complex excitation ($a_i = x_i + jy_i$, $i = 1 \ldots N$) *such that*

$$-Real(E(\underline{r_0})) \text{ is minimum} \quad (4)$$

with the following constraints

$$Imag\left(E\left(\underline{r_0}\right)\right) = 0 \qquad (5)$$

$$\begin{cases} \left|E\left(\underline{r_1}\right)\right|^2 \leq SLL(\underline{r_1}) \\ \vdots \\ \left|E\left(\underline{r_L}\right)\right|^2 \leq SLL(\underline{r_L}) \end{cases} \qquad (6)$$

where L is the number of observation points of the dens grid in the domain. In equation (4) the sign minus is due to the fact that the reference phase of the field for direction $\underline{r_0}$ is π.

Since $\left|E\left(\underline{r_1}\right)\right|^2$ is a positive semidefinite quadratic form (a hypercylinder) as a function of the complex excitations, each constraint in (6) define a convex set in the space of the unknowns. Moreover, the constraint of (5) is linear in terms of the excitation so that it also defines a convex set (an hyperplane) in the space of the unknowns. The intersection of convex sets in turn is a convex set, and therefore the (5) and (6) define a convex set. Finally, the objective function (4) is a linear function of the unknowns and consequently the overall problem is equivalent to the minimization of a linear function in a convex set C. This kind of optimization problem has been analyzed in operations research, and it can be demonstrated that it admits a unique minimum value, which is the global optimum, and it is achieved in a single point or in a connected (convex) subset of C.

In order to assess this fast deterministic optimal procedure, a numerical example is proposed. A linear arrays radiating a pencil beam with a "null-to-null" beamwidth of 29.5 degrees and a sidelobe level of -30 dB is designed and verified by means of a virtual demonstrator in CST (Computer Simulation Technology).

Performing the synthesis of the linear arrays with equally spaced elements, λ/2 at the frequency of 30 GHz, the following eleven normalized coefficients are obtained, see Tab. 2.1-1 and Fig. 2.1-1.

Id element	Amplitude [dB]	Phase [deg]
1	-7.38	0
2	-4.42	0
3	-2.39	0
4	-1.02	0
5	-0.26	0
6	0	0
7	-0.26	0
8	-1.02	0
9	-2.39	0
10	-4.42	0
11	-7.38	0

Tab. 2.1-1: Antenna coefficients for the pencil beam

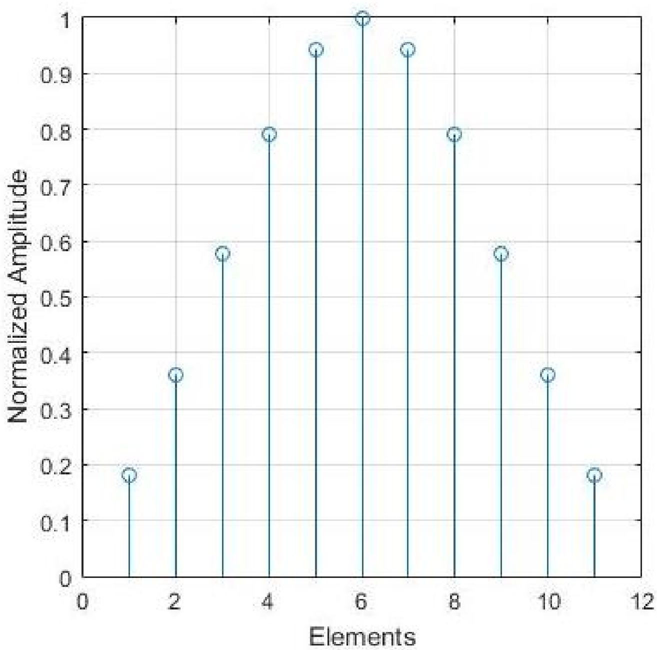


Fig. 2.1-1: Amplitude distribution of the linear array

Fig. 2.1-2 reports the comparison of the radiation pattern between the theoretical pattern achieved implementing the deterministic procedure and the virtual demonstrator pattern achieved by means of a full wave simulation in CST. In order to have a more accurate synthesis, the mutual coupling effect between the radiating elements is considered including in the deterministic procedure the radiation pattern of the feed in embedded condition (the radiation pattern radiated by the only central element of the array constituted by 11 radiating feeds). As demonstrated by Fig. 2.1-2, the results of the optimal synthesis are confirmed by the full wave analysis in CST; a good agreement is obtained validating the design procedure.

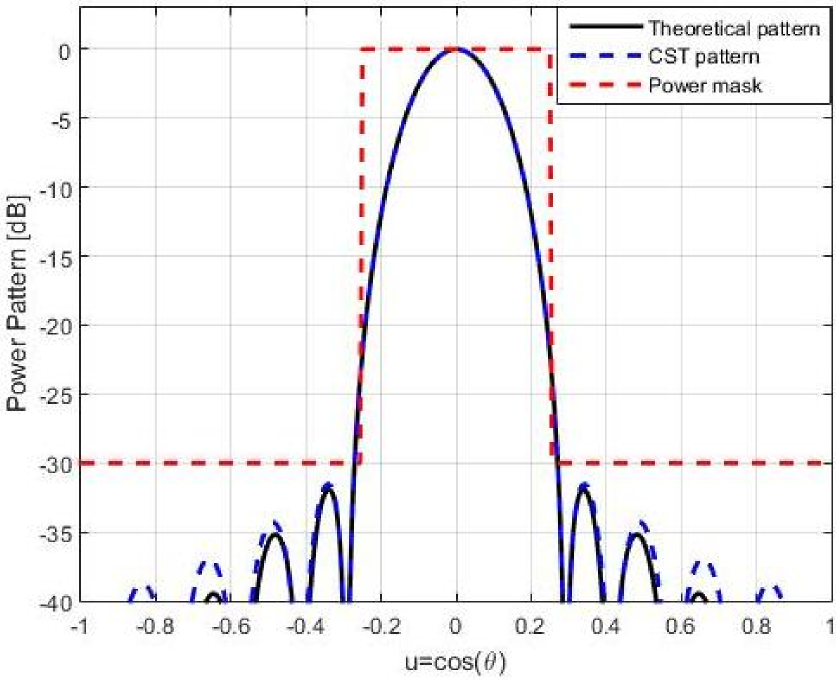


Fig. 2.1-2: Comparison between the theoretical pattern and the CST pattern

Fig. 2.1-3 shows the linear arrays modeled for the full wave analysis. Eleven rectangular waveguides with an inner dimension of 8.36x4.132 mm are used. A ground plane is considered in order to avoid the appearance of the back lobe.

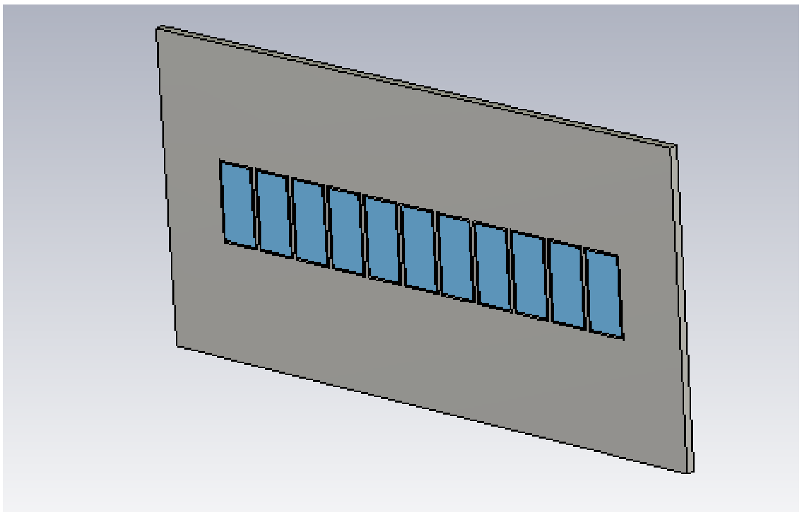


Fig. 2.1-3: Linear arrays with 11 radiating elements simulated in CST

After the validation of the synthesis procedure for the case of uniform sidelobe level, a second example is carried out considering the case of radiation pattern with asymmetric sidelobes. Fig. 2.1-4 shows the normalized asymmetric power mask constraints for the synthesis. The "null-to-null" beamwidth is 12.5 degrees. Implementing the synthesis procedure, the minimum number of radiating elements to meet the requirements is 21.

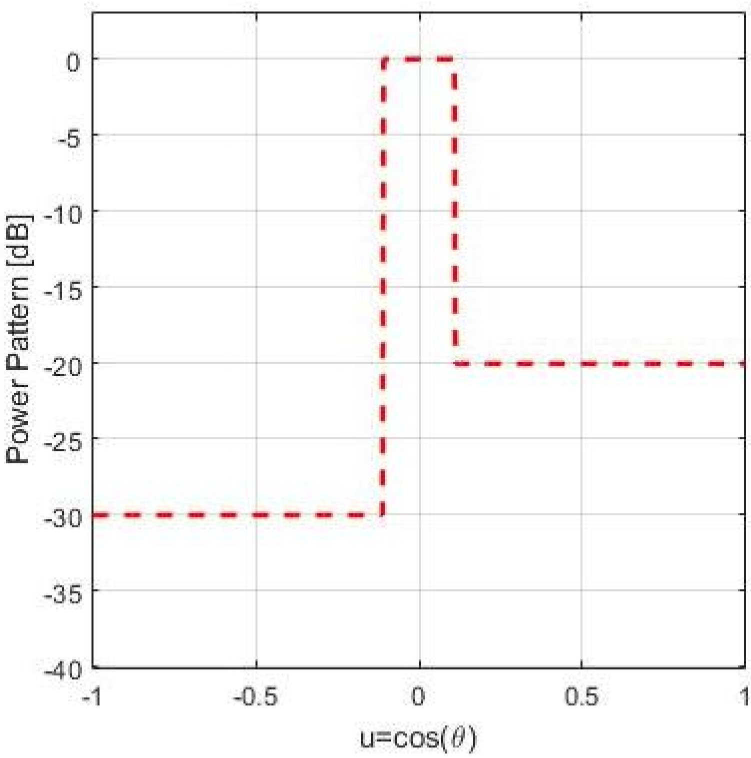


Fig. 2.1-4: Asymmetric power mask constraints

Fig. 2.1-5 reports the amplitudes and phases of the optimal linear arrays synthesized with the proposed deterministic procedure.

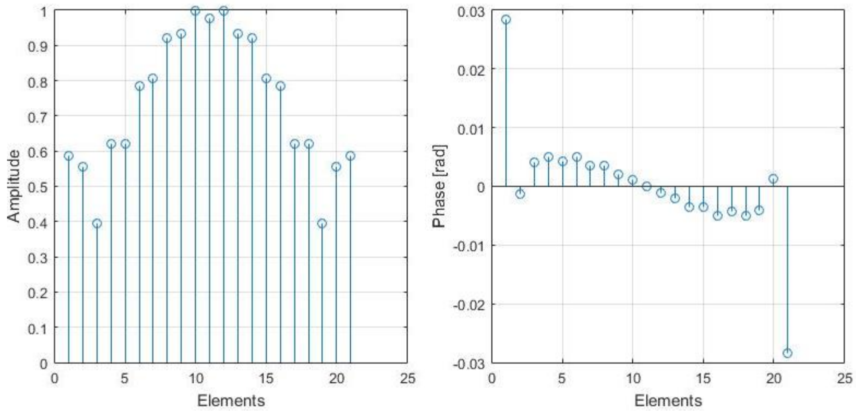


Fig. 2.1-5: Amplitudes and phases of the optimal linear array

Fig. 2.1-6 shows the radiation patterns of the theoretical optimal solution and the simulated pattern evaluated with CST. The agreement between the two patterns demonstrates, once again, the capability of the procedure to design in a fast and deterministic way the optimal solution for the assigned constraints. The CST simulation is performed considering as radiating element a rectangular waveguide working at 30 GHz. The arrays is composed of 21 rectangular waveguides with an inter-element distance of λ/2.

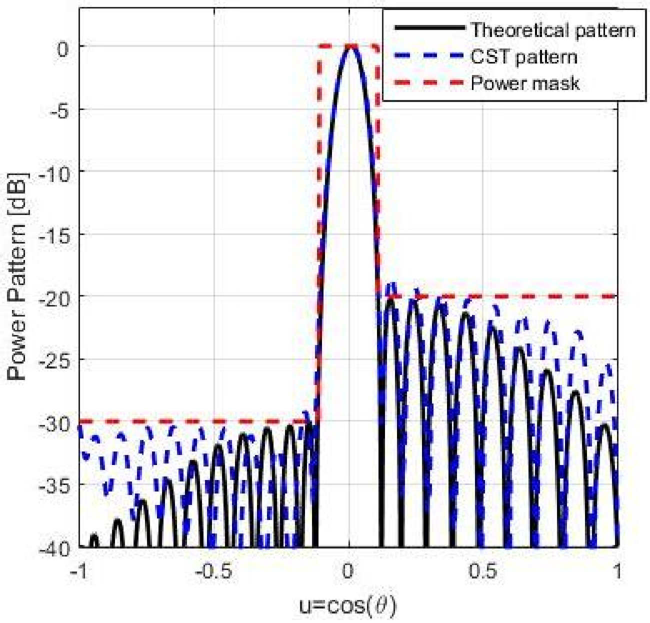


Fig. 2.1-6: Comparison between the theoretical pattern and the CST simulation pattern

2.2 Optimal synthesis of planar arrays with fixed geometry radiating pencil beam

In an equivalent manner as the linear case introduced in the previous section, also for an arbitrary fixed planar arrays geometry the optimal synthesis of a pencil beam pattern with arbitrary sidelobe constraints can be dealt with a Convex Programming problem. In fact, all constraints on the sidelobes are convex in terms of the excitations, and the cost function to be optimized turns out to be convex in both cases where one wants to maximize the field in the target direction [9] as well as in the case one wants to optimize the directivity for a given peak Side Lobe Level (SLL) (for an assigned minimum separation between the peak lobe and sidelobes [11]). As a consequence of convexity inside the problem definition, in both cases, the globally optimal solution can be determined without the need of global optimization techniques.

2.3 Optimal synthesis of linear arrays with fixed geometry radiating shaped beam

Differently from the pencil beam pattern described in the previous section, in the case of a shaped beam pattern the constraints for the power pattern are furnished in terms of lower bound (LB) and upper bound (UB). The UB and LB are functions of the observed angles, and they represent the functions where the power radiation pattern can lay. The upper and lower bound compose the mask power constraints for the desired radiation pattern.

For a linear arrays with fixed geometry the synthesis problem for the shaped beam can be synthesized in the following sentence:

"Define the set of N complex excitations such that

$$LB(\vartheta) \leq |E(\vartheta)|^2 \leq UB(\vartheta) \text{ "}$$

In this case the problem of synthesis and optimization is more complex with respect to the pencil beam case. In fact, the problem is not a linear one in terms of the unknowns but, specifically, it requires to solve a set of non-linear inequalities.

In [12] Isernia and Bucci have defined a general criteria to establish a priori if a given antenna can radiate or not a pattern according to the assigned power mask. To ascertain a priori the feasibility of an assigned power synthesis problem, the mathematical characteristics of a squared amplitude radiated field distribution are exploited.

As can be easy to demonstrate, for a linear arrays with *N* equispaced elements along the z-axis the arrays factor can be represented with the following square amplitude function:

$$P(u) = c_0 + \sum_{p=1}^{N-1}\left[c_p \cos(pu) + s_p \sin(pu)\right] = \sum_{p=1-N}^{N-1} P(u_p) D_{2N-1}(u - u_p) \quad (7)$$

where $u = bd\cos(\vartheta), u_p = \frac{2\pi}{2N-1}$ and d is the separation between adjacent antennas. The formula (7) can be generalized with the following expression:

$$P(\vartheta,\varphi) = \sum_{p=1}^{T}\left[D_p \, \Psi_p(\vartheta,\varphi)\right] \quad (8)$$

Due to the representation of the power pattern of a linear arrays with fixed geometry obtained in (8), it is easy to show which conditions have to be satisfied in order that the pattern respects the power mask constraints. In fact, equation (8) represents all possible patterns radiated from a given class of sources, a necessary condition for the existence of a pattern fulfilling the given constraints is that it satisfies the following system of functional linear inequalities expressed in the variable D_p:

$$\begin{cases} \sum_{p=1}^{T}\left[D_p \, \Psi_p(\vartheta,\varphi)\right] \le UB(\vartheta,\varphi) \\ \sum_{p=1}^{T}\left[D_p \, \Psi_p(\vartheta,\varphi)\right] \ge LB(\vartheta,\varphi) \end{cases} \quad (9)$$

As demonstrated in [39] the function $P(\vartheta,\varphi)$ is a bandlimited function with a band of $2\beta a$ and consequently the equation system (9) can be discretized in a suitable dense manner so that a simplification on the system is obtained:

$$\begin{cases} \sum_{p=1}^{T} \left[D_p \, \Psi_p(\vartheta_i, \varphi_j) \right] \leq UB(\vartheta_i, \varphi_j) \\ \sum_{p=1}^{T} \left[D_p \, \Psi_p(\vartheta_i, \varphi_j) \right] \geq LB(\vartheta_i, \varphi_j) \end{cases} \quad (i = 1 \dots M_1, j = 1 \dots M_2) \qquad (10)$$

The mathematical system (10) is a common linear inequality system expressed in the variable $\mathrm{D_p}$. The solution of this class of problem is well known and it is equivalent to assess the existence of a feasible point for a linear programming problem. As demonstrated, this existence criterion requires the solution of a linear problem which is clearly simpler than the initial system of functional inequalities. It is obvious that solving a system of linear inequalities is faster than solving a system of functional inequalities with the same number of unknowns. Another aspect is that the existence criterion is able to work with any constraints if they are expressed in terms of linear functions of the squared amplitude distributions. Moreover, any type of constraints which is convex with respect to square amplitude distribution can be added at the initial equation system without affecting the possibility to find a solution and defining the feasibility point. Due to the fact that $\mathrm{T} \geq 2\mathrm{C}$, where C is the number of complex degree of freedom of the field, the set of feasible pattern is a subset of the full space determined by (9) and therefore the fulfillment of (10) is usually necessary but not sufficient for the existence of a pattern satisfying the mask constraints. However, there is a special case where the existence criterion is well necessary and sufficient and it is the case of linear array.

Regarding the linear arrays with fixed geometry, the equation (8) can be specialized in the following one:

$$P(u) = \sum_{p=-N+1}^{N-1} \left[D_p e^{jpu} \right] \quad with \; D_p = D^*{}_p \qquad (11)$$

Where the power pattern is a real function and therefore being the $P(u)$ a non-negative trigonometric polynomial and, according to the Fejér-Riesez theorem [40], the $P(u)$, it can be factorized as

$$P(u) = F(u)F^*(u) \qquad (12)$$

Where

$$F(u) = \sum_{p=0}^{N-1} \left[F_p e^{jpu}\right] \qquad (13)$$

which can be considered as an arrays factor of a linear arrays with N radiating elements. Therefore if exists a $P(u)$ as expressed in (11) which satisfies the power mask constraints assigned to the antenna designer, it certainly exists a set of coefficients able to radiate this pattern. An important aspect related to the factorization of the $P(u)$ need to be addressed. In particular, the factorization is not unique and the flipping of the zeros which lying outside the real axis of the complex *u* plane permits to identify 2^{N_0} (N_0 is the number of zeros not lying on the unit circle) distinct sets of coefficients able to radiate $F(u)$.

As described above the existence criterion, for the specific case of linear arrays with fixed geometry, is a necessary and sufficient criterion able to determine if a pattern lying in a specific power mask can be radiated by an arrays of a given size. The described synthesis method can be considered as a modified case of the classical pattern synthesis method based on the "zero location" (see [41]).

The possibility to identify 2^{N_0} possible equivalent solutions gives the opportunity to select the most convenient solution according to some project constraints.

To give a prove of the goodness of this synthesis procedure and to verify the performance an example is carried out.

Fig. 2.3-1 reports the constrained power mask for the synthesis of a linear arrays with fixed geometry, in red the upper bound constraint and in blue the lower bound constraint. The power mask asks to synthesize a shaped beam with:

- a ripple maximum of 1.5 dB,
- a flat region in the angles $-1 \leq u \leq 1$
- -20 dB of SLL and a suppression region in $|1.7 \leq u \leq 2|$ with a SLL of -30 dB
- Inter-element distance of 0.5 λ

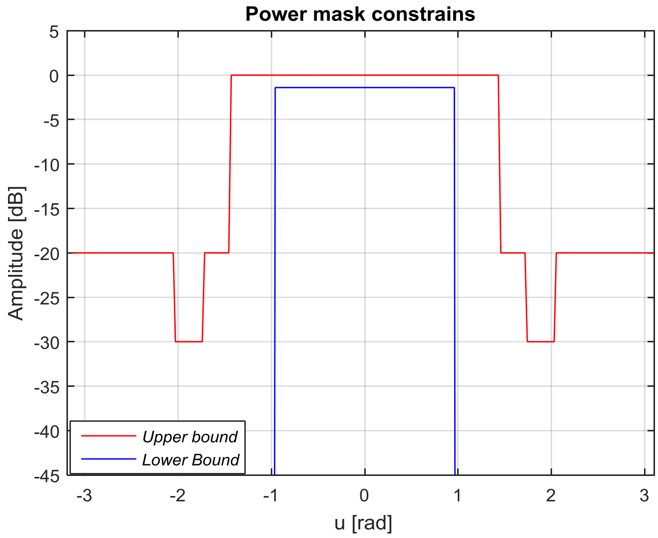


Fig. 2.3-1 Assigned constrained power mask

Performing the existence criterion the power pattern which lying in this power mask is found as shown in Fig. 2.3-2. The criterion permits to define the minimum number of elements in order to satisfy the assigned power mask.

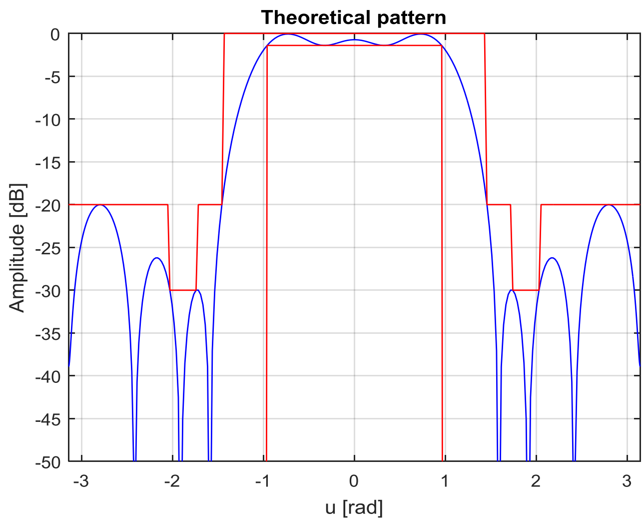


Fig. 2.3-2: Theoretical power pattern satisfying the assigned power mask

As described in the previous part of this section, the theoretical solution is not a unique solution but 2^{N_0} possible equivalent ones are available. For this specific example $N_0 = 10$ and therefore 1024 equivalent solutions are valid to satisfy the request of the problem in terms of square amplitude far field, see Fig. 2.3-3.

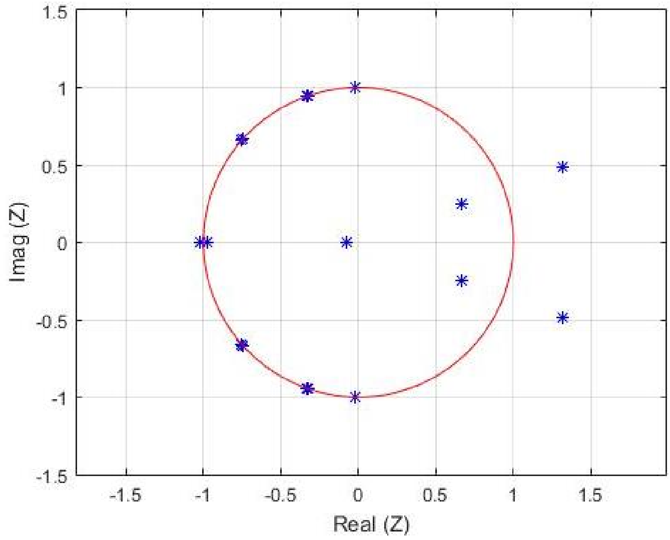


Fig. 2.3-3: Zeros distribution on the complex plane

Tab 2.3-1 reports the set of the synthesized complex coefficients for the linear arrays able to fulfill the constraints.

Id element	Amplitude [dB]	Phase [deg]
1	-4.18	0
2	-1.39	0
3	-0.00	0
4	-1.41	0
5	-7.53	0
6	-5.50	180
7	-4.69	180
8	-12.29	180
9	-9.82	0
10	-7.07	0
11	-18.45	0

Tab 2.3-1: Amplitude and phase of the designed arrays radiating a shaped beam

Fig 2.3-4 reports the radiation pattern calculated by means of a full wave simulation in CST. It can be noted a good agreement between the pattern calculated with the existence criterion and the one calculated with the full wave simulation.

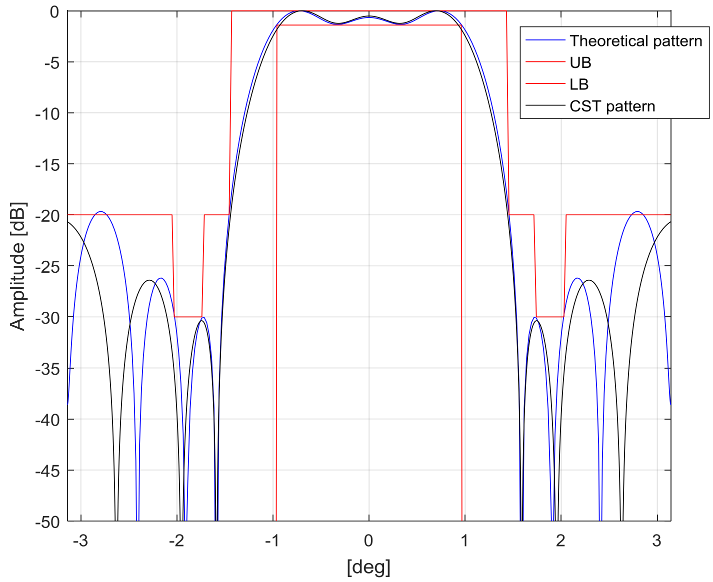


Fig 2.3-4: Radiation pattern of the synthesized array (blue line) and the comparison with the full wave analysis (black line)

2.4 Optimal synthesis of planar arrays with fixed geometry radiating shaped beam

In the previous paragraph the feasibility criterion is introduced for the synthesis of a linear arrays with fixed geometry able to radiate a shaped beam. As demonstrated for the one-dimensional case the feasibility criterion is at the same time necessary and sufficient. In fact the feasibility criterion permits to assess a linear arrays can satisfy the power mask constraints (necessary condition), or not, and then by means of the factorization, it permits to synthesize the complex coefficients (sufficient condition) of the arrays in order to radiate the desired far-field. For the two dimensional case there is an important difference respect the one-dimensional case. In fact for the two dimensional case a factorization rule analogous to the one for 1-D arrays does not exist.

$$P(u,v) = \sum_{q=1-N}^{N-1} \sum_{p=1-M}^{M-1} P(u_p) D_{2N-1}(u-u_p)\left[D_{2M-1}(\mathrm{v}-v_p)\right] \qquad (14)$$

In fact, if the equation (14), which is an explicit form of equation (8) and it represents the power pattern of a NxM equispaced element array, satisfies the power mask constraints the feasibility criterion is necessary. But, at the same time it is not possible to factorize the power patter and therefore a feasibility solution is not available and the criterion is not sufficient.

However, there exists a particular exception: power pattern mask which can be factorized as product of two masks along the principal axis, Fig. 2.4-1. In this case, for each of the principal cuts the procedure presented for the linear case can be used. It has to be noted that in this cases the criterion is sufficient but not necessary. In fact, while it provides a solution to the synthesis, it looks for factorizable excitations, which represent a subset of all the possible ones.

Nevertheless, wherein the sufficiency is not guaranteed, the criterion can be used to discard those problems which are certainly unfeasible.

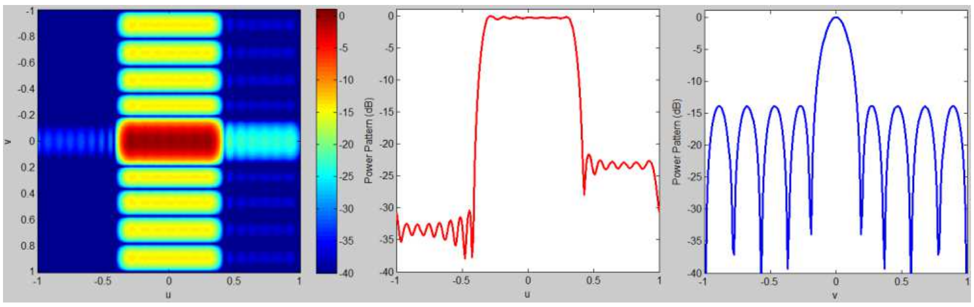


Fig. 2.4-1: Example of factorable pattern: flat-top pattern along u-axis and pencil beam pattern along v-axis

In the "feasible" cases, the pattern furnished by the criterion will be quite certainly not factorizable. However, exploitation of representation (14) allows to state the power synthesis problem in a linear space as small as possible, thus drastically reducing the set of possible patterns with respect to the much larger set of all generic functions compatible with the constraints. In such a way, an effective synthesis procedure can be devised.

To this end, let us note that the operator which relates the unknown excitations to the squared amplitude distribution of the radiated field can be regarded as a quadratic operator, Q, acting on the vector, $\underline{a}$, of the real and imaginary parts of the excitations coefficients. Defining with P the set of realizable power pattern furnished from the existence criterion. The synthesis procedure can be seen as the minimization of the distance between $Q(\underline{a})$ and the convex set of patterns P

$$\Theta(\underline{a},\underline{D}) = ||Q(\underline{a}) - \sum_{p=1}^{T} D_p \, \Psi_p(\vartheta,\phi)\, || \qquad 14.c$$

wherein $\underline{D}$ is the vector of the D_p coefficients.

2.5 Linear arrays with fixed geometry synthesis radiating reconfigurable beam

The design of a single antenna aperture able to radiate different kind of beams is a very interesting problem in the sector of the antenna synthesis due to the large number of application from remote sensing, radar antenna or satellite communications. The satellite communication is the principal field of application where the request of reconfigurable single aperture antenna is demanding. In fact, the satellite operators increase in the last years the request of reconfigurable beams in order to adapt the beam dimension in according to the traffic load in a specific area of the Earth as shown the Fig. 2.5-1 where large beam and very small beam are used in order to contrast the load traffic requests.

As described in the introduction of this thesis, among the different kind of antenna technological solutions, the phased arrays antenna is the solution which permits to implement this kind of flexibility because they can be controlled by means of a completely electronic technique and in a very fast manner. The problems which affect the phased arrays is that regarding the complexity in the BFN (beam forming network) design. In order to maintain the cost and the complexity of the phased arrays architecture, a solution is to realize the reconfigurability acting only in the phase, the excitation sets corresponding to the various patterns differ only in their phase distributions. This has a great advantage because new hardware is not required and, moreover, using in the BFN a single power-divider network it makes the solution cheaper and more efficient than those which dynamically modify the amplitude of the weight coefficients.

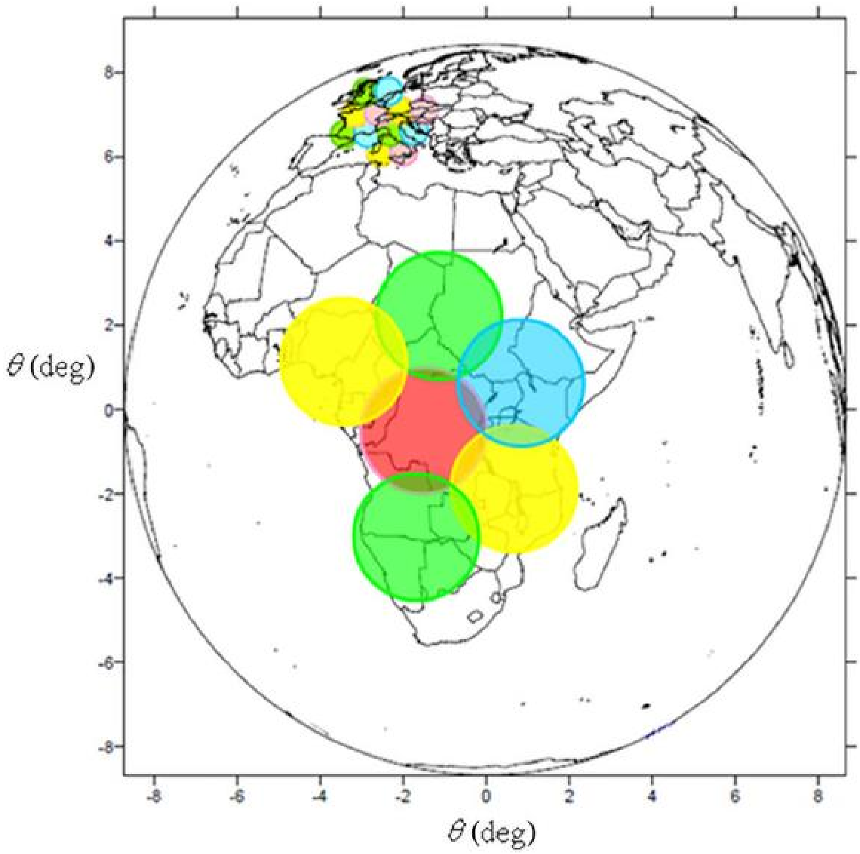


Fig. 2.5-1: Multibeam coverage with reconfigurable beams

The main drawback of the only-phase reconfigurability concerns the intrinsic difficulty in solving the corresponding synthesis problem in an optimal fashion, i.e., in fulfilling given design goals by exploiting the minimum number of elements or optimizing given performances for a fixed number of elements. Indeed, the 'common amplitude' requirement on the excitations results in non-linear and non-convex constraints, which implies considerable additional difficulties in the development of effective synthesis procedures.

In the literature, the problem has been approached both using the so-called 'alternating projections' technique [42]-[44] and exploiting global optimization strategies [45]- [49]. Because of the non-linear constraints and the arising non-convex sets involved in the alternating projection technique, the optimization procedure can be trapped into local minima far from the actual optimal solution. Differently, for 'global optimization' algorithm the computational cost increases with the problem size.

In the recent years A. F. Morabito et al. proposed in [50] an effective approach for the linear arrays only-phase reconfigurable antenna making use of only deterministic procedure. The approach benefits from theoretical results and optimal solutions available in the separate synthesis of pencil and shaped beams with fixed geometry introduced in the previous section of this chapter. This approach is very interesting to face in order to find a first deterministic valid strategy which will be modified to solve the problem of the synthesis of isophoric sparse only-phase reconfigurable array, which is the subject of this thesis work.

Recalling some results for the synthesis of linear arrays radiating pencil beam or shaped beam with fixed geometry, it is important to underline that the optimal synthesis of a pencil beam subjected to arbitrary sidelobe constraints on the power pattern can be formulated as a Convex Programming problem. In fact, all constraints on the sidelobes are convex in terms of the excitation coefficients, and the cost function to be optimized is also convex in both cases where one wants to maximize the field in the target direction and in the case one wants to optimize the directivity for a given SLL peak. As a consequence of convexity, in both cases, the globally optimal solution can be determined without the need of global optimization techniques.

For the shaped beams case where the power pattern distribution is subjected to both upper and lower bounds , the synthesis can be effectively performed in a globally optimal fashion by means of efficient local optimization strategies. Moreover an important aspect for the shaped beam is that the found excitation is not unique, and a number of different possible solutions as large as 2^{N_0}, N_0 is the number of zeroes of the Schelkunoff polynomial not lying on the unit circle, can be determined through the so called 'zero-flipping' procedure. As a consequence, the set of possible solutions to the synthesis problem is not constituted by a single point. After that a reference power pattern has been adopted, distinct solutions exist in terms of arrays excitations.

According to this results the strategy proposed in [50] carries out the following steps:

- Perform the separated optimal synthesis of the required pencil beam which entails to find a unique set of antenna weights
- Perform the separated optimal synthesis of the required shaped beam which implies to find a set of complex weights
- Find from the set of complex excitations of the shaped beam the excitations such that the amplitude distribution is as near as possible to the one corresponding to the solution of the pencil beam synthesis problem

The last step is the crucial and innovative part of this strategy. If $\underline{a}^{S,k} = (\underline{a_0}^{S,k} \dots \dots \underline{a_{N-1}}^{S,k})$ and $\underline{a}^{P} = (\underline{a_0}^{P} \dots \dots \underline{a_{N-1}}^{P})$ define the k-th possible excitation set for the shaped beam pattern and the unique excitation coefficients for the pencil respectively, the approach is to minimize over the 2^{N_0}different possible values of k the possibly weighted distance amongst the excitation amplitudes of the pencil and shaped beam:

$$\Phi(k) = \left\| |\underline{a}^{S,k}| - |\underline{a}^{P}| \right\| \qquad (15)$$

The set of excitations which minimize (15) will the one more easily leading itself to be reconfigured from the shaped to the pencil beam (see Fig. 2.5-2).

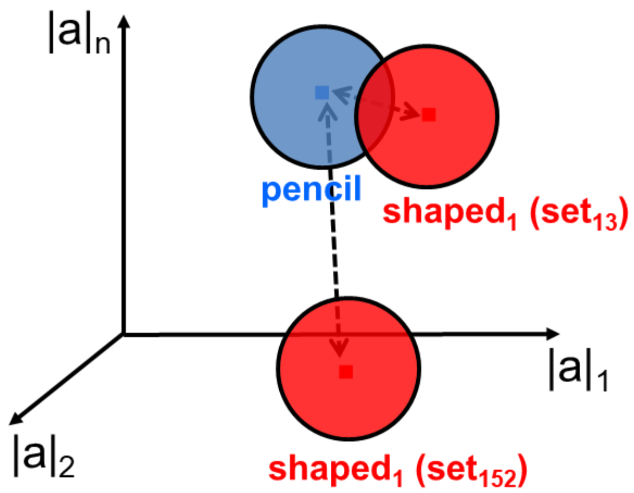


Fig. 2.5-2: Representation of the admissible solutions on space of all possible amplitude distributions

Once the minimum of (15) is determined, a local optimization technique can be used to perform the actual choice of the excitations common to both operative modes. One possible choice is to exploit an alternating projections procedure which acts here just as a final local optimizer.

To understand better the second and third step of the procedure an example is furnished.

Considering the only-phase reconfigurable fixed geometry arrays which radiates a pencil beam and a shaped beam. The shaped beam pattern, reported in Fig 2.5-3, is a cosecant power pattern.

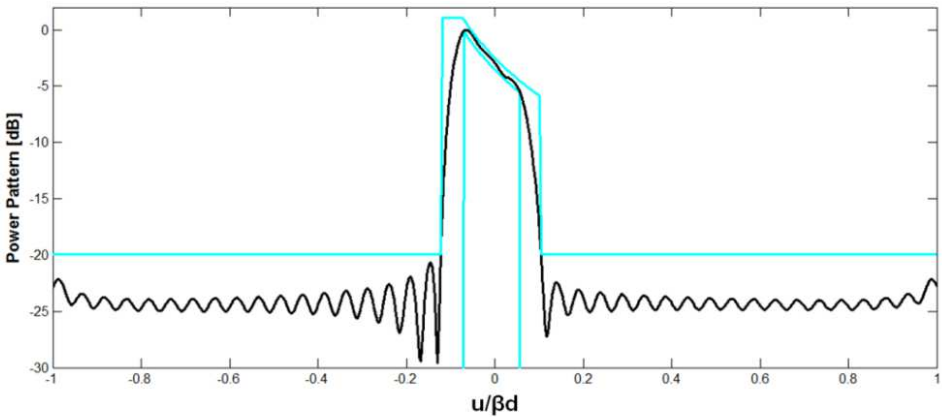
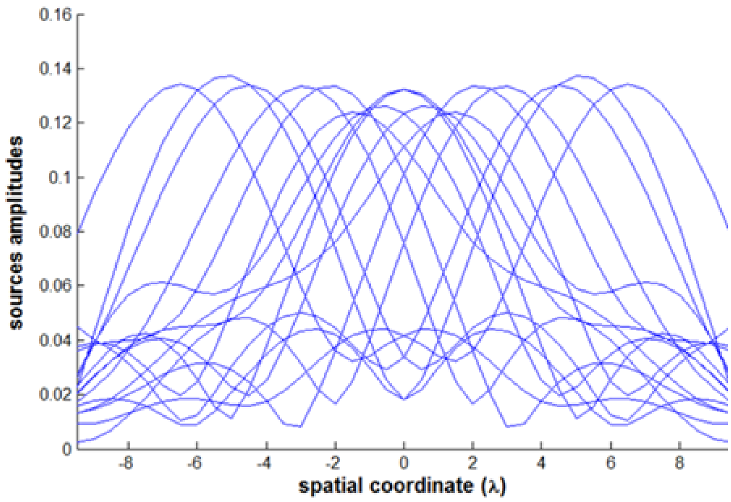
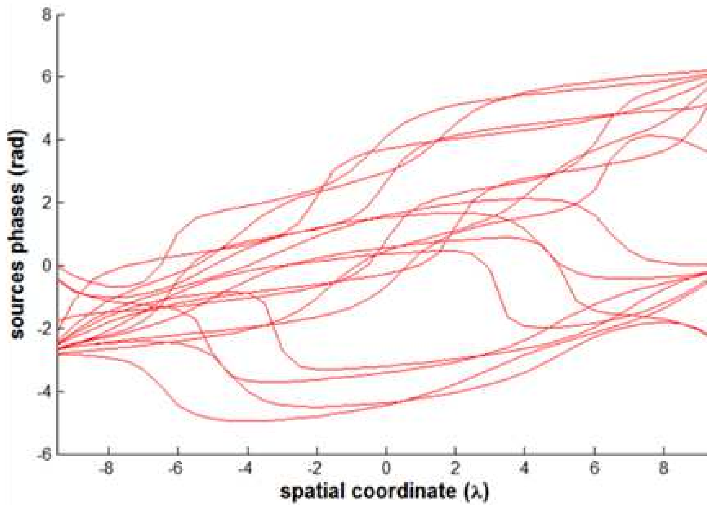


Fig 2.5-3: Shaped beam radiated by a fixed geometry array

Applying the feasibility criterion first, the factorization and flipping of the zeros it is possible to define all the admissible excitation distributions in terms of amplitude and phase. For this specific example the complete set of complex distributions which permit to fit the assigned mask power pattern for the shaped beam are shown in Fig 2.5-4.



(a)



(b)

Fig 2.5-4: The complete set of admissible amplitude (a) and phase (b) distribution

Regarding the pencil beam a unique real solution is admissible to fit the power mask constraints. Fig. 2.5-5 shows the starting point for the optimal choice of the only-phase reconfigurable distribution. From the complete set of the complex distributions of the shaped beam case only the one which is nearest to the amplitude of the pencil beam is selected.

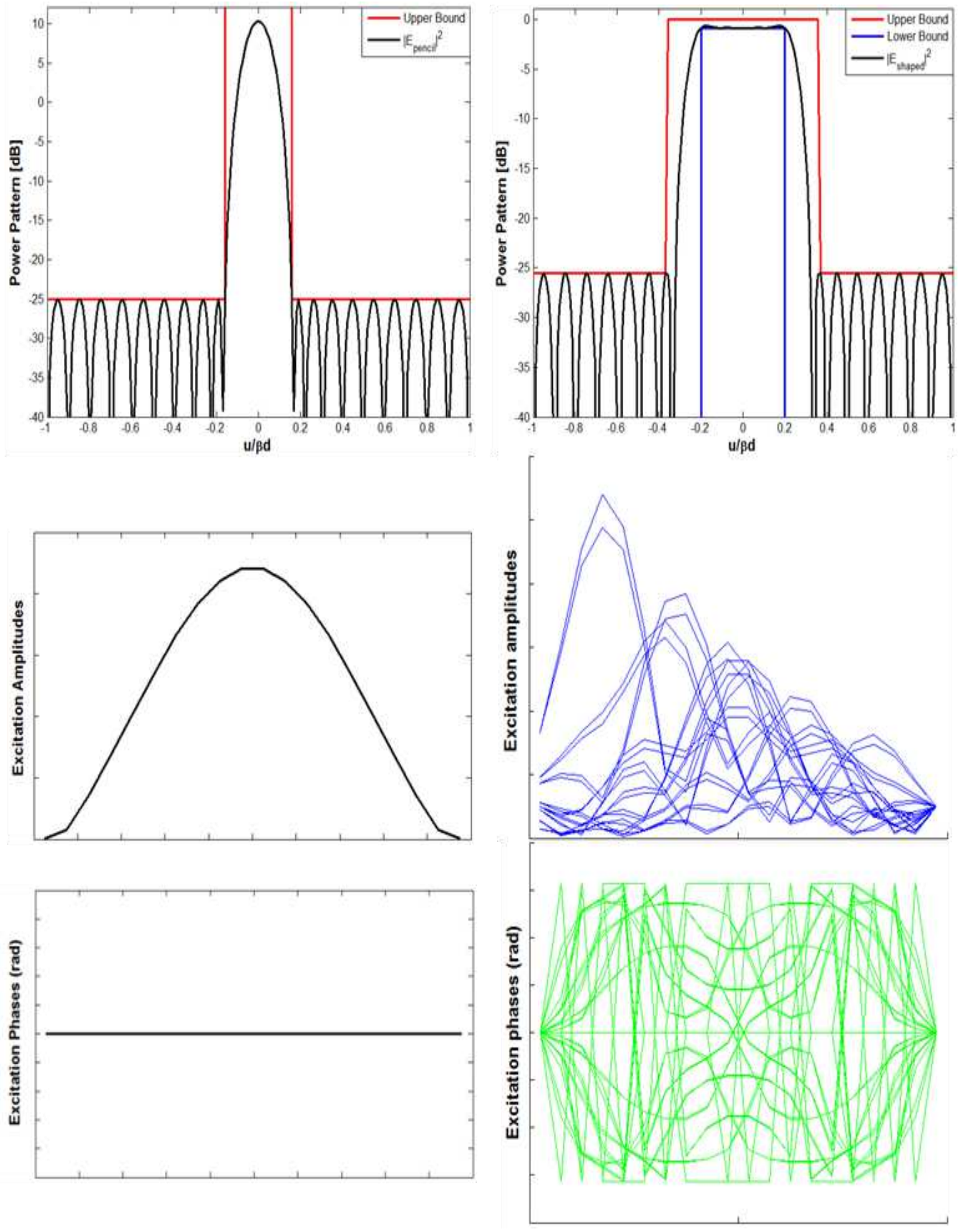


Fig. 2.5-5: Complete set of complex distribution which fit the shaped beam power mask and the unique real solution for the pencil beam pattern

Fig. 2.5-6 reports the two nearest amplitude distributions for the two operative modes: blue line is for the shaped beam and dark line for the pencil beam.

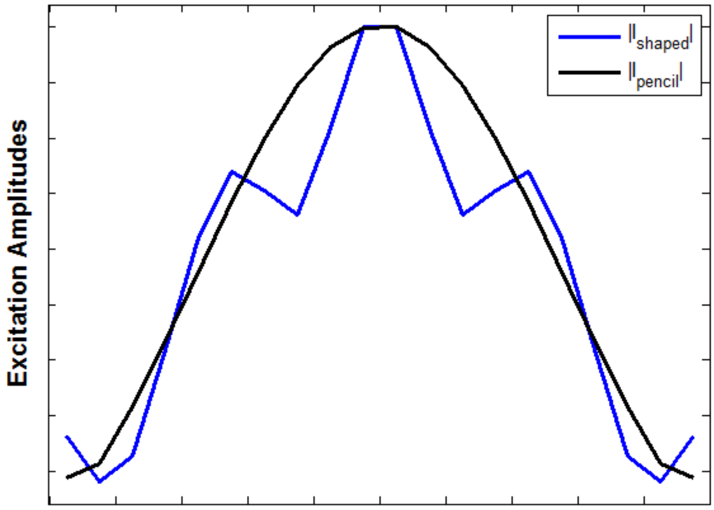


Fig. 2.5-6: The nearest amplitude distribution for the shaped operative mode (blue line) and the pencil beam (dark line)

Acting a local optimization on the two selected amplitude distribution a final and unique amplitude distribution is used to find the excitations of the fixed geometry linear array.

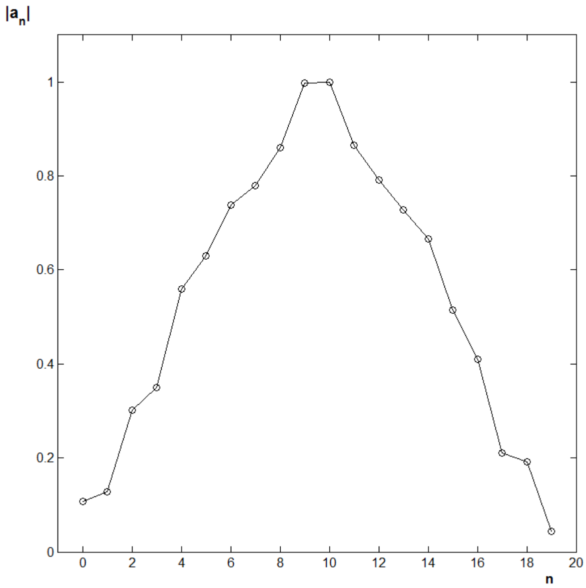


Fig. 2.5-7: Final amplitude distribution for the two operative modes

After the definition of the amplitude distribution for the linear arrays the reconfigurability is obtained changing the phases in according to the operative mode desired by the operator. The two phase distributions are shown in Fig. 2.5-8 wherein blue there is the phase distribution for the shaped beam operative case while in red the phase for the pencil beam case (which is obliviously equal to zero).

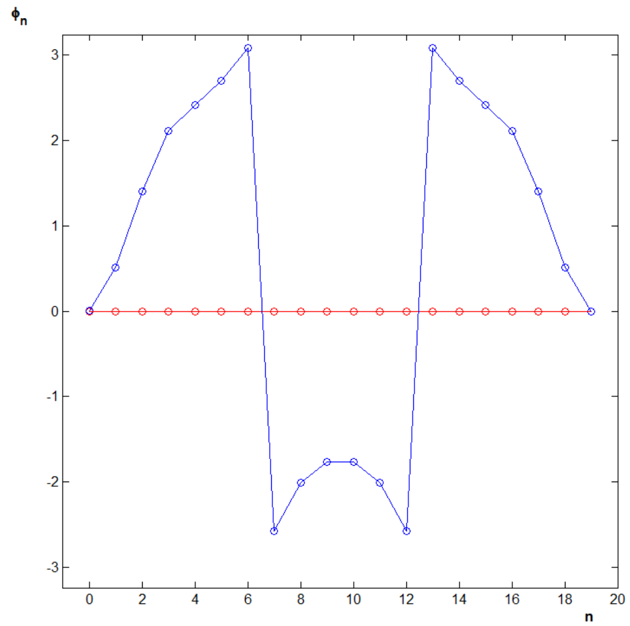


Fig. 2.5-8: Final phase distribution for the two operative modes: blue the phase for the shaped beam and in red the phase for the pencil beam

This technique is a general approach and it can be used in order to create several cases of reconfigurable beams; an antenna which radiates pencil beams and shaped beam, an antenna which radiates two different shaped beams or antenna which radiates more than two patterns of general shape (more details in [50])

2.6 Planar arrays with fixed geometry synthesis radiating reconfigurable beam

As demonstrated in the previous paragraph a deterministic and fast approach for the synthesis of a linear array with fixed geometry radiating a reconfigurable beam exists. The fundamental aspect of the synthesis procedure is the possibility that the shaped beam pattern can have a multiplicity of solutions in terms of complex coefficients to radiate the same desired power pattern.

In some particular cases this approach can be extended to the case of planar arrays, the circularly symmetric array, as demonstrated in [50]:

i. Planar arrays with symmetries which can be dealt with as one-dimensional polynomials
ii. Planar arrays having a factorable pattern

If the planar array does not exhibit one of these two conditions, then the factorization and the relative flipping of the zeros are not applicable and the proposed approach cannot be used.

The approach based on global optimization algorithm represents the solution for the general case of planar arrays with fixed geometry radiating a reconfigurable beam.

2.7 Isophoric sparse direct radiating array

In the first part of this chapter the deterministic synthesis for linear and planar arrays radiating pencil, shaped or reconfigurable beams when the geometry is fixed in advance is dealt with. As discussed in the first part of this thesis work, the deterministic power mask constrained synthesis of fixed-geometry arrays is the more simple case to manage because the far field has a linear relationship with the unknowns, the excitation coefficients of the array. After the case of fixed geometry the problem is to consider if it is possible to define a deterministic strategy for the synthesis of an isophoric sparse array. The isophoric sparse arrays are a particular class of arrays which is very appealing in the satellite communications.

Isophoric arrays mean that all the radiating elements of the arrays are fed with the same level of signal, see Fig. 2.7-1. This condition can introduce important advantages for an antenna which works in a hard environmental condition where the management of the resources, as the payload delivered power, is a critical aspect. In fact, in the isophoric arrays the same class of amplifiers is used and they work all at the same level of power and consequently there is an optimal DC to RF conversion. Moreover, the BFN which controls the array can be simplified because only simple equal amplitude T dividers are required.

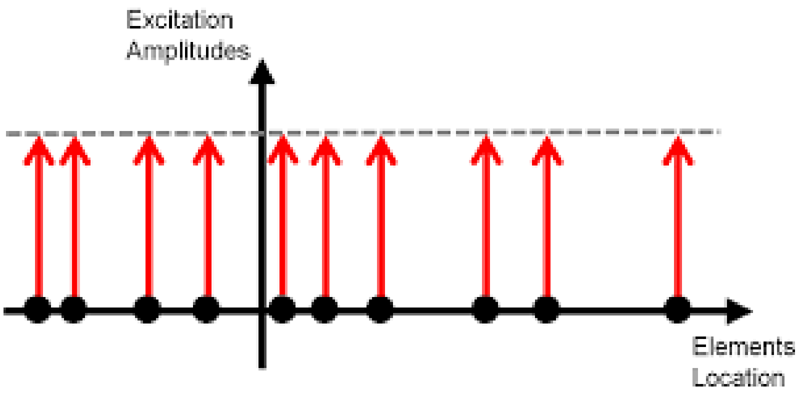


Fig. 2.7-1: Representation of the isophoric sparse array

Another important advantage for this class of arrays is the sparsity in the geometry architecture of the radiant panel. Sparsity means that radiating elements are not uniformly distributed on the antenna aperture, regular lattice, but they are non-equally spaced creating an aperiodicity on the distribution, irregular lattice. The use of sparse arrays introduces some important consequences. At the first the number of radiating elements is smaller than the uniform equally-spaced arrays and therefore the number of active control points is reduced. This aspect entails a reduction of the manufacturing costs, of the mutual coupling inside the antenna aperture (due to the increase of the inter-element distances) and a reduction of the antenna weight and mass. Another important advantage of the sparse arrays is the capability to avoid the appearance of grating lobes which are present in the uniform array with an inter-element distance greater than $\lambda/2$ (see Fig. 2.7-2).

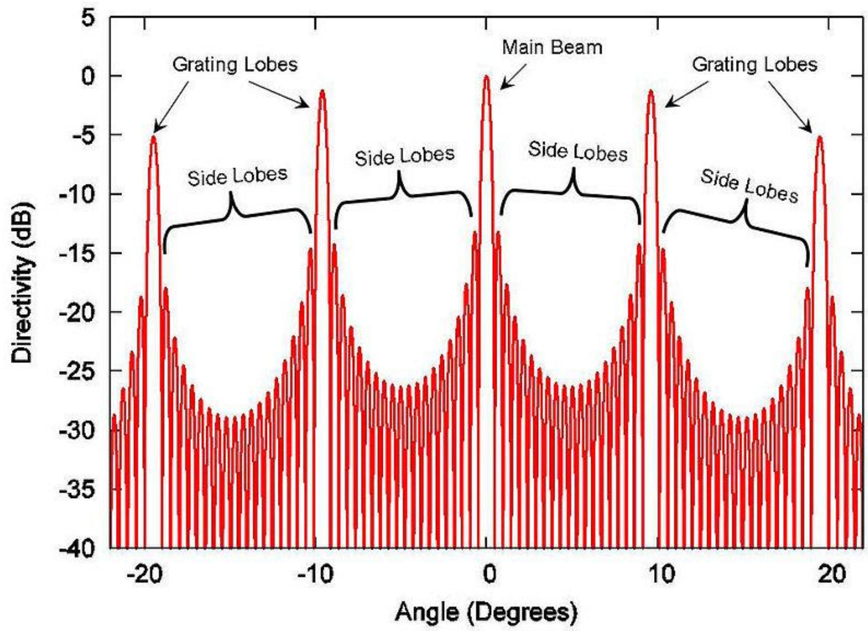


Fig. 2.7-2: Gratin lobes for an array with regular lattice and an inter-element distance greater than half wavelength

The sparsity introduces also some drawbacks as the reduction of the antenna aperture efficiency and consequently the reduction of maximum achievable directivity([51]-[56]). In some cases this drawback can be mitigated using the sub-arrays or element size taper techniques ([57], [58]).

Nevertheless, the synthesis of isophoric sparse arrays is much more difficult than the synthesis of periodic arrays or not isophoric ones. The difficulty of this synthesis is related to the non-linear dependency of the power pattern with the locations of each radiating element which are the unknowns of the synthesis problem as demonstrated by the following formula of the square amplitude of the arrays factor.

$$|AF(\vartheta)|^2 = \left|\sum_{i=1}^{N} w_i e^{-ik*r_i}\right|^2$$

Moreover the set of the constraints of the synthesis problem represent a non-convex set (when a lower bound is applied to the gain) and therefore the problem is a non-linear one in a non-convex set of constraints. Another difficulty of this synthesis is due to the dimension of the array; large arrays involve a higher number of unknown. To overcome this aspect local optimization cannot be used because they may be trapped in a local minimum which is far from the optimal one, and the global optimization suffer the high computational cost.

To tackle these difficulties a deterministic approach constituted by two distinct steps is introduced. This method permits to reach the optimal solution in a deterministic manner and consequently it outperforms the global optimization approaches ([15], [62]-[64]) in terms of computational cost.

This dual step approach takes advantage of the density taper technique proposed by Doyle in [59], reported in [2] and optimized in [60]. The two steps of this technique are:

- Definition of an optimal continuous source able to fulfill the constraints in terms of power mask
- Definition of a fast and deterministic method to discretize the optimal continuous source found in the previous step.

This deterministic procedure permits to circumvent the non-linear problem for the synthesis of a uniform excited sparse array and it is characterized by several advantages. At first, the synthesis of a continuous source without the excitation constraints is simpler than that of a sparse isophoric arrays, in fact, in this circumstance there is a linear dependency between the radiated far field and the aperture distribution. At second, if lower bound on the gain constraints are imposed, for a sufficiently narrow beam, as that of the satellite communication case, the synthesis of the corresponding continuous source can be carried out by means of the minimization of a closely convex functional under convex constraints.

This procedure works with the linear isophoric sparse arrays but also with the isophoric planar arrays. The extension to the planar ring is due, at first, to Milligan in [65] where the density taper is carried out along the radial direction using the Doyle approach which allows locating a number of fixed a-priori ring. In [18], Bucci developed a deterministic new approach that permits to exploit a Doyle-based rationale for determining the entire concentric circular ring geometry. All the above-mentioned approaches are able to work with continuous sources which are real (i.e only for the pencil beam pattern at the boresight), but are not able to perform the deterministic optimal discretization of a complex continuous source(i.e. optimal design of shaped beam). The aim of this thesis is to cover gap. As described in the introduction, this thesis introduces a *deterministic optimal design of shaped beams* by means a *discretization of the complex continuous source*. Moreover, also the *reconfigurability in a fast and deterministic way* can be achieved exploiting this discretization method.

To fulfill these objectives, the following chapter of this work foreseen to give attention to the design of the optimal continuous sources able to radiate pencil beams, shaped beams and reconfigurable beams (the last one is an innovative contribution of the thesis). In the fourth chapter the optimal synthesis of isophoric sparse arrays (ISA) by means of a discretization of the continuous source is treated in an exhaustive manner. The philosophy of the next two chapters is to introduce at first the literature results (which are the bases for the novelty of the thesis) and then to refer to the results carried out in this PhD research activity. Several numerical examples are furnished to support the theoretical aspects.

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Synthesis of circularly symmetric continuous sources radiating pencil beams, shaped beam and only-phase reconfigurable beams

3.1 Introduction to the synthesis of circularly symmetric continuous sources

The problem of the synthesis of circularly symmetric continuous sources able to fulfill the prescribed constraints is a topic argument in the antenna theory as demonstrated by [66] and [67]. In fact, as discussed in the previous chapter, the design of continuous source has practical utility as to state about the maximum performance of an antenna aperture with a given size and, moreover, the continuous source can act as a reference for the synthesis of the arrays antennas.

In the past, the Taylor continuous distribution was the principal reference for the antenna synthesis despite some limitations. In fact, it is not able to take into account possible notches in the far field mask, or aperture fields wherein a hole has to be present at the center of the structure. To overcome these issues and to have the desired flexibility in the continuous source, Bucci – Isernia – Morabito ([68],[69]) introduced in the last years a full theory for the optimal deterministic synthesis of circular symmetric continuous source able to radiate a pencil beam or a shaped beam. Exploiting these results, the thesis will expand the theory of the concentric symmetric continuous source to the case of antenna aperture able to reconfigure the radiation pattern by means of the only-phase reconfigurability.

The chapter is organized as follows. The first part is dedicated to the optimal synthesis of circularly symmetric source able to radiate the pencil beam; the second one is dedicated to the synthesis of continuous aperture radiating the prescribed shaped beam and the last part is concerned to the synthesis of a circularly symmetric source able to perform the only-phase reconfigurability of the radiation pattern.

3.2 Optimal synthesis of a circularly symmetric source radiating pencil beam: statement of the problem and design procedure

In this section the focus is on the optimal synthesis of circularly symmetric source radiating a pencil beam, a pattern able to guarantee a minimum level of directivity in an angular sector (spot beam) and to ensure a sufficiently low directivity level in any other directions. The problem can be considered as to find an aperture distribution with a circular symmetry able to maximize the minimum directivity inside a fixed angular sector and, at the same time, ensuring elsewhere a prescribed upper mask for the sidelobes level. The problem, which will be dealt with, concerns to the case of pencil beam pointing in the boresight direction.

Assuming ϑ as the spanning angular variable from the boresight, let ϑ_0 be the angle corresponding to the edge of the coverage (EOC), the sector where the directivity must not to be lower than a desired value, and ϑ_1 ($> \vartheta_0$) the angle such that for any $\vartheta > \vartheta_1$ the pattern will be constrained to be lower than a desired value (see Fig. 3.2-1).

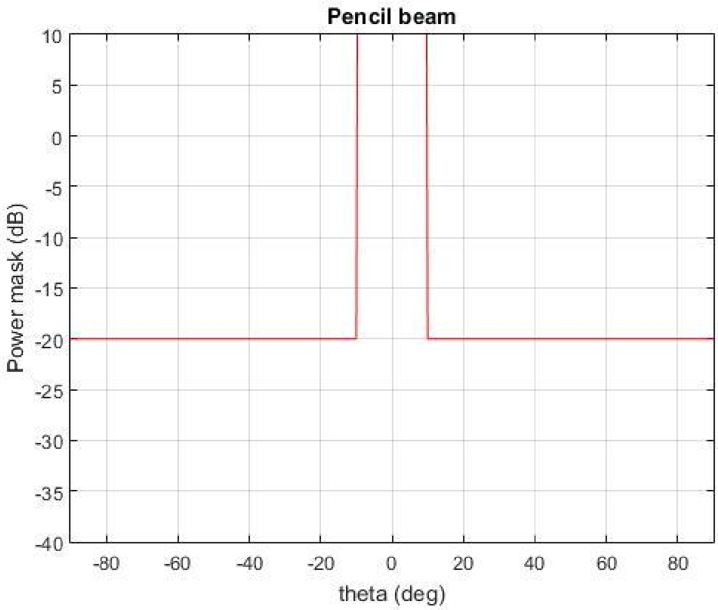

Fig. 3.2-1: Example of pencil beam power mask with a sidelobe of -20 dB

Denoting with R the maximum aperture radius, with $f(\rho)$, ρ is the normalized radial coordinate, the aperture distribution function and with

$$E(\vartheta) = \int_0^1 f(\rho) J_0\big(\rho\beta R sin(\vartheta)\big)\rho d\rho \qquad (15)$$

the associated far-field where J_0 is the Bessel function of the first kind order zero, and β the free-space propagation constant. The problem of the synthesis can be formulated in the following manner:

Find an aperture distribution $f(\rho)$ such that the associated radiating power pattern

$$\int_0^{\frac{\pi}{2}} |E(\vartheta)|^2 \, sin(\vartheta)\, d\vartheta \qquad (16)$$

Is minimized, while the field satisfies the constraints

$$\begin{cases} |E(\vartheta)|^2 > 1 \quad for\ \vartheta \leq \vartheta_0 & (17) \\ |E(\vartheta)|^2 \leq c(\vartheta) \quad for\ \vartheta \geq \vartheta_1 & (18) \end{cases}$$

where c(ϑ) denotes the sidelobes upper mask normalized to the value of the EOC directivity.

In the previous formulation the problem is ill-posed due to the compactness of the radiation operator of (15) and therefore a regularization is required. In this case the SVD (Singular Value Decomposition) is the technique of regularization which permits to remove the ill-posed condition([70]). However, in this case of interest regarding large focusing aperture, the approach of reaching a minimal norm solution, a distribution fulfilling the constraints (17) and (18) and minimizing the functional:

$$\int_0^1 |f(\rho)|^2 \rho d\rho \qquad (19)$$

instead of the radiation power (16) can be adopted. In fact, for focusing sources of large size and directivity, even a small increase of the directivity with respect to that of the minimal norm solution leads to a dramatically large, not physical, increase of the amplitude and variation of the aperture distribution. Accordingly, looking for nonsuperdirective sources, the problem becomes that of minimizing the squared norm (19), which is a strong convex functional, under constraints (17) and (18). The constraint (18) defines a convex set, while constraint (17) is a non-convex one. However, if it assumes that in such an interval the minimum of the field intensity occurs at the EOC, and taking into account that one is minimizing (19) and that a multiplication of the field by an arbitrary complex constant does not affect the requirements, constraint (17) can be replaced by the convex condition

$$E(\vartheta_0) = 1 \qquad (20)$$

and the problem turns in a Convex Programming (CP) one. Now, being (19) strictly convex, it admits a unique solution (unless the constraints are so strict to prevent the existence of a solution). Also, because functional (19) and conditions (20) and (18) are invariant under conjugation and the kernel of (15) is real, it is easy to verify that the unique optimal aperture and far field must be real. If such a solution satisfies also the original constraint, it is clearly that it is the desired solution, otherwise condition (17) needs to be enforced. In this last case, if the solution is forced to be real, the problem remains convex, but the solution is not optimal. In this case, it can be looked for the real solution of the 'relaxed' problem, which is a simple quadratic programming one, verifying a posteriori the fulfillment of constraint (17).

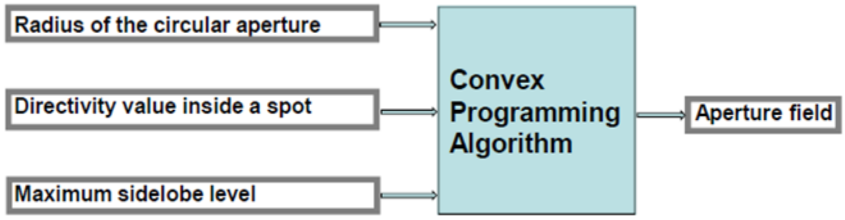

Fig. 3.2-2: Synthesis procedure of the continuous source radiating pencil beam

It can be concluded that in the case of high directivity pencil beams, the optimal synthesis problem can be reduced to a convex optimization problem, which admits a unique, real optimal solution. The same conclusion stays valid if further convex constraints are enforced besides (20) and (18) as for example constraint in the dynamics in the aperture field.

Fig. 3.2-2 shows the synthesis procedure in a build box diagram where the input are the radius of the desired aperture, the minimum directivity inside the spot and maximum sidelobes level. Others convex constraints can be included according to the desired design of the antenna engineer.

3.3 Optimal synthesis of a circularly symmetric source radiating shaped beam. The proposed procedure

As previously described, the synthesis of circularly symmetric aperture continuous sources able to radiate far fields fulfilling given constraints is a typical inverse problem in antenna theory. The identification of such aperture fields furnishes an assessment of the possible performance achievable by an aperture antenna of given size and at the same time it is a reference distribution for the synthesis of arrays antennas ([65-68] and [71-72]). A deterministic approach to the optimal synthesis of continuous aperture sources for the pencil beam case has been proposed in the previous paragraph taking inspiration from [68]. Convex Programming (CP) has been exploited to define circularly symmetric aperture distributions such to maximize the corresponding minimum directivities in the spot beam, while ensuring the fulfilling of the prescribed upper masks for the sidelobes.

In this paragraph the attention is focused on the optimal synthesis of circularly symmetric aperture able to radiate a shaped beam, that is, a pattern lying in a given power pattern mask constituted by two distinct arbitrary functions, one for the upper bound and the other one for the lower bound, in all the visible domain. Moreover, in order to guarantee both the physical feasibility of the source, as well as bandwidth and gain performances, it is required that the spectral content of the source is negligible in the invisible part of the spectrum. Note this requirement automatically excludes the occurrence of the so-called 'super-directive' sources.

Shaped patterns are required in many applications, from telecommunications to remote sensing, for satellite antennas, WLAN and radar applications, or even base transceiver stations for mobile communications. In the most part of the available literature, the power synthesis of this kind of patterns is carried out by looking for a nominal far field pattern or by means of global optimization schemes. The global optimization algorithm are extremely heavy from the computational point of view, so that they can not be applied in case of a large number of degrees of freedom.

In order to take profit from 'ideal' continuous sources to design 'optimal' arrays, Bucci-Isernia-Morabito ([69]) proposed a deterministic and straightforward solution strategy. This solution does not require the exploitation of global optimization techniques, and, also allows one to ascertain a priori, excluding superdirective sources, the feasibility of given mask constrained power pattern synthesis problems (for given dimensions) and it guarantees the identification of the globally optimal solutions (if a solution exists at all). The proposed synthesis procedure is able to give several alternative optimal solutions to the same problem, so that one can adopt additional performance parameters in order to choose the most convenient one.

In order to synthesize shaped power patterns by means of circularly symmetric continuous aperture distributions of minimum size, the effective and convenient procedure can be summarized in the two main steps below:

1. Once an arbitrary power pattern mask has been assigned, find the minimum aperture radius which is needed for the existence of a continuous aperture source able to fulfill the requirements. Moreover, determine the slice (i.e., the elevation cut) of (one of) the power patterns fulfilling constraints;
2. perform a suitable factorization of the above one-dimensional power pattern (i.e., of the power pattern along the slice), and determine the corresponding aperture field distribution by applying the inverse zero-order Hankel transform [73] to the resulting far field (so that one goes back from the slice to the overall circle).

The whole design procedure consists in a Linear Programming (LP) [74] algorithm and in a number of analytical transformations exploiting Abel and Fourier transforms [73] and both space-band limitedness [75] and polynomial representations [12, 40] of the square amplitude radiated fields.

The two design steps are discussed in the following. At the first, the Q ratio factor is needed to be introduced in order to control the feasibility of the solution, and it is defined as the ratio between the reactive power (P_r) and active power (P_a) associated respectively to the invisible spectrum and the effective radiated power:

$$P_r = \int_1^{\infty} P(u)udu \qquad (21.a)$$

$$P_a = \int_0^{1} P(u)udu \qquad (21.b)$$

$$Q = \frac{P_r}{P_a} \qquad (22)$$

A Q factor with a value more than 1 corresponds to a super-directivity source, and therefore during the design of the aperture source the scope is to reduce the amount of power associated to P_r in order to maximize the directivity of the continuous aperture.

Defined the maximum value of Q_{max} (the maximum allowable Q factor), the first step of the above mentioned synthesis consists of find the minimum aperture diameter for which a pattern fulfilling the mask does exist. To perform this task the problem for the synthesis of the shaped beam reported in section 2.3 needed to be solved iteratively increasing the dimension of the source until a solution is found, therefore:

Minimize

$$\int_0^1 P_0(u)udu \qquad (23)$$

Subject to

$$LB(u) \leq P_0(u) \leq UB(u) \quad for\ 0 \leq u \leq 1 \qquad (24.a)$$

$$\sum_{n=-N_0}^{N_0} p_n x^n \geq 0 \qquad for -1 \leq x \leq 1 \quad (24.b)$$

$$\int_0^{u_0} P_0(u)udu \quad \leq \ Q_{max} \ \leq \int_0^1 P_0(u)udu \qquad (24.c)$$

Where P_o is the power pattern of a virtual linear arrays with equispaced $N_0 + 1$ elements which acts as the power of the continuous source, P_o is the discretized version of the ideal power pattern P (detailed information in [69]). The p_n coefficient are the auxiliary unknown of the problem that permits to consider the problem as a linear programming one.

An important aspect in the synthesis of circularly symmetric continuous aperture source is that the maximum achievable directivity is not only dependent on the aperture size but also on the beam shape. The scope of the synthesis is to find a trade-off between the minimum size and maximum achievable directivity. In order to have a full comprehension of the role of the Q factor and of the antenna size an example is carried out in the following. It is required to design a circularly symmetric continuous aperture source radiating a shaped beam with the following characteristics:

- Flat top beam shape
- Maximum allowable ripple ±0.25 dB in the region -0.25≤u≤0.25 deg
- Maximum sidelobe level -35 dB for $|u| \geq 0.35\ deg$

Fig. 3.3-1 shows the constraints in terms of power pattern mask for the synthesis of the required shaped beam. In this figure the upper and lower bound functions and the visible space limit are reported.

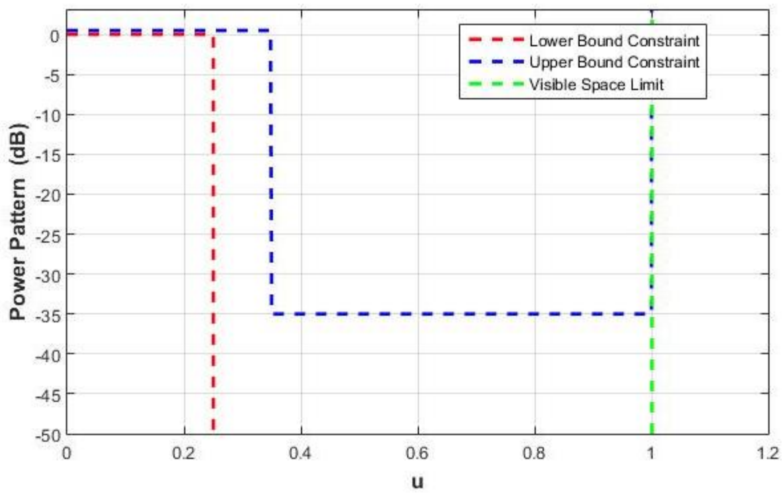

Fig. 3.3-1: Power mask for the synthesis of the continuous source

Solving iteratively the optimization problems reported in equations (23)-(24), it means to change the dimension of the antenna aperture, and calculating for each case the Q factor, the following results are collected, see Fig. 3.3-2.

As it can be noted the Q factor decreases exponentially with the diameter D, about 40 dB per wavelength, and therefore the transition between unfeasibility and feasibility is very sharp, and it has a minimum value in the range of D/λ equal to 14. For a value of D/λ greater than 14 an oscillating behavior is observed.

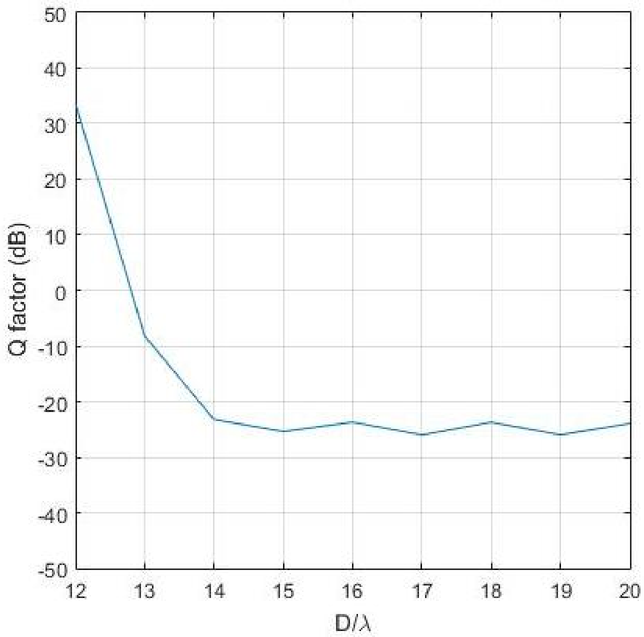

Fig. 3.3-2: Q factor as a function of the antenna aperture diameter

The influence of the source size on the directivity is very modest and it is also demonstrated by the following figure, see Fig 3.3-3 where the Within Beam Directivity (WBD), i.e., minimum directivity in the main beam angular region, is reported as a function of the aperture diameter.

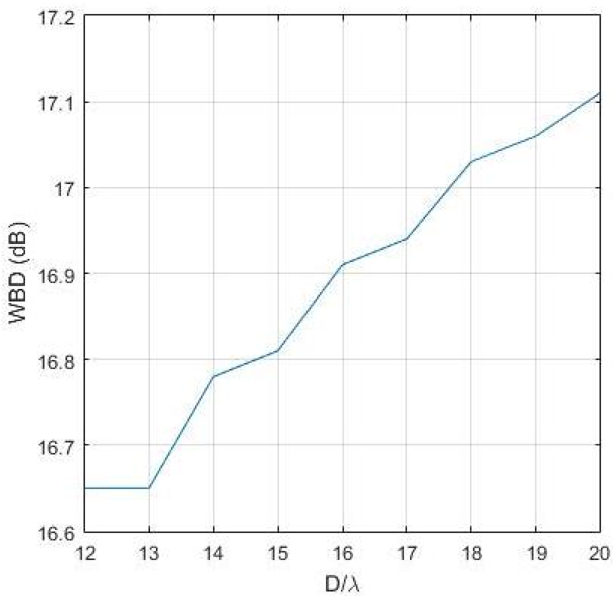

Fig 3.3-3: WBD as a function of the source diameter

As demonstrated by the dependency of the WBD with the antenna size, an high increase of the aperture source diameter does not entail an high increase of the directivity. In particular, doubling the aperture source the directivity increases only by quite 0.5 dB.

After the definition of the minimum continuous aperture size exploiting the virtual linear arrays technique and the optimal factorization of the P_0 is performed (see equation 24.b) the second step foresees the reconstruction of the continuous source by means a sequence of transformations. A Fast Fourier Transform of the P_0 gives the coefficient of the following polynomial

$$P_0(z) = \sum_{n=-N_0}^{N_0} p_n z^n \qquad (25)$$

whose zeros can be found using a simple roots-finding algorithm allowing the factorization. As already described, by coupling in different ways the complex zeros couples outside the unit circumference, 2^N different field cuts of $E_0(u)$, all having the same power pattern are obtained. This fact can be exploited to select one of the solutions according to some further convenient criterion as, for example, source dynamical range and/or phase variation across the aperture. Once $E_0(u)$ has been chosen, the aperture distribution $f(\rho)$ can be recovered ensuring that the corresponding field slice E is the same from E_0 in the visible spectrum.

Due to the Fourier-Bessel relationship between f and E ($E(\vartheta) = \int_0^1 f(\rho) J_0(\rho\beta R sin(\vartheta)) \rho d\rho$) the best estimation of $E(u)$ in terms of mean square norm is obtained by:

1. performing an inverse Hankel transformation of $E_0(u)$ truncated to u_0
2. truncating the Hankel transform to βa, which gives the optimal source $f(\rho)$
3. performing a final Hankel transform of $f(\rho)$

Fig. 3.3-4 reports the flow chart of the overall synthesis procedure for the definition of the circularly symmetric continuous source able to fulfill the assigned constraints.

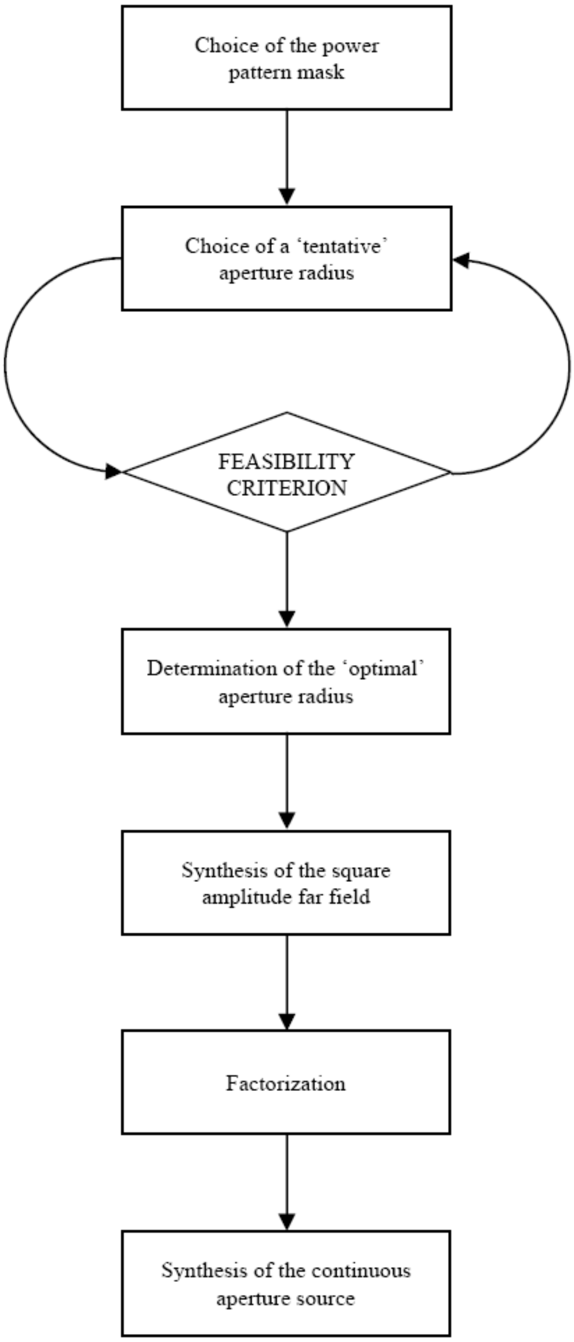

Fig. 3.3-4: Flow chart concerning the different steps of the overall synthesis procedure

Fig. 3.3-5 shows the shaped beam radiated by a circularly symmetric continuous aperture and three different distributions, amplitude and phase, given the same radiation pattern. The requirements for the pattern are:

- SLL ≤ -40 dB
- Ripple +/- 0.5 dB for $|\vartheta| \leq 20°$

The final aperture has a radius of 9.0 λ and the total possible aperture solutions are 2^{12} . For example, only three solutions are reported in the figure below.

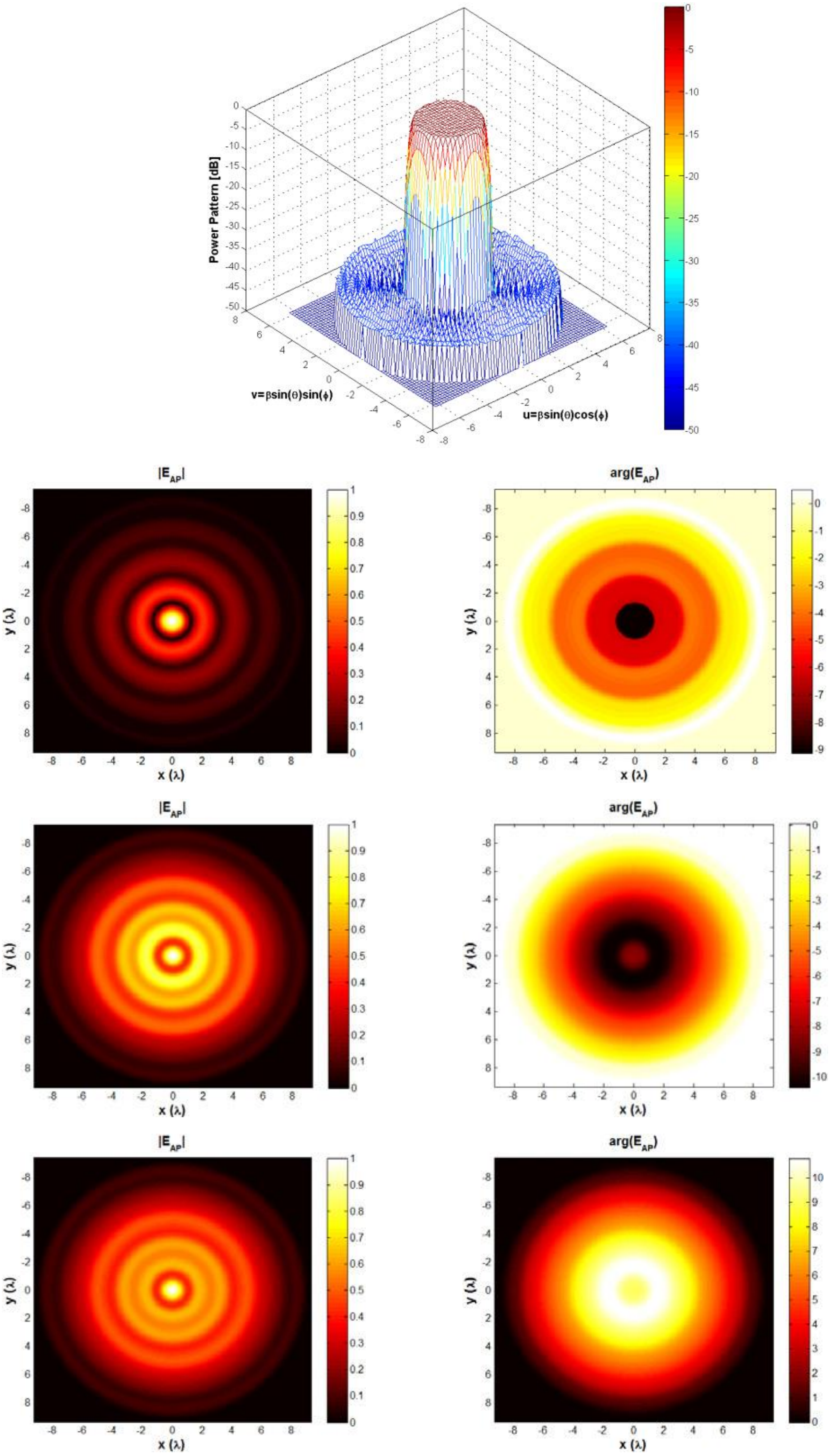

Fig. 3.3-5: Different amplitude and phase continuous aperture distribution given the same shaped beam

3.4 Optimal synthesis of a circularly symmetric only-phase reconfigurable aperture source

The capability of easily reconfiguring the radiation behaviour of a direct-radiating arrays antenna is crucial in several applications, including radar and satellite telecommunications ([20], [27], [35], [42], [43], [45], [76]-[81]). Amongst the different kinds of reconfigurability, a very effective one is undoubtedly represented by phase-only control, as it allows both a simplification of the beam forming network and an increase of the amplifiers' efficiency. This paragraph is focused on the synthesis of a circularly symmetric continuous source able to be phase-only reconfigurable in order to use it as a reference in the synthesis of phase-only reconfigurable isophoric arrays.

An innovative and general approach is proposed to the optimal mask-constrained power synthesis of circular continuous aperture sources able to dynamically reconfigure their radiation behavior by just modifying their phase distribution. As shown in Fig. 3.4-1 acting on the phase distribution on the antenna aperture different kind of radiation patterns are achievable.

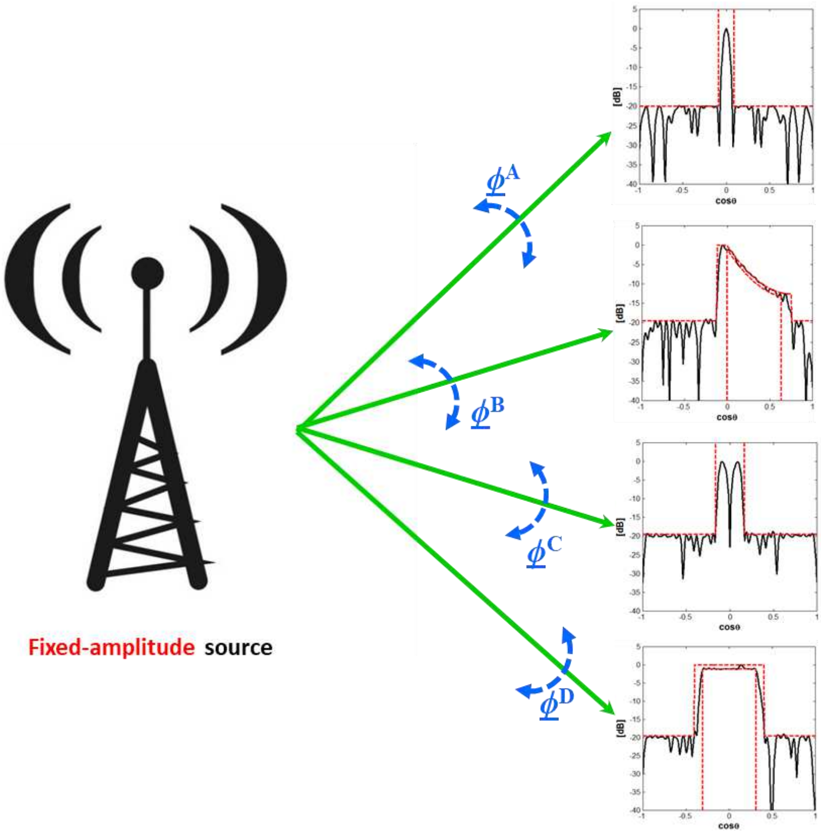

Fig. 3.4-1: Only-phase reconfigurable source concept. Acting on the phase distribution different radiation patterns are available

The design procedure relies on an effective a-priori exploration of the search space which guarantees the achievement of the globally-optimal solution. The synthesis is cast as a convex programming problem and can handle an arbitrary number of pencil and shaped beams. The achieved solutions are then exploited as reference and benchmark in order to design phase-only reconfigurable isophoric circular-ring sparse arrays.

The approach takes decisive advantage from the circumstance that, in the power synthesis of a shaped beam, a multiplicity of equivalent source distributions can be identified. This holds true not only in the case of equispaced 1-D arrays [12], but also in the case of circularly symmetric continuous sources as described in the previous section [69].

To describe the approach, let us focus on the general case wherein the power pattern must be phase-only reconfigured between a desired pencil beam $E_1(\vartheta)$ and a desired shaped beam $E_2(\vartheta)$, ϑ denoting the observation angle with respect to the boresight. Concerning the radiation constraints, according to canonical definitions used in the previous sections of this thesis, the pencil beam is conceived as a far field whose square amplitude must be equal to a fixed value A^2in a target direction θ_T and, at the same time, lower than an upper-bound function UB_1 in the sidelobes region (say Ω). Moreover, the shaped beam is conceived as a far field whose square amplitude must lie in a prescribed mask, $LB_2(\vartheta) \leq |E(\vartheta)_2|^2 \leq UB(\vartheta)_2$, LB_2 and UB_2 respectively denoting the upper and lower bound functions pertaining to the technical requirements.

Under these hypotheses, the proposed procedure for the synthesis of a phase-only reconfigurable circularly symmetric continuous aperture source is as follows:

1. identify the optimal (real and non-negative) source generating the desired pencil beam;
2. identify the multiplicity of optimal (complex) sources generating the desired shaped beam;
3. select, amongst all the source distributions coming out from step 2, the closest, in terms of amplitude, to the one coming out from step 1;
4. set the source coming out from step 3 as the final aperture field pertaining to the 'shaped beam' radiation modality, say $f_2(\rho)$, with $f_2(\rho) = |f_2(\rho)|e^{i\varphi(\rho)}$, φ and ρ respectively denoting the source's phase and the radial coordinate spanning the aperture;
5. set the amplitude of the source coming out from step 3 as the aperture field pertaining to the 'pencil beam' radiation modality, say $f_1(\rho)$, with $f_1(\rho) = |f_2(\rho)|$.

A number of comments are given in the following concerning the above procedure.

Steps 1 and 2 can be performed through the techniques respectively developed from Bucci and co-workers in [28] (pencil beam case) and [29] (shaped beam case). These approaches guarantee the global optimality of the solutions achieved in the two separate synthesis problems.

Step 3 derives from the general philosophy in [50] and allows identifying the shaped beam's source which most easily lends itself to be reconfigured into the other radiation modality. In order to gain a better understanding of the aim of this step (as well as of the way in which it is fulfilled), Fig. 3.4-2 is provided where the following case is represented. It is requested to design an antenna aperture able to be reconfigured in three different manners:

- one pencil beam P_1 (green point in the space of the amplitude)
- one shaped beam S_1 (red points in the space of amplitude)
- one shaped beam S_2 (blue points in the space of amplitude)

As it is demonstrated in the previous chapter of this thesis, the pencil beam is represented by a unique real solution in the space of the amplitude, while the shaped beam can give different solutions in terms of amplitude and phase distribution actin in the zero flipping procedure. For this example it is supposed that the shaped beam S_1 and shaped beam S_2 have three different solutions. Representing them in the space of the amplitude distribution, each solution is represented by a specific point. The solution which are 'more easily' lends itself to be reconfigured are closed in an hypersphere having the minimum radius and containing at least one sample for each pattern.

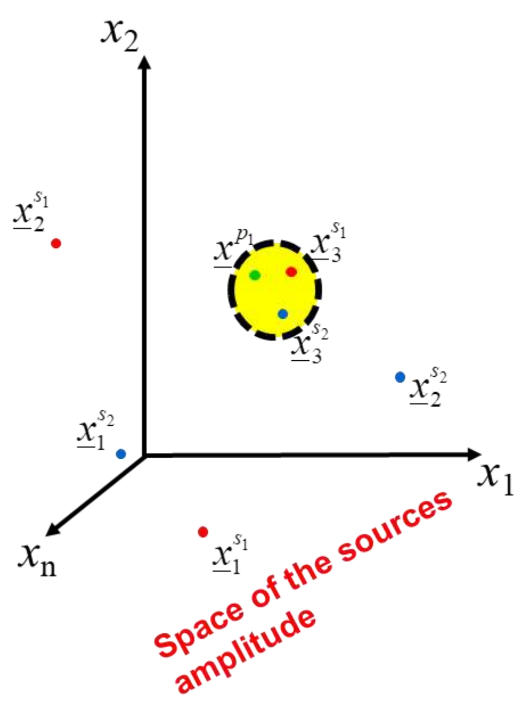

Fig. 3.4-2: Representation of the solutions in the space of source amplitude

Finally, step 4 guarantees that the reconfigurable source radiates a shaped beam fulfilling the given mask, while step 5 is expected to provide a pencil beam very close to the optimal one given by step 1 (and hence fulfilling the initial constraints as well). In fact, the source radiating the shaped beam comes in a straightforward fashion from step 2, while (due to step 3) the pencil beam's source just consists in a 'slight' modification of the optimal one coming out by step 1.

Summarizing, at the end of the overall procedure no radiation-performance losses will be experienced by the shaped beam, and the only price to be paid in order to achieve the phase-only reconfigurability will consist in slight losses on the pencil beam's performances.

These losses will be proportional to the difference (in terms of amplitude distribution) between the source coming out by step 1 and the one coming out by step 4, whose extent will be expressed by the radius of the hypersphere depicted in Fig. 3.4-2.

It is worth noting that, whatever the mission scenario at hand, the overall five-steps procedure above essentially consists in the solution of

Convex Programming problems plus a number of spectral factorizations (each one being an instant operation [12],[69]), with the inherent advantages in terms of solutions' optimality and computational time.

Notably, the approach can be exploited also in the cases where the amplitudes of the sources pertaining to the two radiation modalities must be equal to each other over only a limited portion of the aperture. This special condition permits to recover the losses in the pencil beam radiation modality.

In particular, once $f_2(\rho)$, is determined, 'common core' or 'common tail' [27] architectures can be achieved by substituting the above step 5 with the following CP optimization [in the unknown $f_1(\rho)$,]:

$$\min_{f_1(\rho)} \int_0^a |f_1(\rho)|^2 \rho d\rho \qquad (26)$$

Subject to

$$\begin{cases} |E_1(\vartheta)|^2 \leq UB_1(\vartheta) & (27) \\ E_1(\vartheta_T) = |A| & (28) \\ f_1(\rho) = |f_2(\rho)| \ \forall \rho \in \Gamma & (29) \end{cases}$$

Where

$$E_1(\vartheta) = \int_0^a f_1(\rho) J_0(\frac{2\pi}{\lambda}\rho \sin(\vartheta))\rho d\rho \qquad (30)$$

Denoting with *a* and λ the aperture radius and the wavelength. In fact, problem (26)-(28) is equivalent to the one in section 3.2, wherein the minimization of the aperture power (26) allows to maximize the directivity without incurring into super-directive solution, while the linear constraint (29) enforce the desired common core or common tail behavior as long as the Γ region denotes the central or outlying area within the aperture.

3.4.1 Numerical assessment of the only-phase reconfigurable circular symmetric continuous aperture source

In order to assess the proposed design technique, in the following the outcomes achieved in the synthesis of a phase-only reconfigurable circularly symmetric continuous aperture source providing, from a geostationary satellite, an uniform coverage of both the Europe (in the pencil-beam modality) and the Earth (in the shaped-beam modality) are shown, see Fig. 3.4.1-1.

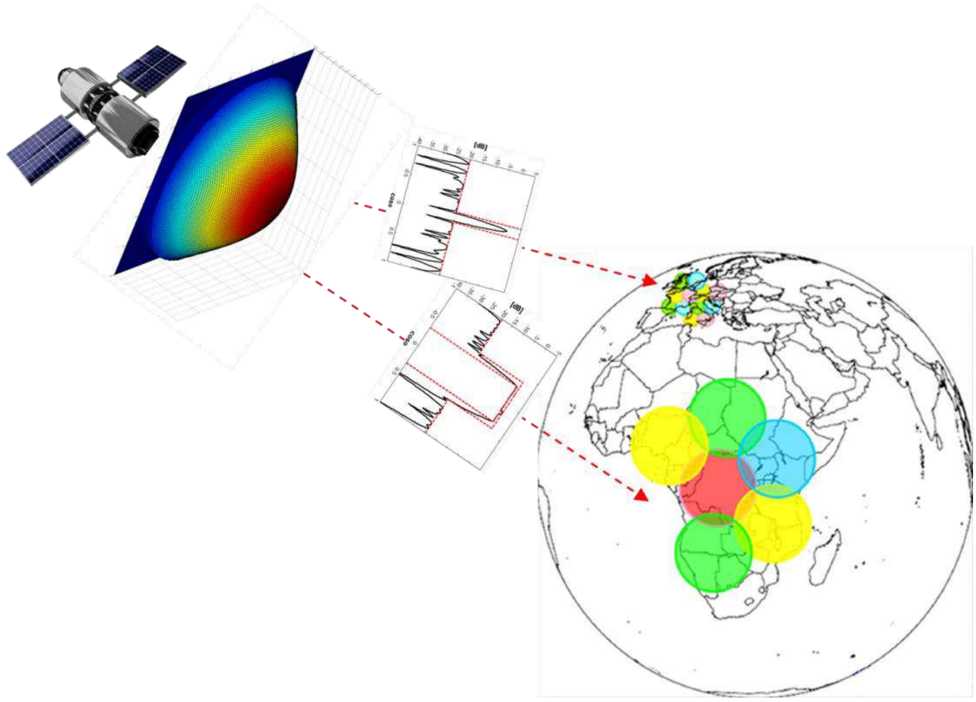

Fig. 3.4.1-1: Only-phase reconfigurable source covering the Earth surface with a different kind of radiation pattern

The constraints in the radiation pattern are derived from a recent ESA ITT [82] which requested two radiating operation modality:

- the pencil beam has a Peak Sidelobe Level (PSL) equal to -26.2 dB for $|\theta|\geq 3.5°$;
- the shaped beam has a maximum ripple not larger than ±0.5 dB for $|\theta|\leq 9°$, and a maximum PSL lower than -20 dB for $|\theta|\geq 14°$.

The power patterns coming out from steps 1-3 for a=11λ are shown in Fig. 3.4.1-2.

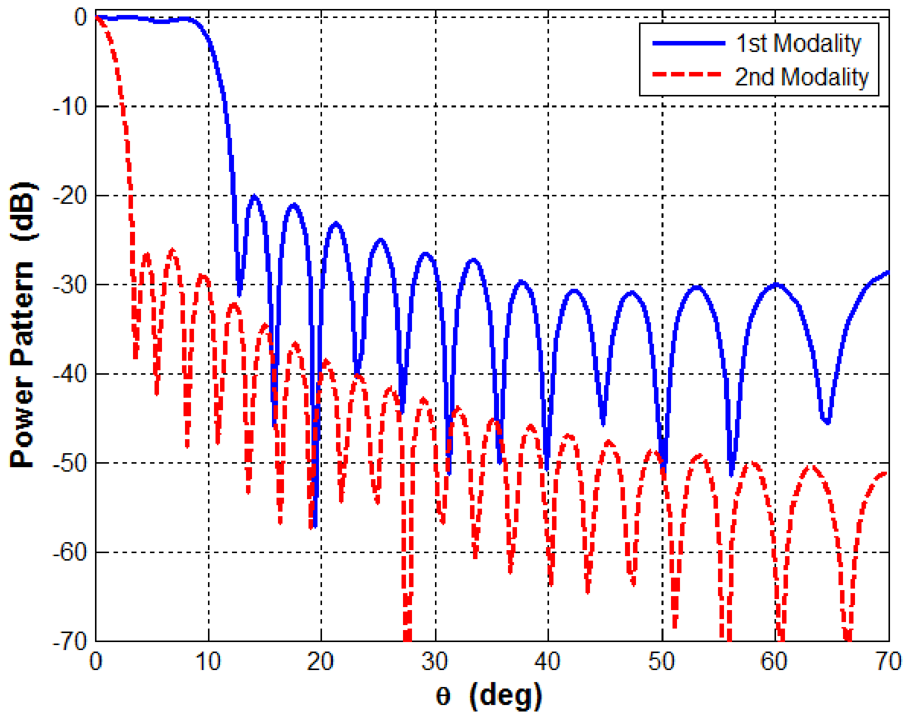

Fig. 3.4.1-2: Power patterns coming out from the steps 1-3 of the procedure

Fig. 3.4.1-3 shows the amplitude of the 1024 equivalent sources generating the shaped beam depicted in Fig. 3.4.1-2.

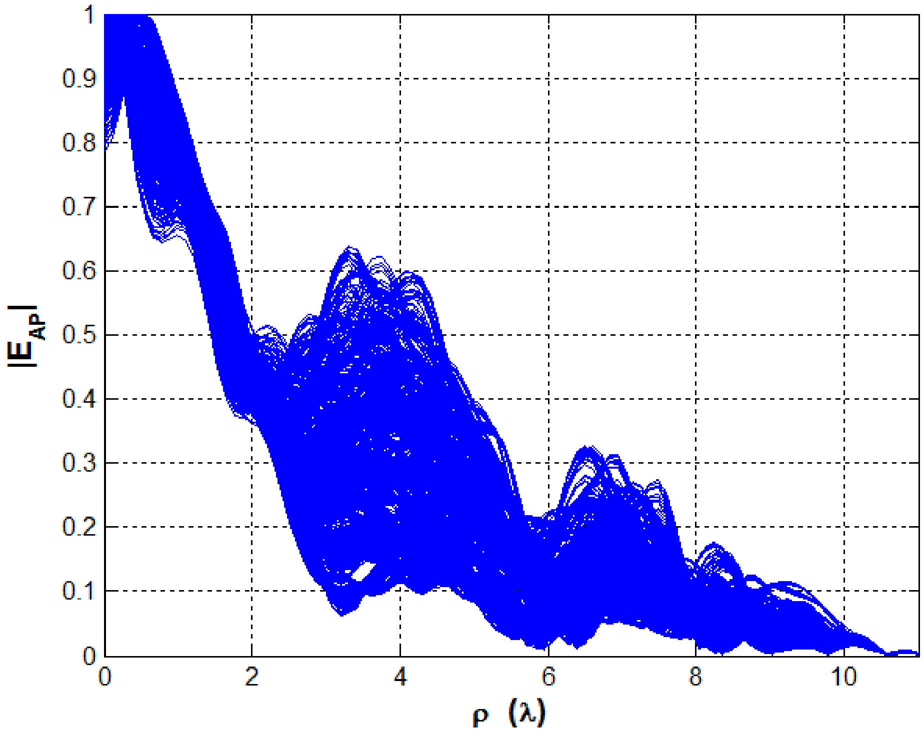

Fig. 3.4.1-3: 1024 equivalent sources generating the desired shaped beam

The source pertaining to the pencil beam modality as well as the one coming out from step 3 are shown in Fig. 3.4.1-4. Notably, despite they are able to fulfill quite different radiation constraints, the two sources have a very similar amplitude distribution. This circumstance attests that, in the space of the sources' amplitude (see Fig. 3.4-2), the minimal hypersphere containing at least one solution for each desired beam has indeed a small radius.

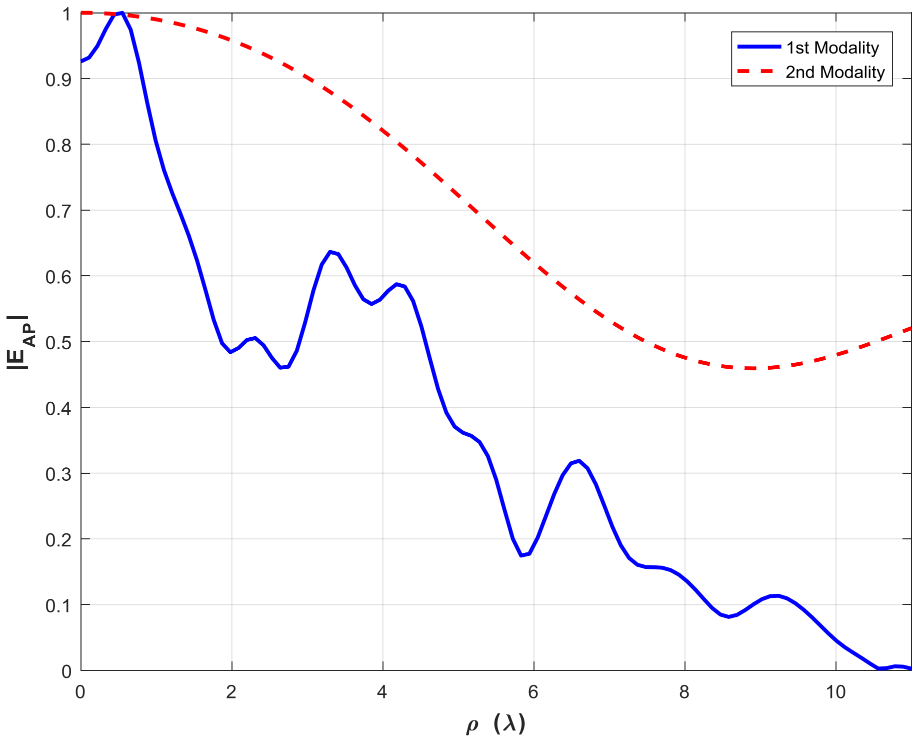

Fig. 3.4.1-4: Amplitude distribution for the pencil beam (red curve) and shaped beam (blue curve)

The final phase-only reconfigurable power patterns are shown in Fig. 3.4.1-5 while the final circularly symmetric continuous aperture source coming out from steps 4 and 5 of the procedure are shown in Fig. 3.4.1-6 and Fig. 3.4.1-7, respectively. As expected, the shaped beam is identical to the reference one shown in Fig. 3.4.1-2 while, as a proof of the high effectiveness of the overall approach, the only effect induced on the pencil beam by reconfiguration is a slight beamwidth increase while SSL performance results even better than the reference ones.

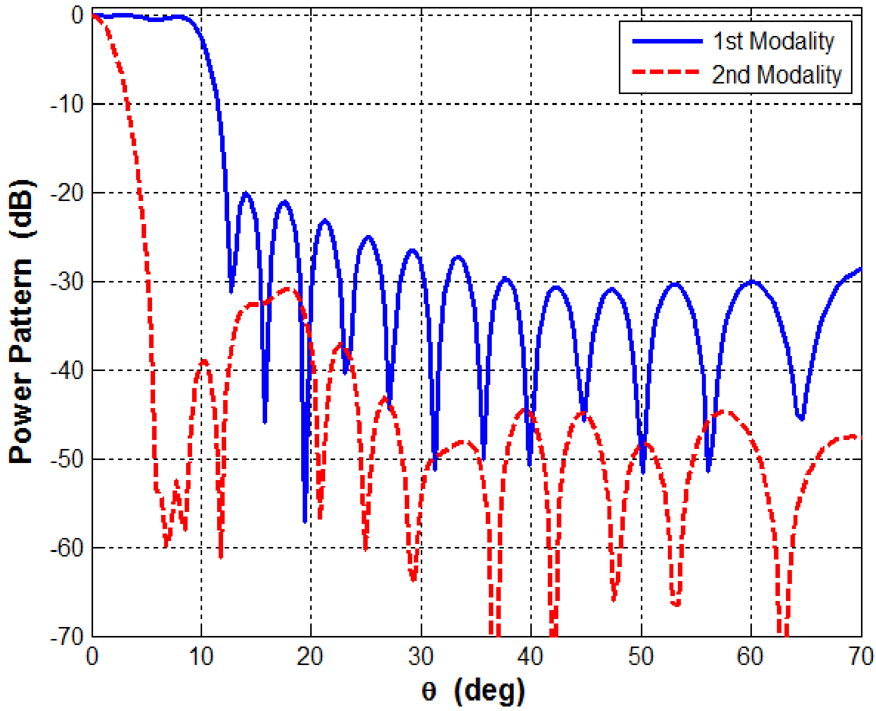

Fig. 3.4.1-5: Final phase-only reconfigurable power patterns

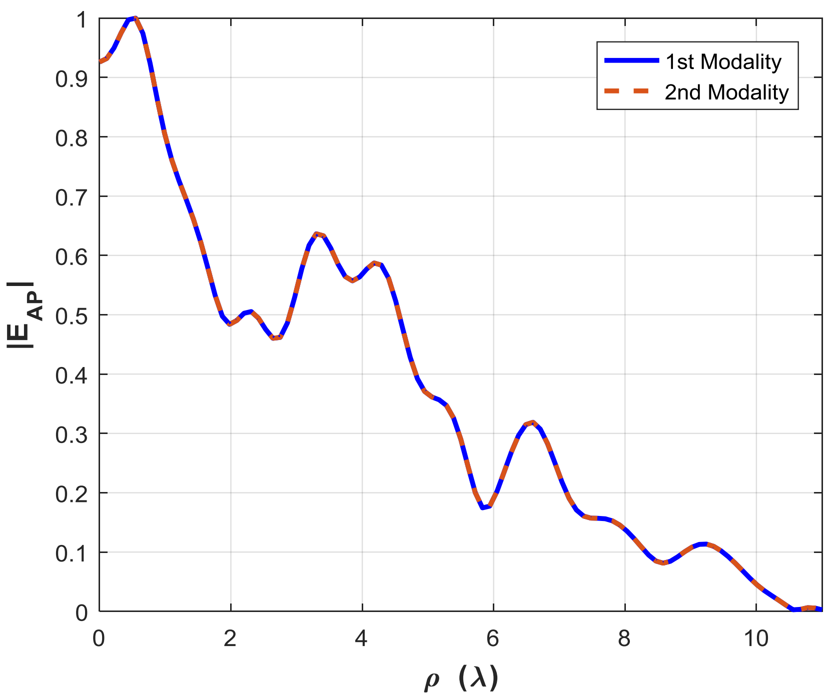

Fig. 3.4.1-6: Common amplitude distribution for the only-phase reconfigurable circularly symmetric continuous aperture source

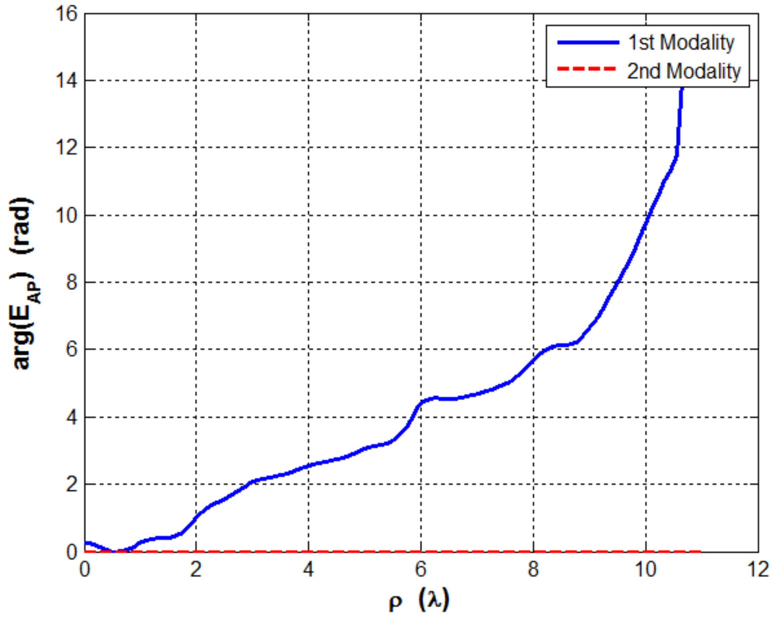

Fig. 3.4.1-7: Phases distribution for the only-phase reconfigurable circularly symmetric continuous aperture source

As a second test case, formulation (1)-(4) has been assessed. In particular, the previous experiment has been repeated by also enforcing a 'common core' behavior of the sources, by setting Γ as the region $\rho \in [0, 6\lambda]$. The achieved power patterns for the shaped and pencil beam radiation modality are shown in Fig. 3.4.1-8 and Fig. 3.4.1-9, where a comparison with the reference field is carried out. The relative amplitude and phase distribution for this kind of reconfigurable aperture source are reported in Fig. 3.4.1-10 and Fig. 3.4.1-11.

This kind of circularly symmetric continuous aperture source is useful in those cases where it is allowed to generate the reconfigurable fields by varying not only the excitation phases but also the outer part of the circular ring isophoric sparse arrays layout (see Fig. 3.4.1-12).

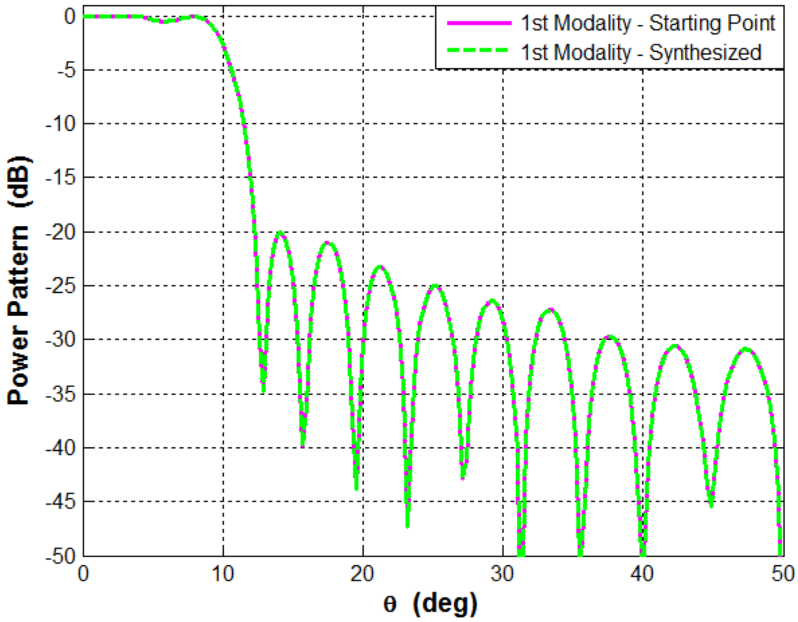

Fig. 3.4.1-8: Comparison between the reference and final reconfigurable power patterns achieved for the shaped beam

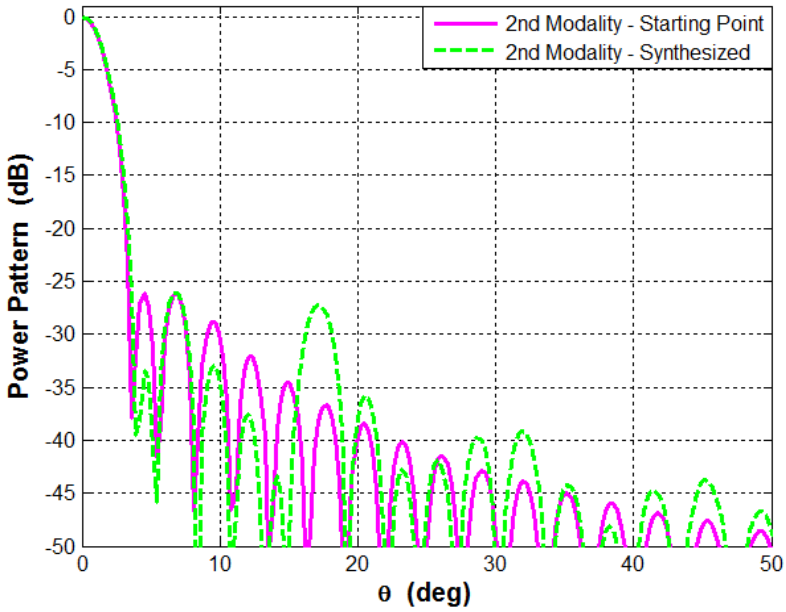

Fig. 3.4.1-9: Comparison between the reference and final reconfigurable power patterns achieved for the pencil beam

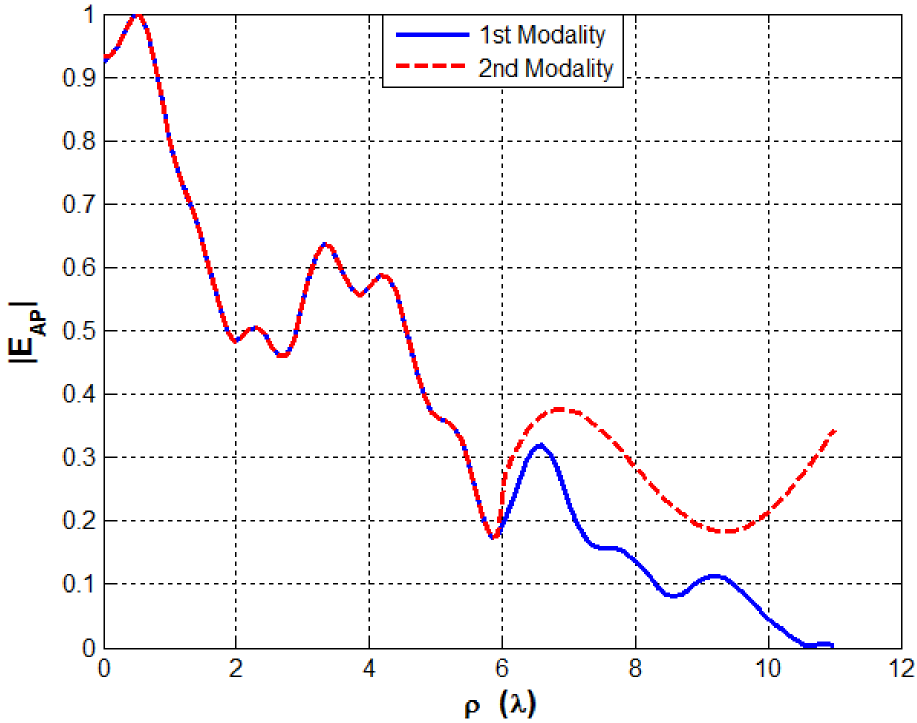

Fig. 3.4.1-10: Amplitude distribution with common core

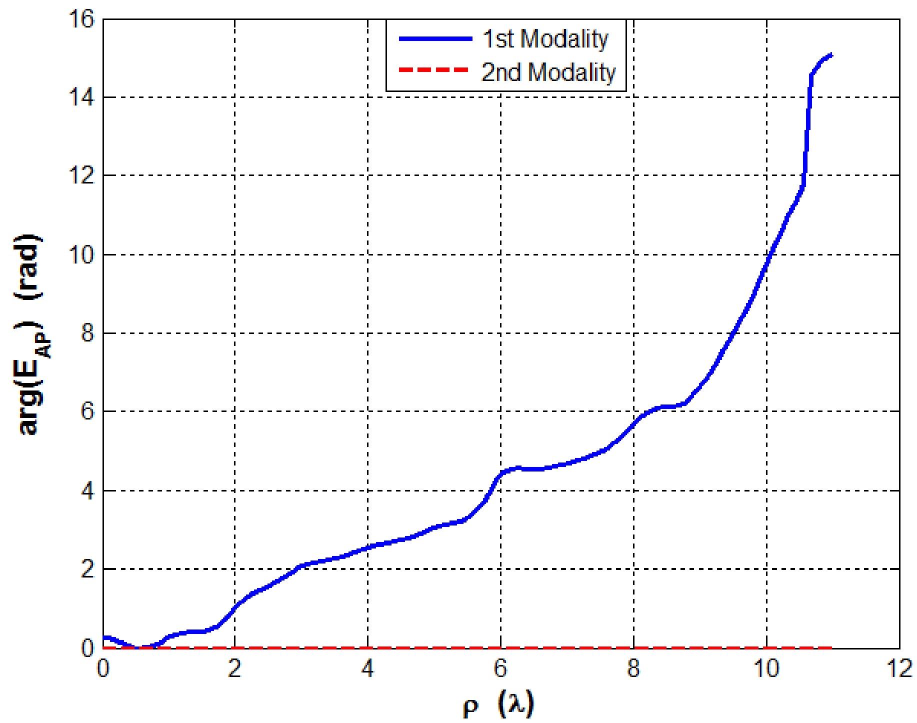

Fig. 3.4.1-11: Phases distribution with common core

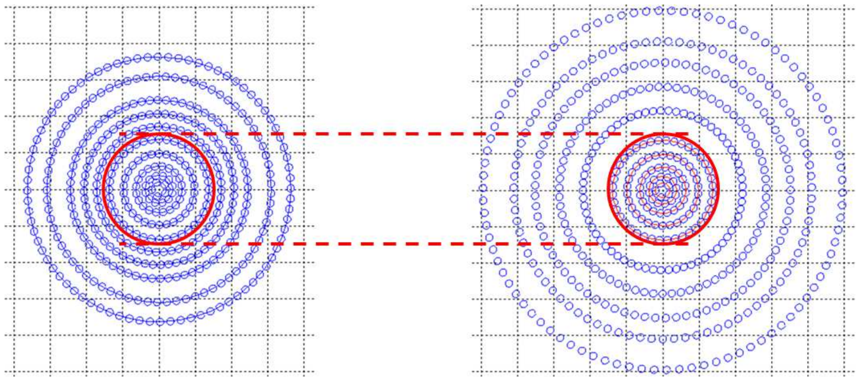

Fig. 3.4.1-12: Sketch of Circular ring isophoric sparse arrays layouts derivable from 'common core' reconfigurable sources

The proposed technique is able to deal with an arbitrary number of shaped and pencil beams, and it exploits at best all the knowledge available in the separate synthesis of the different patterns. The engine of the procedure is based on convex programming optimizations and fast spectral factorizations, guaranteeing at the same time a low computational burden and the achievement of globally-optimal solutions.

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4

Optimal deterministic discretization of the continuous source aperture into isophoric sparse array

4.1 Introduction to the optimal and deterministic discretization of a continuous source

Sparse arrays antennas, i.e., arrays whose equi-amplitude excited elements are properly located onto a non-regular grid, represent a suitable architecture able to address the problem of flexible and reconfigurable antennas achieving a directive behaviour of the overall arrays while reducing as much as possible the number of control points (i.e., amplifiers and phase shifters).

In such an architectural solution, the amplitude tapering which is required on the source in order to get the required far field shaping in terms of sidelobes level is realized by means of a proper choice of the locations of the equally-excited radiating elements, thus coming to a density taper [2] over the arrays aperture. Two possible draw-backs in this kind of antenna solution need to be addressed. First, a too large spacing amongst the elements can induce pseudo-grating lobes. Second, a large spacing can imply a poor overall aperture efficiency if a single kind of radiating element is adopted (which is the usual case in order to reduce the complexity of the overall antenna architecture). Both circumstances affect the possibility of actually fulfilling the required constraints.

An additional, important difficulty to overcome to get effective solutions is the strong non-linearity and non-convexity of the optimization problem to be faced, as the radiated pattern and locations of the different array elements are related by means of complex exponential dependencies. As a consequence, in the case of large antennas, local optimization algorithms are likely to be trapped in false solutions, whereas global ones can become ineffective and/or computationally prohibitive [83].

A large part of the work presented in the following has been devoted to discuss a possible strategy to overcome these difficulties.

In particular, starting from the simple observation that the synthesis of a continuous source is much more simple than that of an isophoric sparse arrays ([17],[84-85]), the following two-steps strategy for an effective design procedure has been devised:

First, identify a continuous source fulfilling 'at best' the required specifications;

Second, by using a fast and effective analytical 'density taper' technique synthesize the layout of the array

While in the previous Chapters a general and effective approach to the optimal synthesis of two dimensional circularly symmetric continuous aperture sources is discussed, this Chapter aims to show how the knowledge of this kind of sources can be exploited in order to perform the power pattern synthesis of uniformly-excited sparse circular ring arrays, from one side, and to present a completely new DRA architecture guaranteeing the achievement of the optimal radiation performance.

Notably, as it will be shown in the following, the overall synthesis procedure is fully based on analytical calculations, and it allows one to solve the overall synthesis problem with a negligible computational burden.

The chapter is organized as follows. At the first the Density Taper approach for isophoric linear arrays emulating a real source (i.e. pencil beam pointing at the boresight) is presented. After that the extension to the planar case will be addressed ([87], [88]). These two arguments represent the theoretical basis for the extrapolation of the density taper approach to the case of the complex source. In particular, at the first the isophoric linear arrays able to emulate a complex source is presented and discussed ([90]). This is a well-known result in the antenna synthesis literature but will be discussed in order to understand the difficulties of the 2D case. The novelty, and objective, of this thesis will be the introduction of a deterministic, optimal and fast procedure for the discretization of the complex source in the 2D case starting from the results of the 1D synthesis problem. The new procedure permits to achieve an important step in the full deterministic synthesis of isophoric sparse arrays closing the gap in the synthesis of isophoric arrays emulating the behavior of a complex source. This new effective strategy will outperform the results presented in the literature which do not use an optimal deterministic procedure and therefore they are not able to discretize in an optimal and analytical way the complex source ([86], [91]) as the one proposed.

Moreover, this approach can be used in the design of the reconfigurable isophoric sparse arrays where a discretization of a complex source is required in order to achieve the maximum flexibility. Several numeric examples are exposed in order to validate the design procedure.

4.2 Density taper for the discretization of a real continuous source into isophoric sparse linear arrays

The density taper technique for the discretization of a real continuous aperture source into an isophoric sparse arrays was introduced for the first time in the antenna literature by Doyle-Skolnik in [2, 59] and it was revisited in the recent past in [17]. The principal idea of this kind of design is to use the density taper of the uniformly excited elements to emulate the behavior of a properly chosen continuous source acting as a reference. In particular, as some kind of tapering is required on the continuous source to get the control of the sidelobes, this results in a density taper of the elements of the array in a manner such that the elements of the arrays will be more densely accommodated where the continuous source is higher. Therein, one starts from the idea of approximating a given real and positive continuous aperture current density with an array of equal amplitude non-uniform spaced elements. As the optimal synthesis of pencil beam generally implies real and positive aperture function when the beam is at boresight, the method is of interest any time one has to deal with the synthesis of pencil beams.

If $h(x)$ is the ideal current distribution and $I_C(x)$ is the cumulative current distribution defined as:

$$I_C(x) = \int_{-a}^{x} h(\xi)d\xi \qquad (31)$$

where $2a$ is the aperture diameter normalized to the wavelength, the idea consists in matching the ideal cumulative current distribution with the actual cumulative distribution generated from the sparse array. Assuming that $h(x)$ has an unitary integral over the source extension, and *N* is the number of elements array, the following procedure is foreseen:

1. Calculate the ideal cumulative current distribution by means of (31)
2. Divide the interval (-a, a) into *N* intervals each having the same area $1/N$. This division identifies *N+1* points $\{\hat{x}_0, \ldots\ldots \hat{x}_N\}$, such that

$$I_C(\hat{x}_n) - I_C(\hat{x}_{n-1}) = \frac{1}{N} \qquad (32)$$

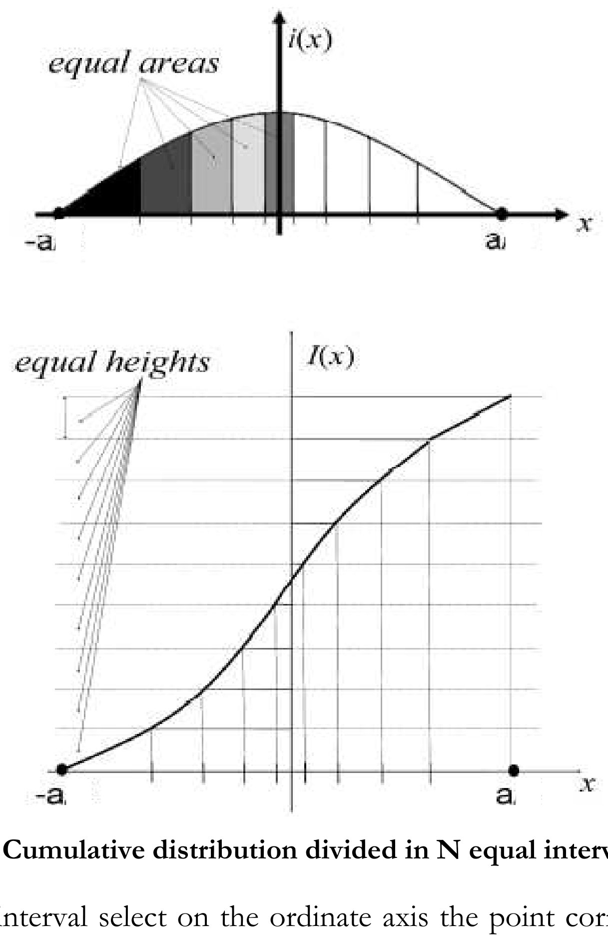


Fig. 4.2-1: Cumulative distribution divided in N equal intervals

3. For each interval select on the ordinate axis the point corresponding to

$$[I_C(\hat{x}_{n-1}) + I_C(\hat{x}_n)]/2 = I_C(x_n) \qquad (33)$$

4. Determine the corresponding abscissa x_n, see Fig. 4.2-2.

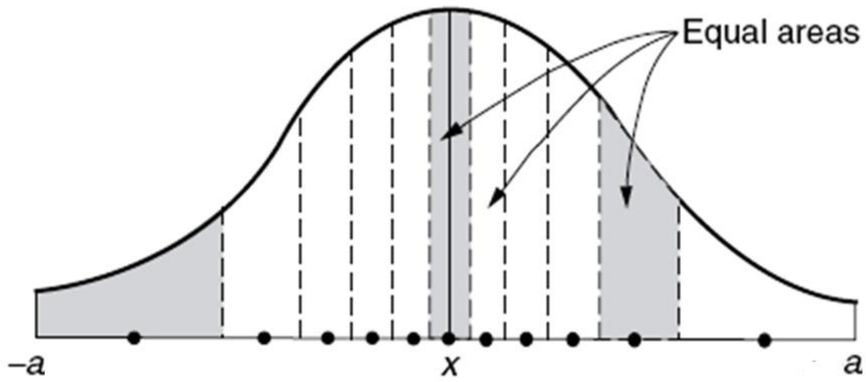


Fig. 4.2-2: Location of the radiating element according to the amplitude current distribution

As indicated in [2] and [59], the synthesis procedure is equivalent to minimize a weighted difference between the desired far-field pattern:

$$f_0(u) = \int_{-a}^{a} h(x) e^{j2\pi x u} dx \quad (34)$$

and the synthesized one,

$$f_a(u) = \frac{1}{N} \sum_{n=1}^{N} e^{j2\pi x_n u} \qquad (35)$$

Where x_n is the algebraic distance of the n-th element from the center of the antenna and $u = \sin(\vartheta)$, see Fig. 4.2-3.

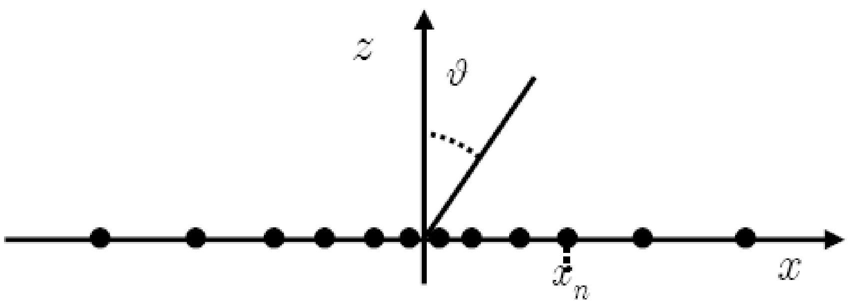


Fig. 4.2-3: Reference linear arrays with density taper approach

More precisely, the technique is equivalent to minimize the following functional:

$$\Theta(x_1, x_2 \ldots\ldots x_n) = \int_{-\infty}^{\infty} |f_o(u) - f_a(u)|^2 W(u)\, du \qquad (36)$$

Where $W(u) = 1/u^2$ is a weighting function depending on the spectral variable u. Also, it corresponds to well defined optimization criteria, as the procedure gives back an actual pattern which is the best approximation to the ideal pattern in an L^2 sense (see [2] and [59]for more details).

The Doyle-Skolnik procedure design for discretizing a continuous real source is very simple and permits to obtain an isophoric sparse arrays emulating the desired aperture distribution. This method contains some drawback as:

- The adopted functional is not the most suitable for several practical applications;
- Considering that the element factor can limit the pattern close to the endifire direction, it could be interesting in realizing a fitting over a reduced angular zone. Besides, it could be interesting to consider a uniform norm rather than an L^2 one;
- This method cannot deal, in an actually globally optimal way, with the case wherein the reference source is by itself a regular, uniformly spaced, array.

In order to overcome this drawback in [17] the functional for the optimization is reformulated in the following manner:

$$\Theta(x_1, x_2 \ldots\ldots x_n) = \left\| f_0(u) - \frac{1}{N}\sum_{n=1}^{N} e^{j2\pi x_n u} \right\| \qquad (37)$$

Where $\|\bullet\|$ denotes either the L^2 or the uniform norm (L^∞) in the generic interval (u_m, u_M) where the matching between the reference pattern and the actual pattern is desired. Performing some mathematician manipulation of equation (37) it is possible to demonstrate that the solution of the minimization problem is

$$x_n = N \int_{\hat{x}_{n-1}}^{\hat{x}_n} x h(x) dx \qquad (38)$$

which represents, in a geometrical sense, the barycenter of $h(x)$ in each interval (x_{n-1}, x_n). In order to obtain a solution of the minimization problem in a closed formula, a condition in the calculation is imposed

$$2\pi(x - x_n)u \ll 1 \qquad (39)$$

This limitation reported in (39) entails that the quantity $2\pi(x - x_n)u$ is sufficiently small and it depends on:

- Maximum observation angle
- Extension of the different intervals in terms of wavelength
- The desired degree of accuracy on final patterns

Therefore, for a given pattern, the analytical solution of equation (38) will guarantee very good results if the spacing in the aperiodic arrays is sufficiently small, and/or when the angular sector of interest is sufficiently small.

4.3 Two-dimensional density taper approach for the discretization of a real continuous source into isophoric sparse arrays

In this paragraph a deterministic approach for the synthesis of planar aperiodic arrays able to exploit continuous reference planar sources is presented, see [87]. Rotationally symmetric beams are considered extending the density taper approach proposed by linear source by Doyle-Skolnik. Concentric ring arrays, which approximate the circular field symmetry, are considered. Differently from other 2-D deterministic technique working with concentric ring arrays ([65]), the proposed approach exploits a Doyle-based rationale for determining the whole arrays geometry, minimizing the properly weighted mean square difference between the patterns produced by the arrays and the continuous reference source. In order to perform the extension in the 2D case of the 1D Doyle approach, the following key concept substitutions are required:

"linear subinterval and equal area will be replaced with angular sector and equal volumes" .

It is needed to stress that the reference continuous source is still a real and nonnegative function, it means that the pencil beam pointing at the boresight is the synthesized pattern. Given the 2D circularly symmetric continuous reference source, in analogy with the 1D case, the first step of the procedure is an appropriate subdivision of the available circle in concentric ring, and of each ring into equal angular sector. In this way, the counterpart of the linear subinterval introduced by Doyle-Skolnik for the 1D were identified in the 2D case. Now it needs to be shown that if all these sectors are forced to subtend the same volume of the 2D reference source, then it can be minimized the properly weighted mean square difference between the patterns of the continuous source and the concentric ring arrays containing one element per each sector.

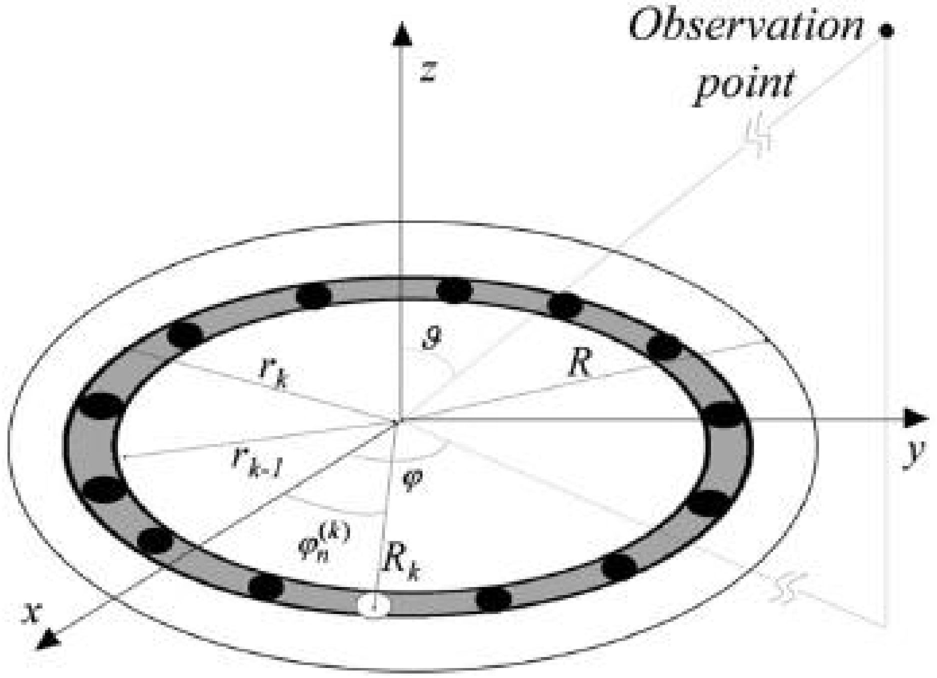


Fig. 4.3-1: Geometry of the problem considering the k-th ring

Considering the geometry of Fig. 4.3-1, the couple (ϑ, φ) define the angular position of a generic observation point in the far field. The antenna pattern radiated by a circular aperture with a radius *R* fed by a circularly symmetric continuous amplitude current $\mathrm{i}(\rho)$, where ρ is the radial coordinate, is

$$f(u) = \int_0^R \hat{\imath}(\rho)\rho J_0(\beta\rho u)d\rho \qquad (40)$$

Where $J_0(\bullet)$ is the zero-order of Bessel function of first kind, $\beta = \frac{2\pi}{\lambda}$ with λ the wavelength, $u = \sin(\vartheta)$, and $\hat{\imath}(\rho)$ is the source $i(\rho)$ scaled by a constant factor.

At the same time, the arrays factor of an isophoric concentric ring arrays with arbitrary spaced rings is expressed by

$$\tilde{f}_a(u,\varphi) = \sum_{k=1}^{K_{ring}} \sum_{n=0}^{v_k-1} e^{[j\beta R_k u\cos(\varphi-\varphi_n^k)]} \qquad (41)$$

where K_{ring} is the total number of rings, v_k is the number of elements of the k-th ring, φ_n^k and R_k represent the azimuth and radial coordinates of the n+1-th arrays element of the k-th ring. Using the following discrete arrays illumination function:

$$\tilde{\imath}_a(\rho,\varphi) = \sum_{k=1}^{K_{ring}} \sum_{n=0}^{v_k-1} \frac{1}{\rho}\, \delta(\rho - R_k)\delta(\phi - \varphi_n^k) \qquad (42)$$

where $\delta(.)$ is the delta of Dirac, inside the equation (41) it is obtained the following arrays factor:

$$\tilde{f}_a(u,\varphi) = \int_0^R \int_0^{2\pi} \tilde{\imath}_a(\rho,\varphi)\rho\, e^{[j\beta\rho u\cos(\varphi-\phi)]} \qquad (43)$$

It has to be noted that the pattern of the azimuthally symmetric continuous source $i(\rho)$ is circularly symmetric as well, whereas the arrays pattern $\tilde{f}_a(u,\varphi)$ is not (see [3]). But for the case of interest of the satellite communication where high directivity antennas are designed, such dependence is negligible in the angular sector wherein the radiated field is high concentrated. Thus, only the zero harmonics of the arrays pattern can be retained

$$f_a(u) = \langle \tilde{f}_a(u,\varphi) \rangle = \int_0^R i_a(\rho)\rho J_0(\beta\rho u)d\rho \qquad (44)$$

where

$$i_a(\rho) = \int_0^{2\pi} \tilde{\imath}_a(\rho,\phi)d\phi \qquad (45)$$

Using the equation (44) and equation (40) the following difference can be computed

$$f(u) - f_a(u) = \int_0^R [\tilde{\imath}(\rho) - i_a(\rho)]\rho J_0(\beta\rho u)\, d\rho \qquad (46)$$

Applying the integration by part on the integral of equation (46) and introducing the following cumulative distribution functions:

$$\hat{I}(\rho) = \int_0^\rho \hat{\imath}(\eta)\eta\, d\eta \qquad (47)$$

$$I_a(\rho) = \int_0^\rho i_a(\eta)\eta\, d\eta \qquad (48)$$

and imposing that $\hat{I}(R) = I_a(R)$, it turns out that

$$\frac{f(u) - f_a(u)}{\beta u} = \int_0^R [\tilde{I}(\rho) - I_a(\rho)]J_1(\beta\rho u)\, d\rho \qquad (49)$$

Equation (49) represents a generalized Hankel transform relationship between the weighted pattern difference and the cumulative distribution difference. Applying the Parseval's theorem (see [92]) the following relationship is possible:

$$\int_0^{+\infty} \left|\frac{f(u) - f_a(u)}{u}\right|^2 du = \beta^2 \int_0^R \left|\hat{I}(\rho) - I_a(\rho)\right|^2 d\rho \qquad (50)$$

Minimizing the mean square difference of the patterns with $1/u^2$ weighting, it is equivalent to minimizing the mean square difference between the cumulative distributions in (47) and (48), provided that these latter distributions satisfy the constraint $\hat{I}(R) = I_a(R)$.

Considering the equation (42), (45) and (48) it seems clear to verify that $I_a(\rho)$ is a sum of discretized step with an height of v_k with $k = 1 \ldots . K_{rings}$. In particular, $I_a(0) = 0$ and

$$I_a(R) = \sum_{k=1}^{K_{rings}} v_k = N \qquad (51)$$

where N is the total number of the array elements. From (51), enforcement of $\hat{I}(R) = I_a(R)$ requires the scaling:

$$\hat{\imath}(\rho) = N \frac{i(\rho)}{\int_0^R i(\eta)\eta \, d\eta} \qquad (52)$$

To be applied to $i(\rho)$, which is equivalent to let

$$\hat{I}(\rho) = NI(\rho) \qquad (53)$$

where the function

$$I(\rho) = N \frac{\int_0^\rho i(\eta)\eta \, d\eta}{\int_0^R i(\eta)\eta \, d\eta} \qquad (54)$$

represents the volume of the 2D reference source $i(\rho)$ subtended by the circle of radius ρ, normalized to that subtended by the whole circle of radius R. Use of (16) allows rigorously deriving the 2D counterpart of the equal areas concept used in [2], [59] for the 1D case. It has to be noted that to minimize the right side of (50), the function $\hat{I}(\rho)$ scaled as reported in (53) should pass through each step of $I_a(\rho)$.

Considering Fig. 4.3-2, the k-th ring arrays with $v_k$elements should be located at a radial coordinate R_k belonging to the interval $[r_{k-1}, r_k]$, where

$$\begin{cases} \hat{I}(r_{k-1}) = v_1 + \cdots + v_{k-1} \\ \hat{I}(r) = v_1 + \cdots + v_{k-1} + v_k = \hat{I}(r_{k-1}) + v_k \end{cases} \qquad (55)$$

To locate the v_k elements of the k-th ring arrays it is needed to subdivide the ring delimited by the radii r_{k-1} and r_k onto v_k adjacent angular sector, and consequently locate one element per each angular sector.

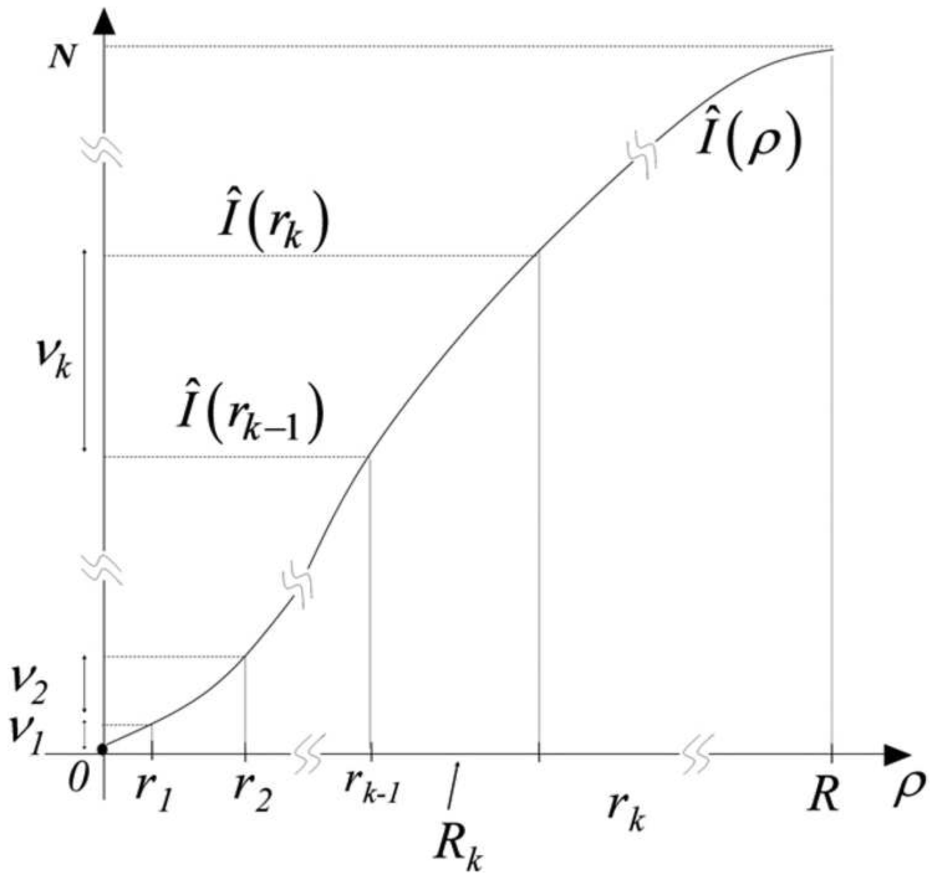


Fig. 4.3-2: Representation of cumulative discretization of the 2D density taper approach

Considering equation (54) and using the relationship (53) and (55), each of the v_k adjacent angular sector subtends the following normalized volume of the reference current source

$$\frac{I(r_k) - I(r_{k-1})}{(v_k)} = \frac{1}{N}\frac{\hat{I}(r_k) - \hat{I}(r_{k-1})}{v_k} = \frac{1}{N} \qquad (56)$$

An important aspect is that the volume expressed in (56) is independent of k, therefore, the application of the same procedure for all the rings leads to a total number *N* of angular sector, each of one subtending the same portion of volume of the reference current source, that is a fraction 1/N of the volume subtended by the whole available circular area of radius R. With this procedure the iso-volume sectors are defined and therefore the similar iso-area concept of the1D is achieved.

At the end of this step, one can summarize that, in the 2D case, to minimize the mean square difference of the pattern originated by the continuous reference source and the concentric ring arrays containing one element for each angular sector, the equal areas concept, derived by Doyle-Skolnik work is substituted by the concept of equal volumes. Thus, at the end of the first step, the procedure permits to find the radii of the different k-rings($\{r_1, \ldots \ldots r_2, \ldots \ldots r_{Krings}\}$), the number of element for each ring and consequently the N iso-volume angular sector.

To find numerically the iso-volume angular sector, the scaled cumulative distribution is divided into N equal increments in the ordinate axis and these points are projected onto the ρ axis. This is equivalent to compute the following set of radii:

$$\{\rho_1, \ldots \ldots \rho_2, \ldots \ldots \rho_N\} \qquad (57)$$

in such a way that

$$\hat{I}(\rho_n) = n \qquad where\ n = 1,2,3 \ldots \ldots N \qquad (58)$$

By extracting the searched set $\{r_1, \ldots \ldots r_2, \ldots \ldots r_{Krings}\}$ from the set (57) and enforcing for each k the following condition:

$$v_k = \hat{I}(r_k) - \hat{I}(r_{k-1}) \qquad (59)$$

Then a number of K_{rings} rings and N iso-volume sectors are achieved. However, at this stage several sets of $\{r_1, \ldots \ldots r_2, \ldots \ldots r_{Krings}\}$ may be extracted from the origin set (57) and therefore a definition of suitable criterion is needed. To this aim, it can be enforced the quite natural requirement that all the isovolume sectors should be as "square" as possible, in other words, with equal radial and azimuth extensions. Of course, such a solution is particularly appropriate for arrays elements having the same dimensions along two orthogonal coordinates. This entails that the following iterative rule can be used :

$$\frac{\pi(r_{k+1} + r_k)}{r_{k+1} - r_k} = \hat{I}(r_{k+1}) - \hat{I}(r_k) \qquad (60)$$

Subject to the constraints:

$$\frac{\pi(r_{k+1} + r_k)}{v_k} \geq d; \quad r_{k+1} - r_k \geq d \qquad (61)$$

where *d* is the dimension of the feed in the array.

Applying iterative the equation (60) with constraints (61) it is possible to define in a deterministic way the radius r_{k+1} once the radius r_k is well known. Consequently, the use of equation (59) permits to define the number of adjacent angular sector for each ring.

This proposed method permits to outperform similar method reported in [65] and [93] where the minimization of the mean square difference of continuous and sampled source pattern is not assured as in the proposed method.

After the numerical definition of the K rings and of the number of angular sectors, the positions of each radiating element inside the angular sector need to be defined. In order to define the position of each element inside each angular sector two parameters have to define:

- φ_n^k with $n \in \{0,1 \ldots \ldots . . v_k - 1\}$
- $R_k \in [r_{k-1}, r_k]$

Due to the azimuth symmetry of the pattern, azimuth distance between adjacent elements of the same ring is kept constant and so $\varphi_n^k = \frac{2\pi n}{v_k} + \varphi_k$, where the reference angle φ_k determines in practise the angular positions of all the adjacent sector of the k-th ring and it can be set randomly. Regarding the parameter R_k, in a similar way as in the Doyle procedure, it is chosen in order to enforce the minimization of the right-hand side in equation (50) and therefore it can be set in order to satisfy the following relationship:

$$\hat{I}(R_k) = \frac{\hat{I}(r_{k-1}) - \hat{I}(r_k)}{2} \qquad (62)$$

At the conclusion of this paragraph, a full deterministic procedure for the discretization of a circularly symmetric continuous real (nonnegative) source in an isophoric planar arrays organized in the concentric ring is exposed. This method is a rigorous extension of the Doyle procedure for the 1D case where the two steps are carried out. The first one for the definition of the concentric rings and their subdivision in angular sectors subtended the same volume, and the second one for the definition of the positions of each radiating element inside each angular sector.

The presented procedure is investigated in a very deep way because it represents an important step for the development of a new full deterministic and fast method for the discretization of a circularly symmetric continuous complex source.

4.4 Density taper for the discretization of a complex continuous source into isophoric sparse linear arrays

As described in the previous two sections, in the case of isophoric sparse arrays radiating pencil beams, the approaches exploiting a real and positive continuous source acting as a reference and benchmark in the arrays layout design is explored in an accurate manner. In particular, the following two-steps strategy has been proposed for the generation of linear or concentric ring layouts:

1. identify the optimal real and positive continuous source fulfilling the required specifications;
2. determine the arrays layout by applying a 'density taper' technique to the continuous source.

Notably, as both steps may be carried out by means of procedures having a linear or quadratic complexity in the unknowns, this strategy gives advantages in terms of effectiveness and computational burden.

The previous approach for the synthesis problem of isophoric sparse arrays radiating shaped beams, power patterns lying in a mask specified by two 'upper' and 'lower' bound functions, is not fully applicable due to the fact that the 'classical' density taper techniques do not work with complex sources. It has to be stressed that it exists a deterministic approach to deal with the step one (see paragraph 3.3 of chapter 3 of this thesis) of the procedure.

Before considering the case of shaped beams radiated by a concentric ring isophoric sparse arrays the attention will be given to the case of linear isophoric sparse arrays radiating shaped beam in order to acquire in a simple way some key concepts which will be used in the definition of a procedure for the 2D case.

The procedure for the synthesis of linear isophoric sparse arrays radiating shaped beam is reported in [90] and it foresaw two separate steps:

1. determination of a continuous complex reference source fulfill the requirements in terms of power mask;
2. determination of the arrays layout and of the elements excitation phases, using the reference source

Regarding the first step it is performed exploiting at the best the procedure reported in [12] for the linear case. It must be stressed that, in the case of shaped beams, the first step provides not one but a set of sources giving rise to the same power pattern [20]. Accordingly, we need some criterion to select a reference solution in such a set. This is a relevant and non-trivial point, as the criterion adopted to select the reference function of the sparse arrays synthesis can significantly affect the final results. In turn, the "goodness" of the reference source depends on the way the sparse array is generated, so that the two problems are indeed related.

Once the first step is performed, it is needed the second step is addressed. The procedure reported in [12] wants to generalize the Doyle procedure used for a continuous real source which ensures the minimization of the weighted mean square difference between the patterns radiated by the reference source and by the isophoric sparse array. This is equivalent to require that both sources radiate the same field in the boresight direction, and to minimize the mean square difference between the corresponding cumulative distributions.

Such minimization is achieved by:

- subdividing the line source into a number of sub-intervals equal to the number of arrays elements, in such a way that the field radiated by each sub-interval in the boresight direction is equal to that of the isophoric array;
- locating the element of the array in each sub-interval in such a way that the mean square difference between the cumulative distributions of the continuous source and of the arrays is minimized.

In order to generalize such procedure to the case of complex continuous sources, the cumulative distribution of the reference function $i(x)$, with $x \in [0, L]$, where L is the maximum linear source define at the first step, is considered:

$$I(x) = \int_0^x i(x')dx' \qquad (63)$$

The behaviour of $I(x)$ respect the coordinate *x* can be represented by a curve in a complex plane or as a curve in a three-dimensional space where the third coordinate is the position on the line source while the other two represent the real and imaginary part of the complex cumulative function (see Fig. 4.4-1)

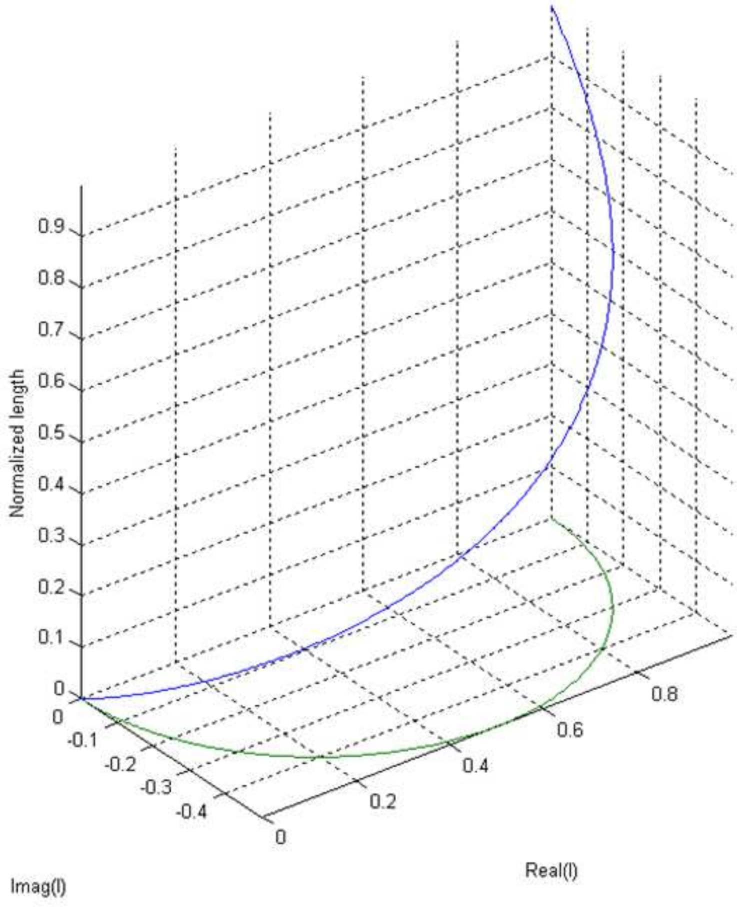


Fig. 4.4-1: 3D representation of the cumulative function, blue and green curve

In the complex plane representation, the boresight contribution of any sub-interval of the source is clearly represented by the segment joining the extremes of the corresponding section of $I(x)$. The isophoricity requirement entails that the lengths of all the segments joining the points of $\mathrm{I}(\mathrm{x})$corresponding to the extremes of the sub-intervals must be equal. This implies that the source subdivision has to be obtained by inscribing in the curve representing $\mathrm{I}(\mathrm{x})$ in the complex plane an equilateral polygonal (red curve in Fig. 4.4-2).

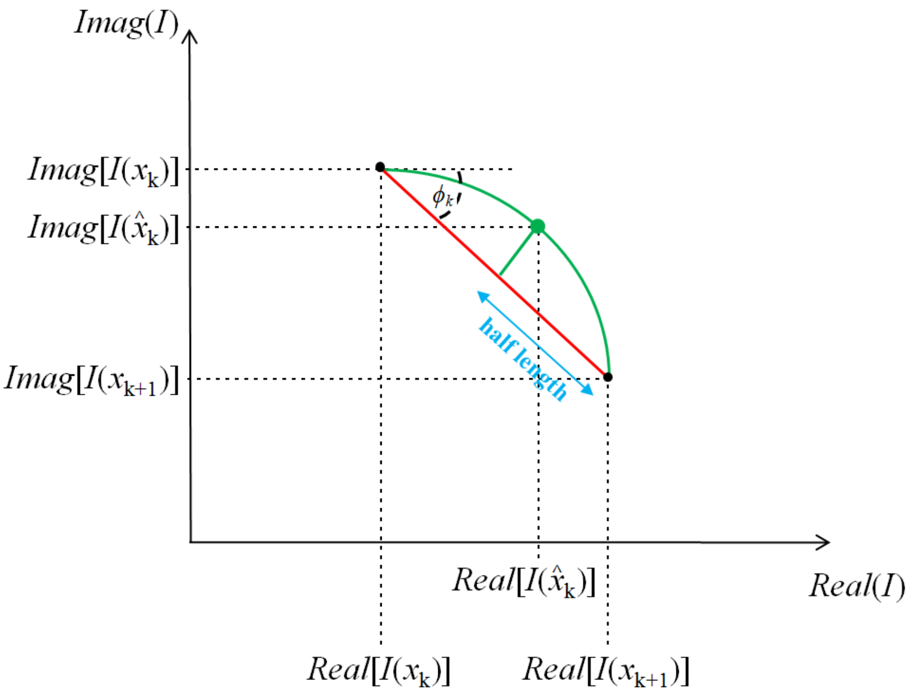


Fig. 4.4-2: Geometrical representation of a segment of the equilateral polygon dividing the continuous source

Once the sub-intervals of the line source have been identified, the locations and excitation phases of the isophoric sparse arrays elements have to be determined as follows.

The extremes of each sub-interval will be denoted with $\mathrm{x}_0, \ldots\ldots \mathrm{x}_\mathrm{N}$ ($\mathrm{x}_0 = 0$ and $\mathrm{x}_\mathrm{N} = \mathrm{L}$ with N the number of elements). Denoting with $\hat{\mathrm{x}}_\mathrm{k}$ the locations of the array elements accommodated in the k.th sub-interval, the cumulative distribution of the arrays will be given by

$$\mathrm{I}_\mathrm{A}(\mathrm{x}) = \sum_{\mathrm{k}=1}^{\mathrm{N}} \mathrm{u}(\mathrm{x} - \hat{\mathrm{x}}_\mathrm{k})[\mathrm{I}(\mathrm{x}_\mathrm{k}) - \mathrm{I}(\mathrm{x}_{\mathrm{k}-1})] \qquad (64)$$

where *u()* is the step function.

To determine the radiating elements positions, it has to minimize the mean square norm of the difference between $I(x)$ and $\mathrm{I}_\mathrm{A}(\mathrm{x})$,

$$||F - F_A||^2 = \sum_{\mathrm{k=1}}^{\mathrm{N}} \int_{x_{k-1}}^{x_k} |I(x) - \mathrm{I_A(x)}|^2 \, dx \qquad (65)$$

The equation (65) after several mathematician operations can be written as

$$||F - F_A||^2 = \sum_{\mathrm{k=1}}^{\mathrm{N}} \{ \int_{x_{k-1}}^{\hat{x}_k} |I(x) - I(x_{k-1})|^2 \, dx + \int_{\hat{x}_k}^{x_{k-1}} |[I(x) - I(x_{k-1})] - [I(x_k) - I(x_{k-1})]|^2 \, dx \} \qquad (66)$$

Equating to zero the derivative of equation (66) respect $\hat{x}_k$ the following relationship is obtained

$$Real\left[\frac{I(\hat{x}_{k-1}) - I(x_{k-1})}{I(x_k) - I(x_{k-1})}\right] = \frac{1}{2} \qquad (67)$$

which provides the position of the array element in each sub-interval.

The geometrical meaning of the equation (67) is reported in Fig. 4.4-2. In the complex $I(x)$ plane, the orthogonal projection of $I(\hat{x}_k)$ onto the corresponding side of the polygon must be its middle point. Projecting the middle points of the polygonal onto $I(x)$, and reading the corresponding abscissae the element position is obtained. After the definition of the position of the element inside each sub-interval of the continuous source, the excitation phase of each radiating element has to be defined. In according to the procedure the excitation phase of each element is represented by the angle in the complex $I(x)$ plane between the real axis and the polygonal.

Such design technique extends to complex electromagnetic sources the Doyle approach, exactly solving a strongly nonlinear problem and providing a completely deterministic synthesis procedure having a negligible computational burden.

The design technique is depicted in Fig. 4.4-3, wherein the isophoric sparse array's element locations are represented by the black points in the "normalized length" axis, which are the projections (through the cumulative distribution in blue color) onto the spatial axis of the green points defined in the complex plane by condition (67).

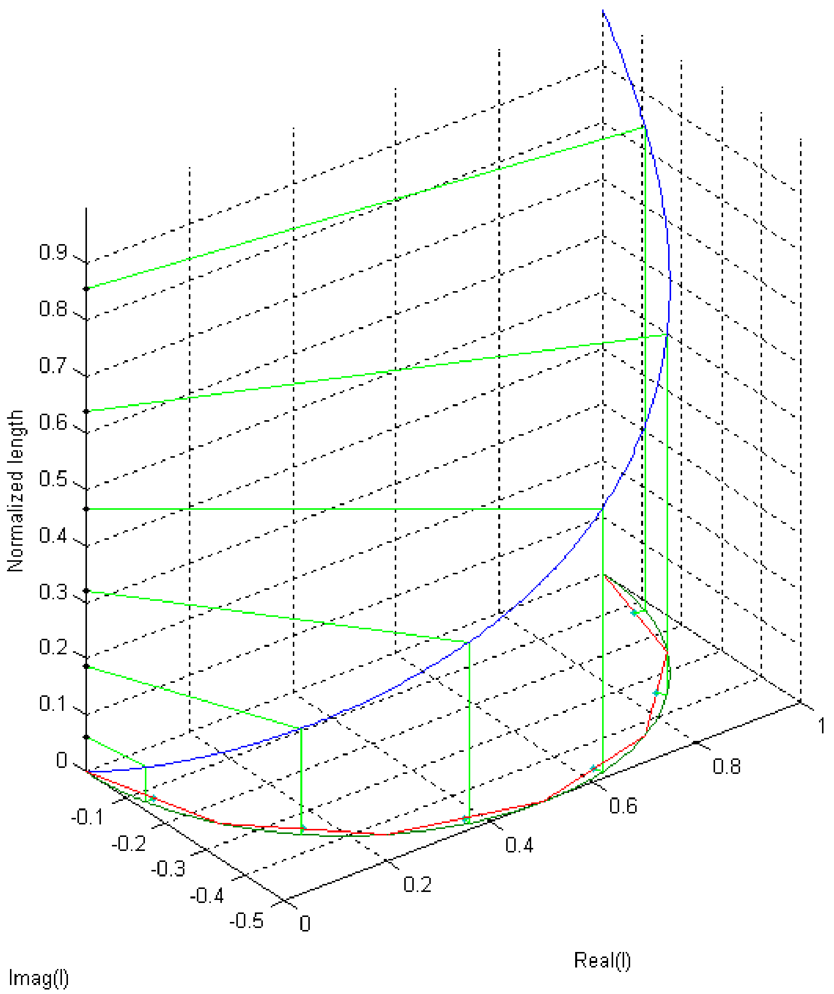


Fig. 4.4-3: Representation of the synthesis procedure for isophoric sparse arrays radiating shaped beam

A last important aspect which needs to be addressed is related to the choice of the reference source due to the multiplicity of complex source radiating the same shaped beam. In the light of the above results, it is clear that we must avoid sources whose cumulative distribution has a spiral-like shape. As a matter of fact, such a shape would require either the use of polygonal with a small side (and hence a huge number of arrays elements) or the acceptance of intersections between the reference cumulative distribution and the polygonal itself.

Such circumstance not only would make the solution of (67) not unique, but would also imply a large discretization error in the high-frequency part of the source's spectrum.

4.5 Optimal Synthesis of Shaped beam through concentric ring isophoric sparse array.

An innovative deterministic approach to the optimal power synthesis of mask-constrained shaped beams through concentric-ring isophoric sparse arrays is presented in this section. The design procedure exploits at best the state-of-the-art techniques respectively available in the cases of circular-ring isophoric arrays radiating pencil beams and of linear isophoric arrays generating shaped beams presented in the first part of this chapter. The technique avoids exploitation of global-optimization algorithms and it allows to significantly outperform all the (few) available procedures. At the end of the section, numerical examples are furnished to validate the complete design procedure.

Amongst all isophoric sparse arrays planar architectures, Concentric Ring Isophoric Sparse Arrays (CRSAs), i.e., Isophoric Sparse Arrays whose elements are disposed onto concentric rings, appear being the most convenient ones due to their capability of uniformly spreading the antenna energy over all azimuth directions [18],[22]-[26]. In fact, CRSAs constitute one of the usual ESA's choices to realize the satellite multibeam coverage of Earth [18],[25],[26].

Of course, isophoric sparse arrays adoption has also its disadvantages, the most critical one being related to the corresponding synthesis procedures. In fact, since the elements' locations are the unknowns of the design problem, the synthesis is unaffordable through Convex Programming (CP) procedures of the kind presented in [27]. Therefore, as done for instance in [13]-[16], the antenna designers often recur to Global Optimization (GO) procedures. However, due to their high computational weight, GO techniques practically result unsuitable for the synthesis of isophoric sparse arrays composed by a large number of elements.

To overcome such difficulties, the following two-steps procedure has been recently devised for the design of Linear Isophoric Sparse Arrays (LISAs) and CRISAs [17]-[24] as demonstrated in the previous section of this chapter:

1. identify a Reference Continuous Aperture Source (RCAS) fulfilling 'at best' the radiation requirements at hand;
2. derive the arrays layout as a discretization of the RCAS.

This procedure allowed to outperform previous approaches [17]-[24]. In fact, a number of well-assessed methods already exist to perform step 1 (e.g., [28],[29]) and, only in the 'pencil beams' case, step 2 (e.g., [2],[17],[18],[22]-[24]).

Unfortunately, much fewer alternatives to perform step 2 are available in the 'shaped beams' case. The reason for such lack derives from a simple circumstance: while the RCASs required to generate sufficiently-narrow pencil beams are real functions [28], in the shaped beams case they result complex [29]. This issue, which drastically complicates step 2 [23], has been recently solved for the case of LISAs in [21] but still results unsolved for CRISAs. In fact, the unique approach currently available to perform step 2 in the CRISAs case is the 'rough' one in [23], which bypasses the problematic of the RCAS's complexity by:

a) identifying the elements' locations by applying the technique presented in [18] only to the RCAS's amplitude;
b) assigning to each array's element an excitation phase equal to value assumed in its location by RCAS's phase.

This procedure neglects the fact that, as discussed in [21], the arrays elements locations must be a function of both the RCAS's amplitude and phase distributions, and hence its performance is considerably improvable. On the other side, beyond [23], the unique contribution addressing the synthesis of CRISAs in the shaped beams case is [16] which, however, relies on GO and hence is exploitable only in case of CRISAs composed by a low number of elements.

In the attempt of filling such a gap, in this section a new approach to the mask-constrained power synthesis of shaped beams through CRISAs is introduced. The technique can be seen as the extension of the approach in [21] to the case of ring symmetric arrays layouts, and results fast and effective even in case of arrays composed by a large number of elements.

4.5.1 Rationale of the design procedure

The proposed approach consists in synthesizing the CRISA by performing a discretization of a RCAS which optimally fulfills a given circularly-symmetric power mask. Denoting with (ρ) such a RCAS, having a circularly-symmetric distribution and covering a disk of radius R over the x-y plane. Moreover, N denotes the overall number of CRISA elements and ϕ the aperture azimuth coordinate.

The CRISA is conceived as the union of M concentric rings on which a given number of radiating elements is located with a uniform angular spacing, see Fig. 4.5.1-1

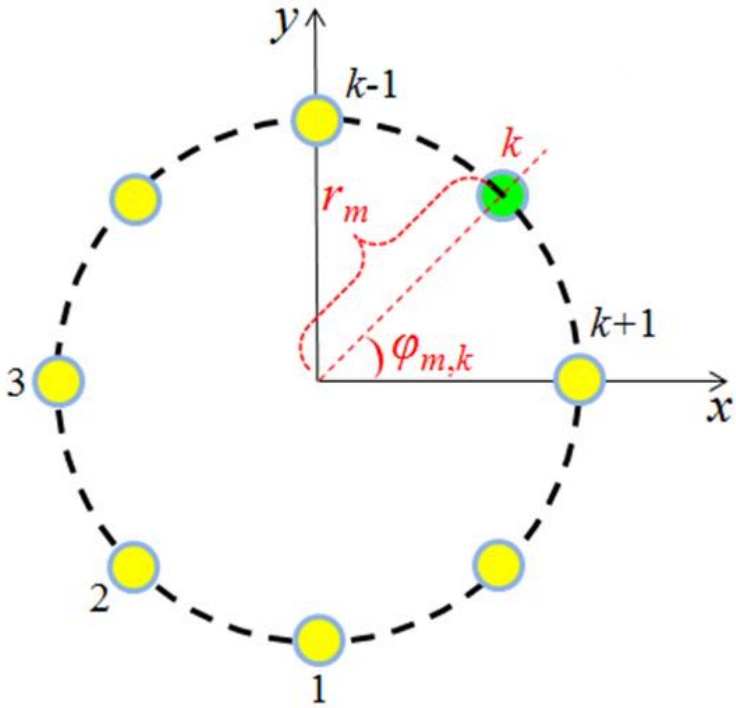


Fig. 4.5.1-1: Pictorial view of the m-th ring of a generic CRISA

The unknowns are the elements' locations and excitation phases and the aim is to determine them in such a way to minimize the mean square difference between the far-field distributions respectively corresponding to the RCAS ($F^{RCAS}(u)$) and the CRISA ($F^{CRISA}(u)$). Apart from inessential constants, these two fields can be written as:

$$F^{RCAS}(u) = \int_0^R s(\rho) J_0(\beta\rho u)\rho d\rho \qquad (68)$$

$$F^{CRISA}(u) = \int_0^R s^A(\rho) J_0(\beta\rho u)\rho d\rho \qquad (69)$$

Where

$$s^A(\rho) = \int_0^{2\pi} \sum_{m=1}^{M} \sum_{k=1}^{N_m} \delta(\rho - r_m)\delta(\phi - \varphi_{k,m}) \left(\frac{1}{\rho}\right) d\phi \qquad (70)$$

Wherein $u = \sin(\vartheta)$ (ϑ denoting the elevation angle with respect to boresight), $\beta = 2\pi/\lambda$ (λ denoting the wavelength), r_m and N_m respectively are the radius of the m-th CRISA's ring and the number of elements located over it, and $\varphi_{k,m}$ is the azimuth coordinate of the k-th element belonging to the m-th ring. It must be noted that differently from the RCAS's far field, the CRISA's arrays factor depends also on the azimuth angle. However, as shown in [9], such dependence is negligible for u<<Nmλ/(2πrm), m=1,…,M. In all examples of the next sections, as well as in very many applications, these angular sectors cover the whole region wherein the power pattern is significant, and hence the circularly symmetric representation (69) can be exploited.

Under these assumptions, it will be:

$$\int_0^{\infty} \left| \frac{F^{RCAS}(u) - F^{CRISA}(u)}{\beta u} \right|^2 du = \int_0^R |S(\rho) - S^A(\rho)|^2 d\rho \qquad (71)$$

Wherein $S(\rho)$ and $S^A(\rho)$ represent the cumulative distributions associated to the function $s(\rho)$ and $s^A(\rho)$:

$$S(\rho) = \int_0^{\rho} s(\xi)\xi d\xi \qquad (72)$$

$$S^A(\rho) = \int_0^{\rho} s^A(\xi)\xi d\xi \qquad (73)$$

Therefore, minimizing the mean square difference between $F^{RCAS}(u)$and $F^{CRISA}(u)$ with $\frac{1}{u^2}$ weighting is equivalent to minimize the mean square difference between the functions (72) and (73). This can be done, once M and N have been chosen, by means of the following procedure:

1. represent S(ρ) as a curve in a three-dimensional space where the first and second coordinates are its real and imaginary parts, respectively, and the third coordinate is ρ (see Fig. 4.5.1-2). Then inscribe in this curve an equilateral polygonal composed by N segments;
2. for m=1,..,M, determine the value of N_m in such a way that the N segments above can be grouped into M contiguous subintervals, the m-th of which is composed by N_m segments and guarantees that the following ratio:

$$V_m = \left|\frac{S(\rho_{m+1}) - S(\rho_m)}{N_m}\right| \qquad (74)$$

is constant (wherein ρ_m and ρ_{m+1} denote the endpoints of the m-th subinterval, with $\rho_1 = 0$ and $\rho_{M+1} = R$);
3. for $m = 1 \ldots.. M$, determine r_m in such a way to fulfil the following equation:

$$Re\left[\frac{S(r_m) - S(\rho_m)}{S(\rho_{m+1}) - S(\rho_m)}\right] = \frac{1}{2} \qquad (75)$$

4. assign to all the elements belonging to the m-th ring, for $m = 1 \ldots.. M$, an excitation phase equal to the angle subtended in the complex plane by the real axis and the segment connecting $S(\rho_m)$ and $S(\rho_{m+1})$.

The motivations underlying the procedure are in the following.

Concerning step 1, it derives from a simple principle: the fact that the CRISA must be composed by N isophoric elements entails that $S(\rho)$ must be partitioned in N segments having the same length.

As far as step 2 is concerned, it must be noted that $S(\rho)$ represents the volume subtended by the RCAS over the circle of radius ρ. Therefore, this step allows to subdivide the aperture into M concentric annular sectors, the m-th of which contains N_m 'iso-volume' sectors (see Fig. 4.5.1-2). This operation can be performed by exploiting the fast iterative procedure in [18], which

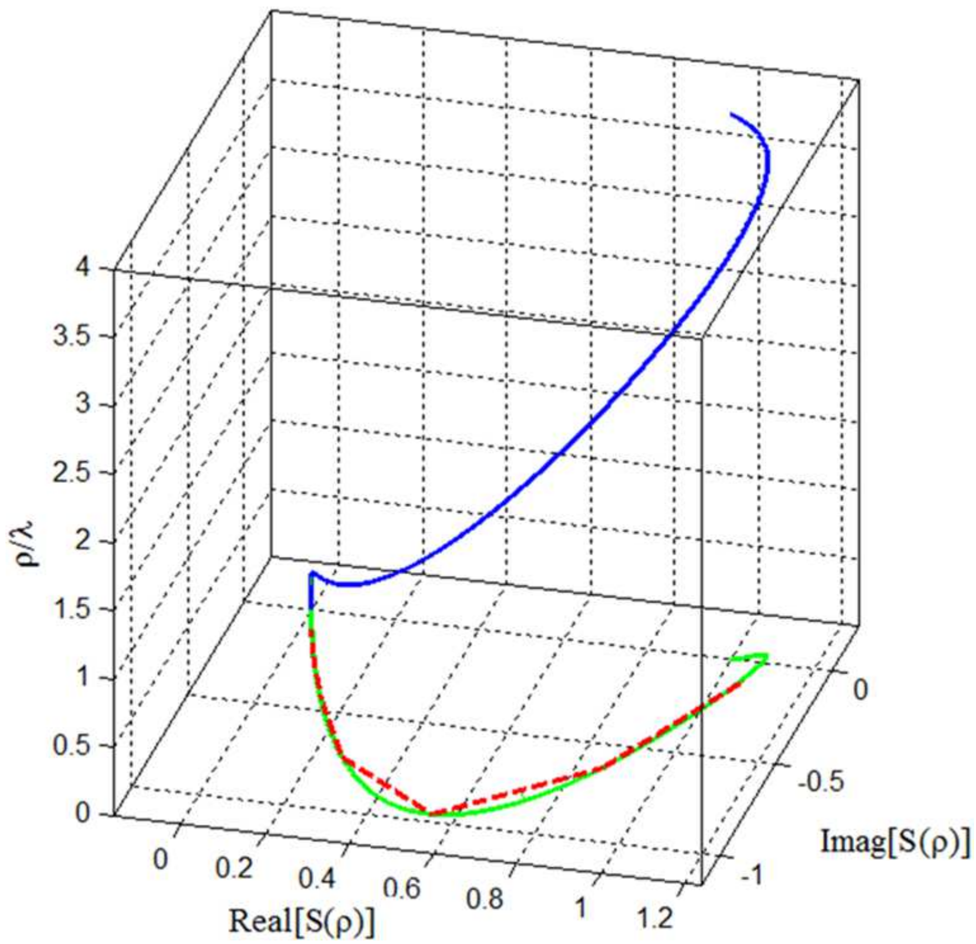


Fig. 4.5.1-2: Representation of S(ρ) in the complex plane (green color: reference function; red color: discretized function) and as a curve in a three-dimensional space where the first and second coordinates are its real and imaginary parts, respectively, and the third coordinate is the radial coordinate (blue curve)

exploits an equation analogous to (74) and provide an analytic way to determine the optimal V_m value. On the other side, it should be noted that [18] addresses only the synthesis of pencil beams through real and positive RCASs, so that the additional steps 3 and 4 are necessary herein.

Step 3 allows identifying, for $m = 1 \dots\dots M$, the optimal radius the m-th CRISA's ring inside the interval $[\rho_m, \rho_{m+1}]$. In fact, by virtue of steps 1 and 2 above, it will be:

$$S^A(\rho) = \sum_{m=1}^{M} H(\rho - r_m)\left[S(\rho_{m+1}) - S(\rho_m)\right] \qquad (76)$$

Where H is the step function. By substituting (76) in (71) it can be achieved:

$$\int_0^R |S(\rho) - S^A(\rho)|^2 d\rho$$

$$= \sum_{m=1}^{M} \int_{\rho_m}^{\rho_{m+1}} |S(\rho) - S^A(\rho)|^2 d\rho$$

$$= \sum_{m=1}^{M} [\int_{\rho_m}^{r_m} \left|S(\rho) - S(\rho_m)\right|^2 d\rho$$

$$+ \int_{r_m}^{\rho_{m+1}} \left|S(\rho) - S(\rho_{m+1})\right|^2 d\rho] \qquad (77)$$

With $S(0) = S^A(0) = 0.$ By equating to zero the derivative of (77) with respect to r_m the equation (75) is exactly obtained. Therefore, the radius of the m-th CRISA ring must be determined by projecting the midpoint of the m-th polygonal segment onto $S(\rho)$ and then reading the corresponding ρ value.

Finally, step 4 is perfectly coherent with the previous steps and allows to assign to the CRISA's elements the required circularly-symmetric excitation-phase distribution.

4.5.2 A numeric example of the deterministic synthesis of the shaped beam

In this paragraph a numeric example is carried out in order to validate the complete design procedure reported in the previous sections of this chapter regarding the optimal synthesis of shaped beams through circular ring isophoric sparse array.

The scope is to start from the power mask constraints and to design a circularly symmetric isophoric sparse array able to radiate a desired shaped beam. The unknowns of the problem are the number of radiating elements, their location on the plane, and their phase excitation.

At the first, it is needed to define the power mask constraints for the desired radiation power pattern:

- Pattern shape: Flat top beam
- Ripple : $\pm\ 0.5\ dB$
- Flat region: $\pm\ 3\ degrees$

- Sidelobes level $\leq -20\, dB\ for\ 5 \leq |\vartheta| \leq 15\ degrees$ and $-15\, dB\ for\ |\vartheta| > 15$ degrees

The constraints are reported in the below power mask which has a circular symmetry in the azimuth plane.

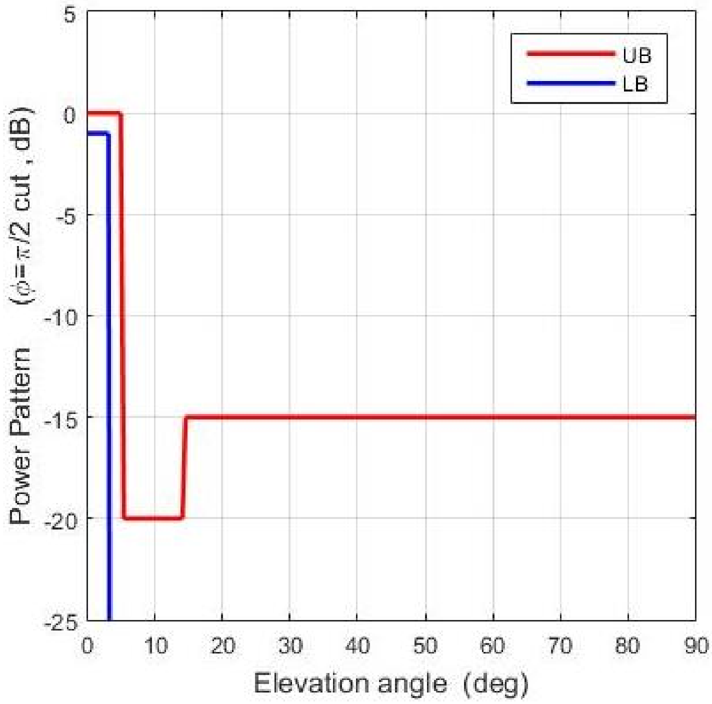


Fig. 4.5.2-1: Power mask constraints for the desired flat top beam

Assigned the designed constraints in terms of desired power pattern, the first step of the proposed procedure foreseen the definition of the continuous complex source able to meet the exposed requirements.

Performing the design procedure for the synthesis of continuous source able to radiate a shaped beam reported in section 3.3 of this thesis the complex source shown in Fig. 4.5.2-2 is obtained. The minimum radius for the theoretical source is 25 λ.

After that the first step of the procedure is performed, in order to discretize the continuous source in a concentric ring isophoric sparse array, the optimal fast and deterministic procedure reported in section 4.5.1 is exploited.

According to the equation (72) the cumulative function of the complex continuous source is calculated and its 3D representation is shown in Fig. 4.5.2-3 and Fig. 4.5.2-4.

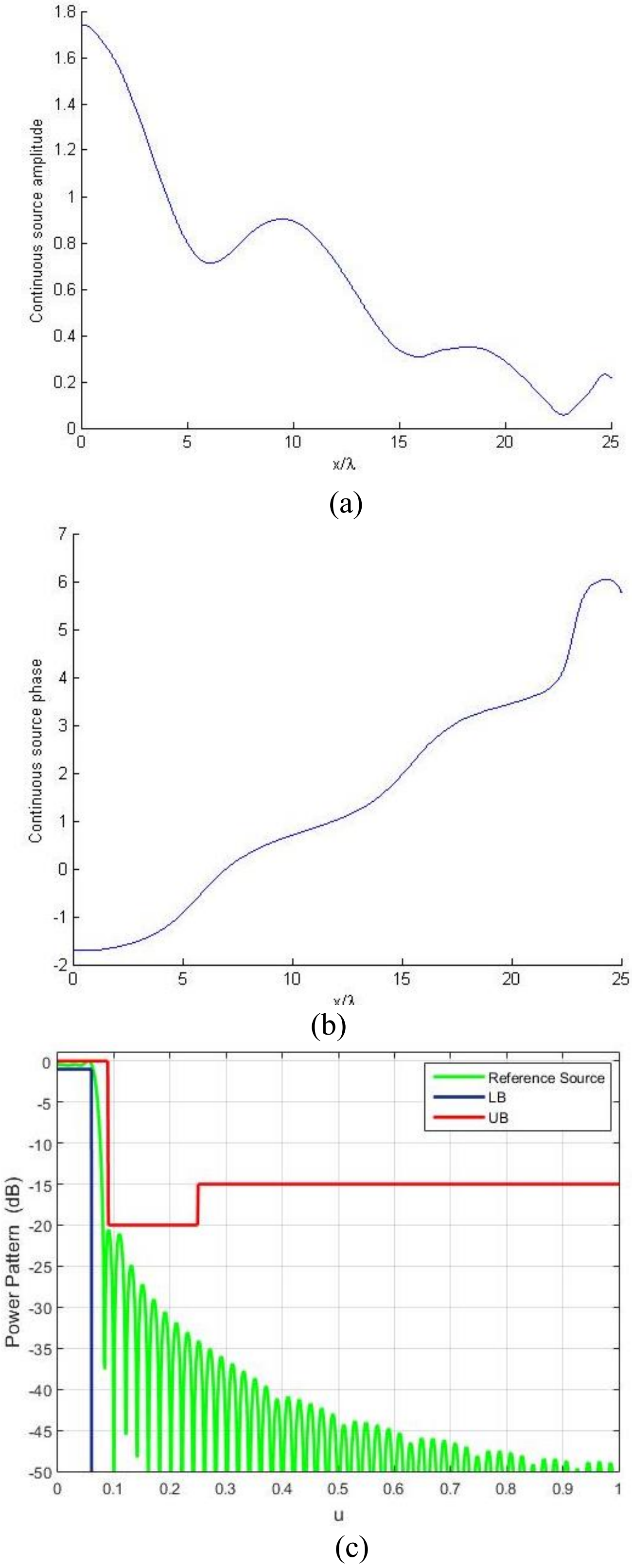


Fig. 4.5.2-2: Amplitude (a) and phase (b) distribution of the optimal continuous source and (c) relative radiation pattern

Defined the cumulative complex source, the procedure foresaw the subdivision of the cumulative function in the complex plane in N equal length segment where N is the number of isophoric elements in the final array. The N elements are organized in M subinterval in a such way that each subinterval has $\mathrm{N_m}$ element in order to satisfy the equal-volume ratio as reported in equation (74). In a few words, this step allows to subdivide the aperture into M concentric annular sectors, the m-th of which contains N_m 'iso-volume' sectors, see Fig. 4.5.2-5.

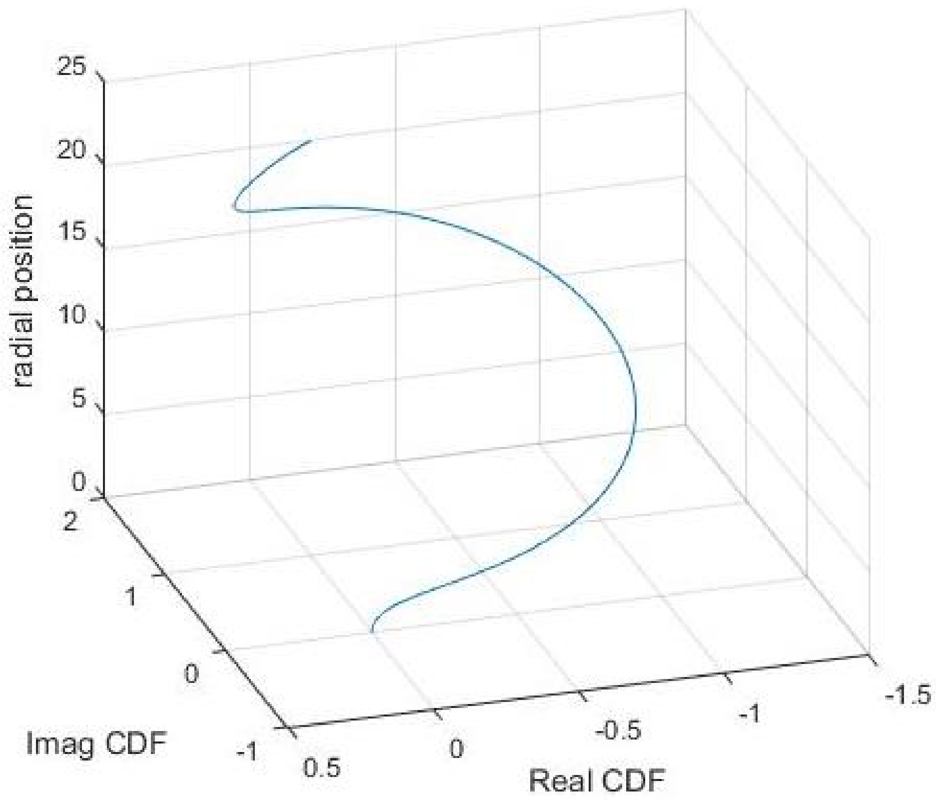


Fig. 4.5.2-3: 3D cumulative function (CDF) of the complex source reported in Fig. 4.5.2-2

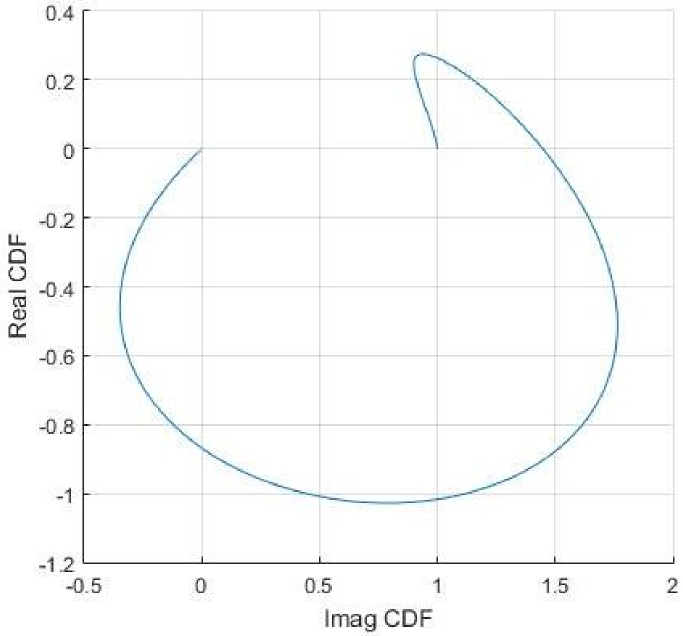


Fig. 4.5.2-4: 2D cumulative function (CDF) of the complex source reported in Fig. 4.5.2-2

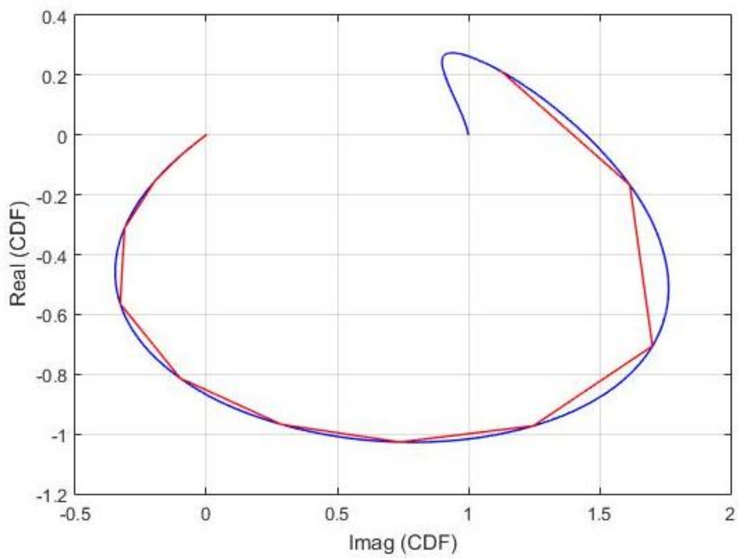


Fig. 4.5.2-5: 2D cumulative function (CDF) of the complex divided in M equal volume sectors

At this stage of the procedure, the M adjacent equal volumes are defined, in particular 12 angular sectors are obtained. The next step is to identify the optimal radius the m-th CRISA's ring and it is carried out using the equation (75) which entails to select the middle point inside each singular sector., see Fig. 4.5.2-6

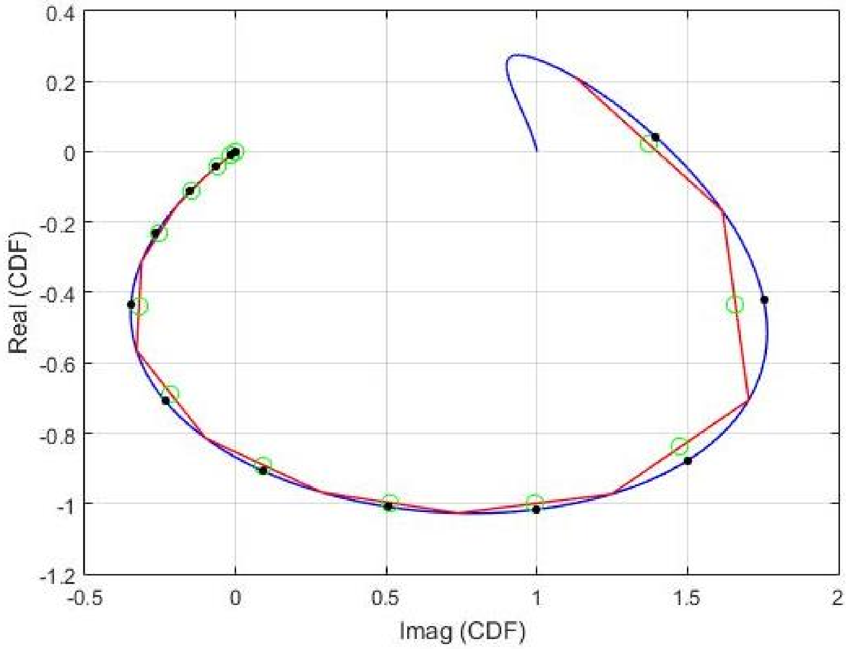


Fig. 4.5.2-6: 2D cumulative function (CDF) of the complex divided in M equal volume sectors and the middle point identifying the radius of each ring

At this stage pf the procedure the M rings, their radius and number of elements for ring are calculated. The last step of the procedure is regarding the definition of the phase coefficient for each radiating element. Each excitation phase is equal to the angle subtended in the complex plane by the real axis and the segment connecting $S(\rho_m)$ and $S(\rho_{m+1})$ (red curves in Fig. 4.5.2-6).

ID Ring	Radius (λ)	Excitation phase [deg]	Nr. of element
1	0.8113	-146.5567	5
2	1.6916	-144.4003	12
3	2.5992	-138.8694	19
4	3.7671	-126.3216	28
5	5.3018	-93.7088	38
6	7.1569	-47.4123	48
7	8.7691	-21.7821	56
8	10.1747	-7.4379	62
9	11.6732	6.0835	68
10	13.5025	30.4134	74
11	16.4066	99.1314	81
12	19.1401	142.2006	87

Tab 4.5.2-1: Radii and excitation phase of the isophoric sparse array

Tab 4.5.2-1 reports the geometrical information (radii and number of elements for each radius) and excitation phase of the isophoric sparse arrays calculated with the proposed procedure.

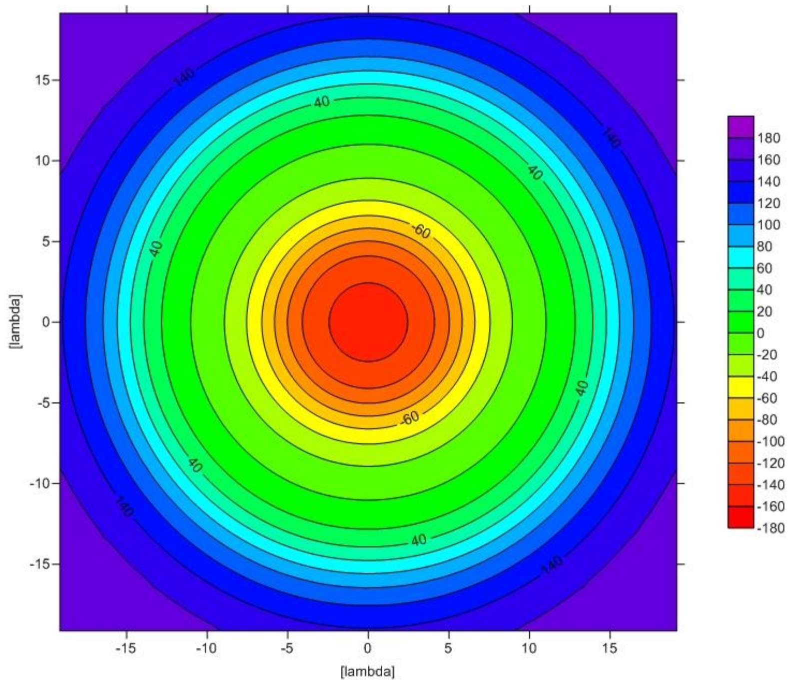


Fig. 4.5.2-7: Final optimal phase distribution on the arrays aperture

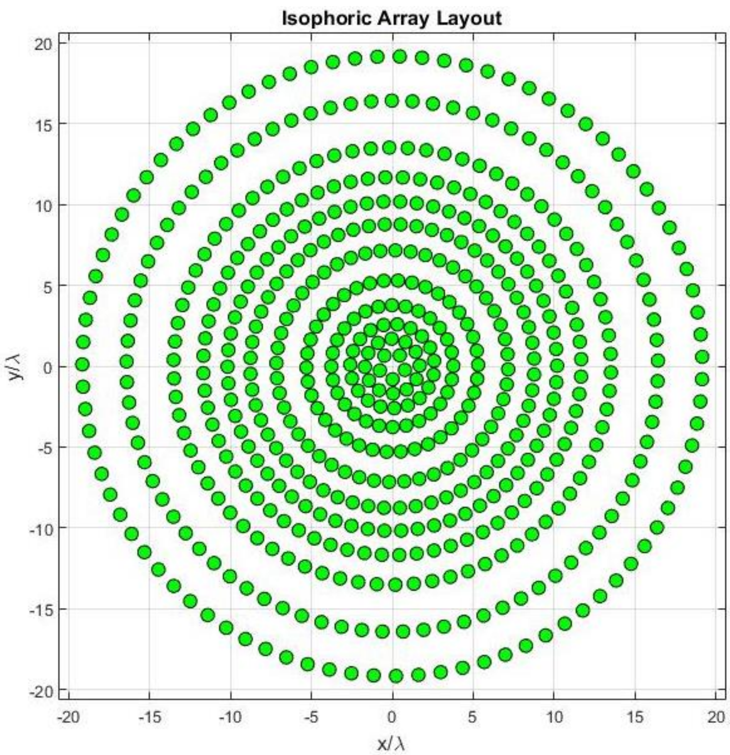


Fig. 4.5.2-8: Final Optimal concentric ring isophoric arrays layout

Fig. 4.5.2-8 shows the final optimal concentric ring isophoric arrays layout obtained with the new proposed full deterministic procedure and **Fig. 4.5.2-7** shows the optimal phase distribution on the arrays aperture.

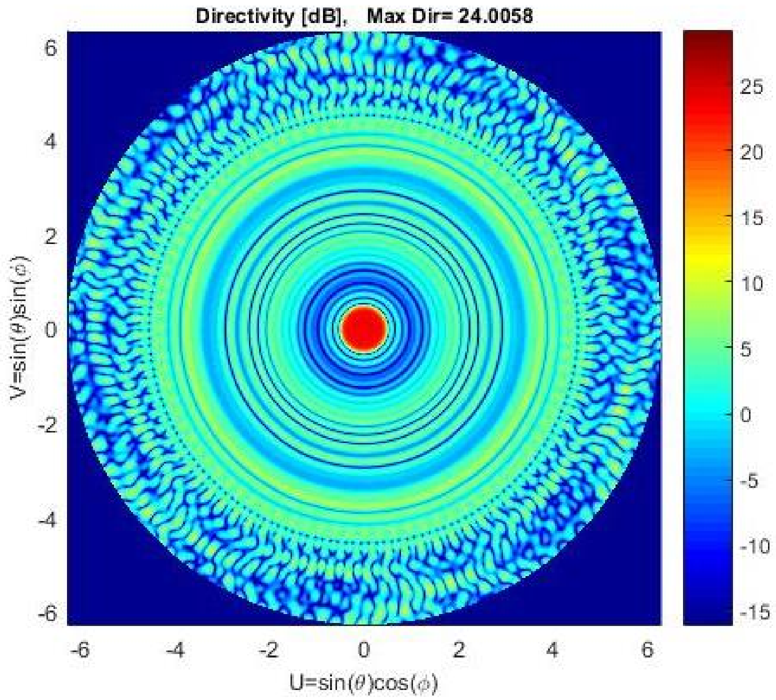


Fig. 4.5.2-9: UV radiation pattern of the concentric ring isophoric arrays layout

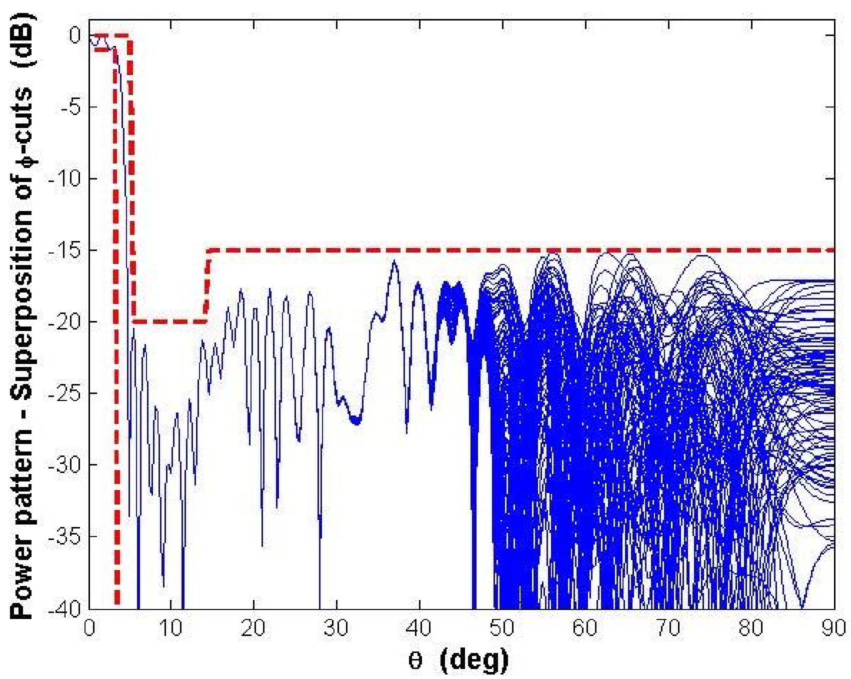


Fig. 4.5.2-10: Superposition of the ϕ-cuts of the achieved radiation pattern

Finally the radiation pattern is reported in Fig. 4.5.2-9 in the UV plane representation while the superimposition of all the φ cuts is shown in Fig. 4.5.2-10 where the circular symmetric is observed. From the concentric ring isophoric sparse radiation pattern it can be noted that the far field constraints are respected for all the phi cuts.

The problem of the optimal synthesis of shaped beams in the presence of completely-arbitrary lower and upper bounds on the power distribution has been solved by exploiting ring symmetric isophoric sparse arrays.

4.5.3 Comparison with the recent literature results

The aim of this paragraph is to compare the new approach for the synthesis of isophoric sparse arrays radiating shaped beam organized in concentric rings with the other approaches presented in the literature. At the first the comparison will be made considering the results reported in [23] where the general procedure for the synthesis of isophoric sparse arrays is exploited. As described in [23] the two-steps procedure consisting in finding the optimal continuous source and then discretizing it into isophoric sparse arrays. The deterministic method reported in [23] is synthesized in the following:

- Calculate the optimal non-superdirective source able to meets the requirements;
- Use the deterministic density taper technique reported in [18] to discretize the amplitude of the complex source;
- Assign to each radiating element, obtained in the previous step, the phase excitation picking in the theoretical phase distribution of the complex source.

The above-mentioned technique is a full deterministic method but represents a suboptimal solution respect the one reported in this thesis because it does not deal with complex sources but only with the amplitude distribution while the proposed method of this thesis considers the entire complex source in the discretization process.

In the following two examples involving constraints of possible interest for communication from geostationary satellites are reported.

In the first example, the goal is that of generating a circularly symmetric flat top having a beam width of 3.25 degrees ensuring a coverage of the Europe. Design specification also included a sidelobe level at least 20 dB below the flat top level in the region corresponding to the earth cone as seen from a geostationary satellite and an allowed ripple in the shaped zone equal to ±0.5 dB. The arrays layout and the radiation pattern reported in [23] are shown in Fig. 4.5.3-1.

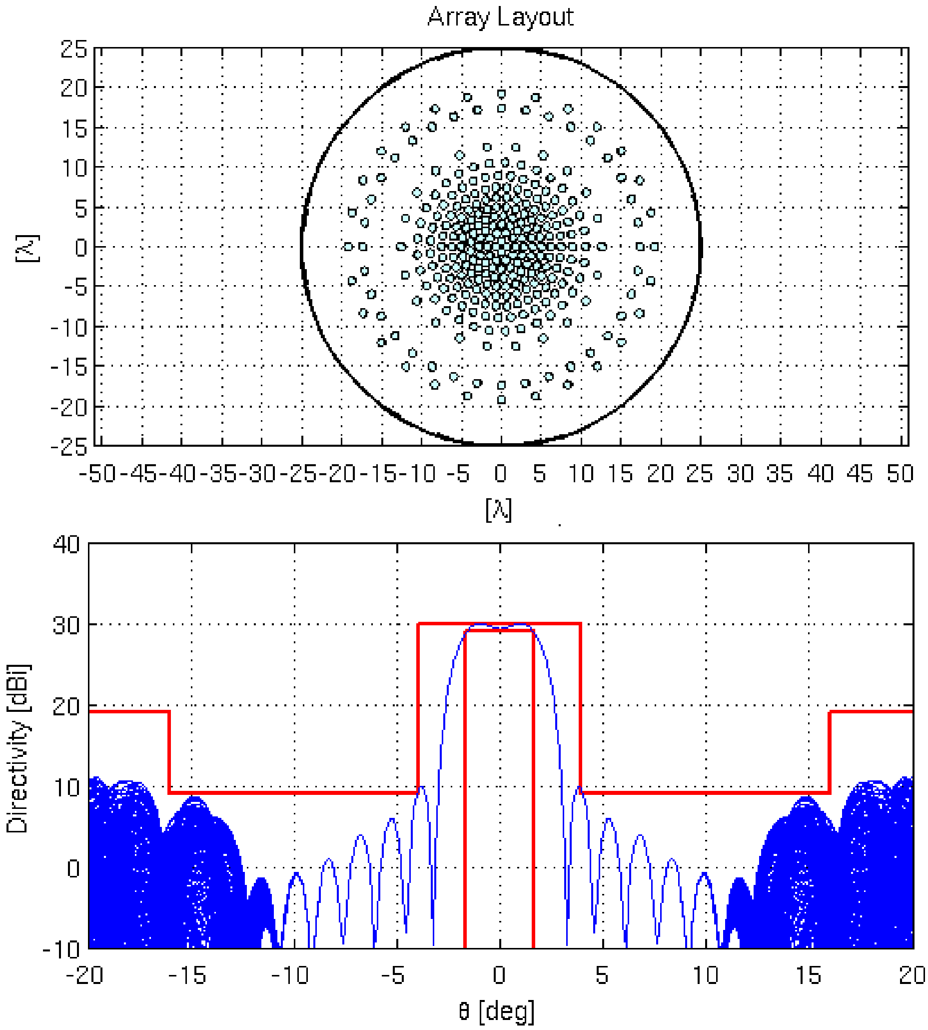


Fig. 4.5.3-1: Arrays layout and radiation pattern of [23]

The maximum achieved directivity is 28.9 dBi using a total number of 295 radiating elements with a radius of 1λ.

In contrast with the arrays layout of [23], Fig. 4.5.3-2 shows the arrays layout obtained exploiting the new CRISA synthesis presented in this thesis.

The total number of elements is 289 with a saving of quite 3% respect to the solution in [23]. The two solutions use the same radiating element with a diameter of 1 λ

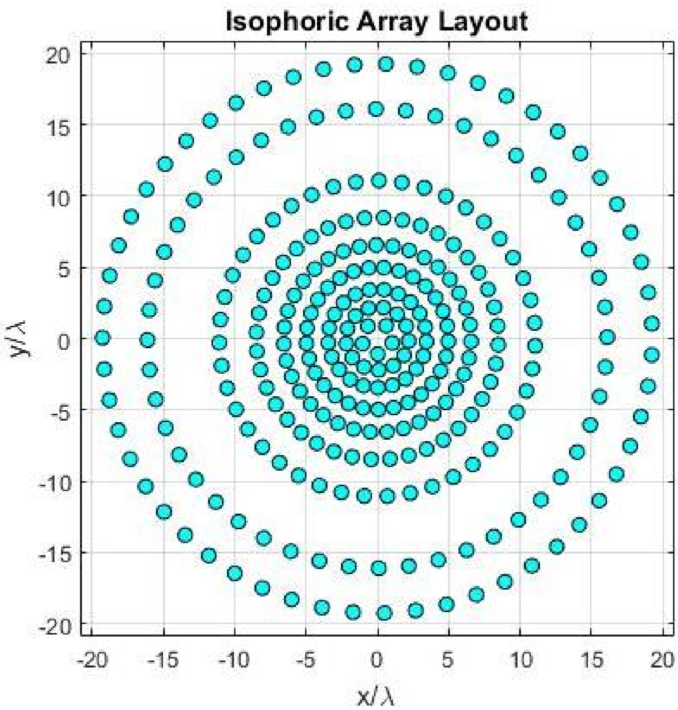


Fig. 4.5.3-2: Arrays layout and radiation pattern of the proposed method

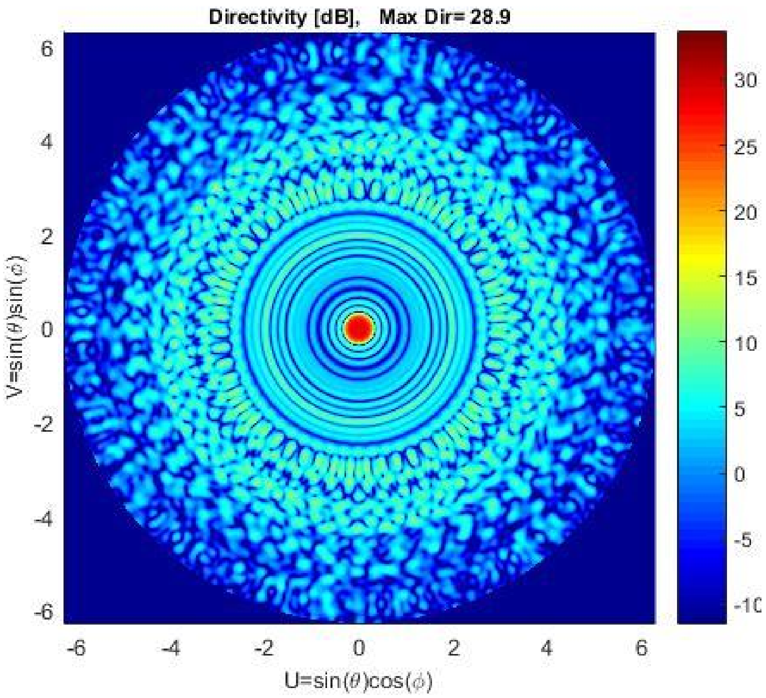


Fig. 4.5.3-3: UV pattern of the shaped beam calculated with the CRASA reported in Fig. 4.5.3-2

The antenna directivity is the same as the one reported in [23] and all the φ cuts of the radiation pattern are reported in Fig. 4.5.3-4 where it can be noted a good circular symmetry and the mask is respected for all the phi cuts.

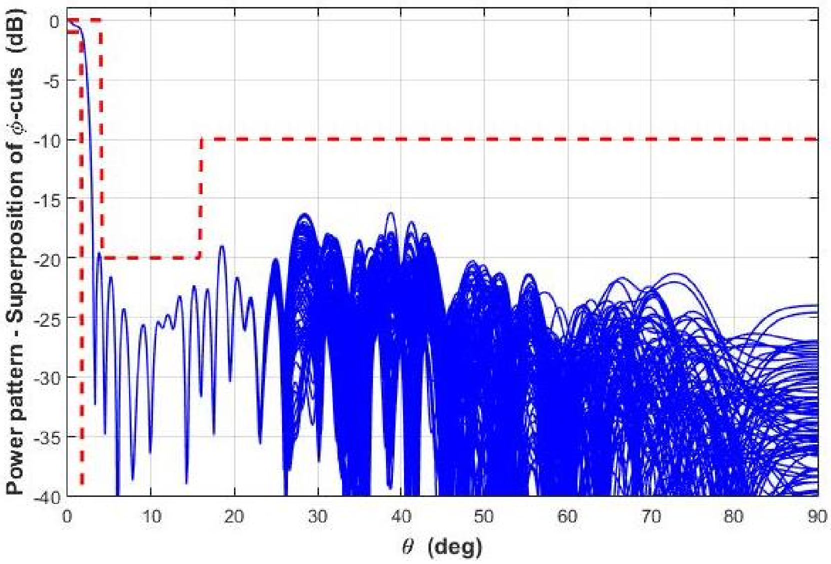


Fig. 4.5.3-4: Superimposition of all phi cuts of the designed isophoric sparse array

Comparing the results of [23] with the one obtained with the proposed method, it can be observed that the results are quite similar. In facts, only a saving of quite 3% in terms of radiating elements is achieved. The similarity in the two solutions has a justification and it is due to the fact that the power mask has a flat zone very tight and the optimal reference source which has been identified is very close to being real. In this condition the method of [23] exploiting only the amplitude of aperture distribution gives good results.

In the second example, extrapolated from [23] conceived by a specific mission scenario of the ESA ITT [94], the power pattern mask has a flat top zone covering almost all of the Earth disc as seen from a geostationary satellite A maximum sidelobe level of -20 dB and an allowed ripple of ± 0.5 dB have been enforced.

The arrays layout and the radiation pattern achieved in the article [23] are depicted in Fig. 4.5.3-5. The array is composed by a total number of 966 elements on an antenna aperture of 42 λ and the minimum antenna directivity is 18.3 dB in the flat zone. The radiating element has a radius of 0.8λ.

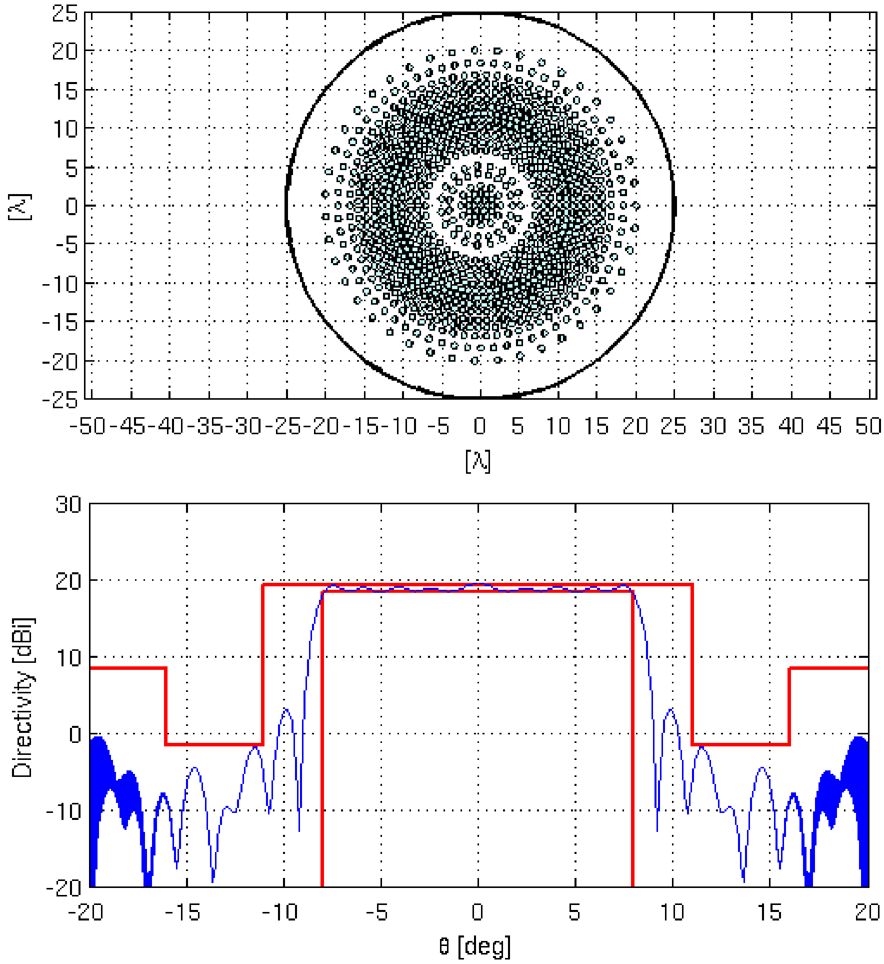


Fig. 4.5.3-5: Arrays layout and radiation pattern of [23]

Using the fast and deterministic method for the synthesis of CRISAs exposed in this thesis, the arrays layout reported in Fig. 4.5.3-6 is designed. The array is composed of 460 radiating elements with a radius of 0.5λ on an antenna aperture of 22 λ. The maximum antenna directivity is 20.8 dB as demonstrated in Fig. 4.5.3-7. Comparing the proposed arrays layout and the one reported in the article [23] a saving of quite 50% is achieved. The great difference in terms of number of radiating elements is due to the fact that, in this time, the initial requirements lead to a reference source exhibiting nonnegligible oscillations in both amplitude and phase distribution. The proposed method, considering the amplitude and the phase of the complex source during the discretization process, is able to outperform the performance achieved in [23], which has a discretization method based on the amplitude distribution.

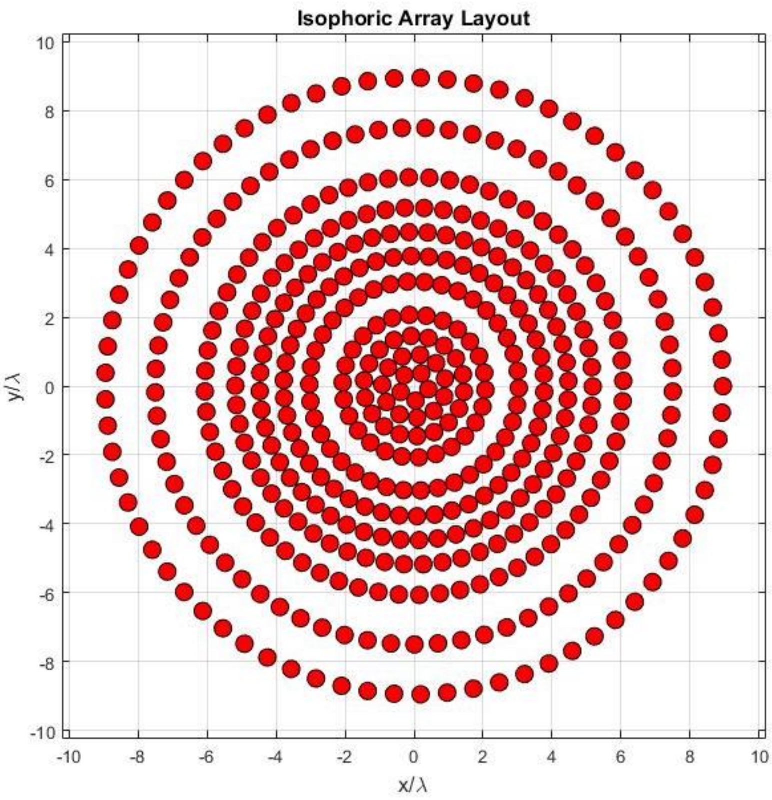


Fig. 4.5.3-6: Optimal arrays solution achieved using the proposed method

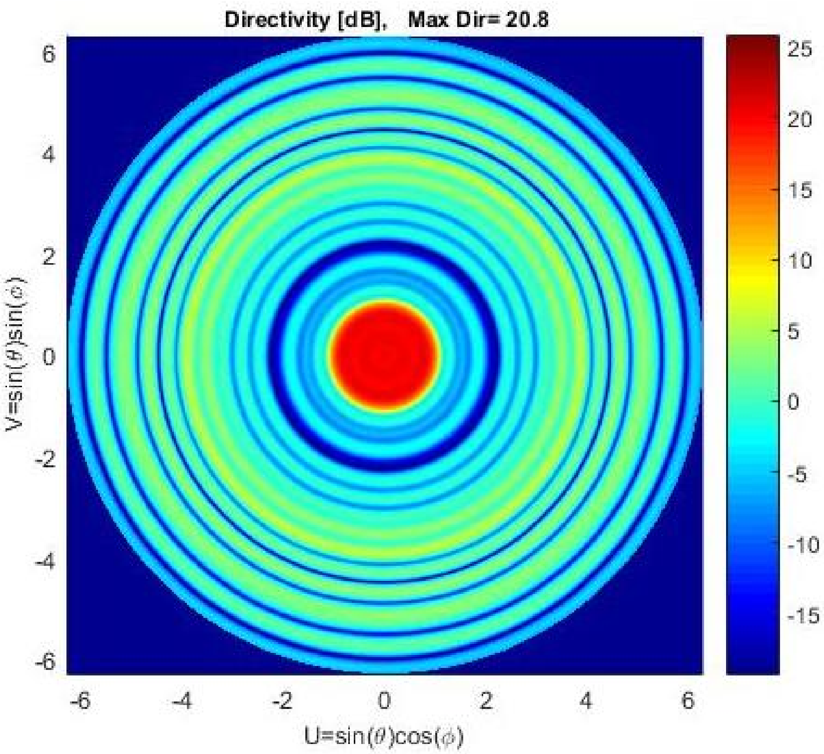


Fig. 4.5.3-7: Directivity pattern in the uv-plane

Fig. 4.5.3-8 depicted the radiation pattern of the optimal solution achieved by means the use of the proposed method. The several φ cuts are indistinguishable in the graphical representation demonstrating the perfect symmetry obtained by the optimal arrays layout.

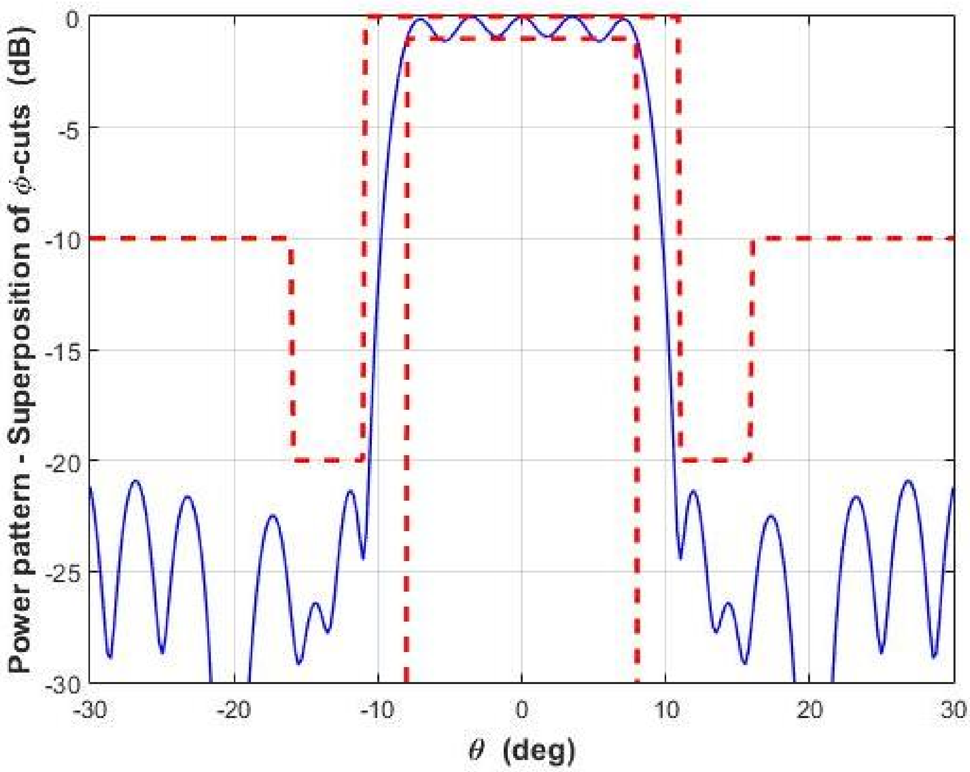


Fig. 4.5.3-8: Superimposition of the phi cuts of the radiated far field of the optimal solution

The last example used to validate the proposed method for discretizing a complex source and obtained the optimal concentric ring isophoric sparse arrays is taken from the article [16]. In this case the desired beam pattern is a flat top beam with a very large flat zone, an Half Power Beamwidth (HPBW) equal to 31.2 degrees, and a maximum ripple equal to ±0.41 dB for $\theta \leq 13.4$ degrees.and SLL equal to -12.18 dB for $19.6 \leq \theta \leq 90$ degrees. The radiation pattern proposed in [16] is reported in Fig. 4.5.3-9 and it is obtained by an array constituted by 220 radiating elements distributed on an aperture of 10λ. The results of [16] are achieved by means a hybrid method based on the simulated annealing optimization algorithm aimed at jointly minimizing the maximum ripple and the maximum SLL. The best proposed solution by the authors is represented by the continuous line in Fig. 4.5.3-9.

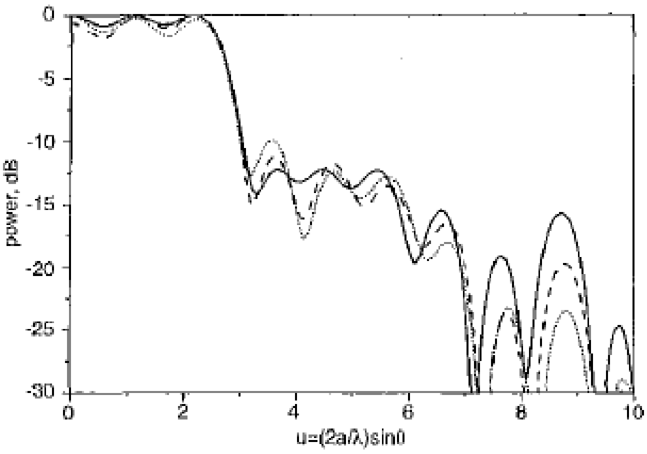


Fig. 4.5.3-9: Proposed radiation pattern in [16]

Applying the new method based in the deterministic discretization of a continuous complex source in an isophoric sparse arrays the following antenna layout is designed, see Fig. 4.5.3-10.

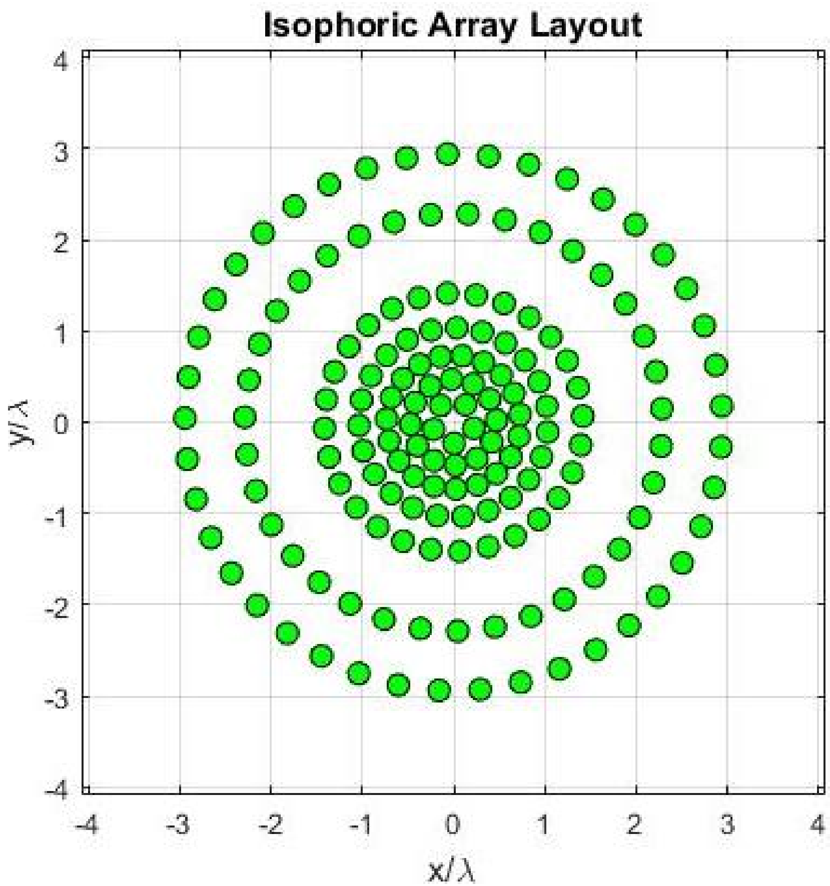


Fig. 4.5.3-10: Optimal arrays layout with isotropic elements

The deterministic proposed method is able to fulfill the same mask of [16] by exploiting a CRISA composed of 163 isotropic elements located over an aperture of diameter 5.86λ. Therefore, the presented approach allowed a 26% reduction in the elements' number without experiencing any radiation loss.

Fig. 4.5.3-11 shows the superimposition of all φ cuts radiated by the optimal arrays layout.

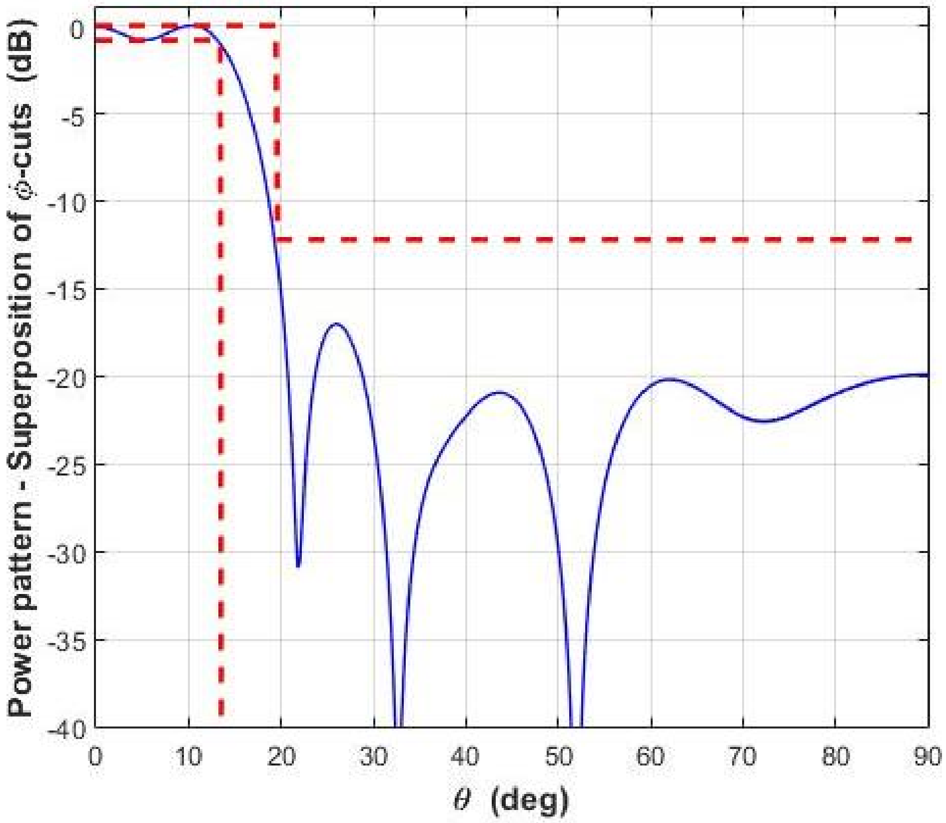


Fig. 4.5.3-11: Superimposition of the phi cuts of the square amplitude arrays factor

The CRISA guarantees in the flat zone a minimum directivity of 15.98 dBi and a maximum directivity of 16.8 dBi (see Fig. 4.5.3-12) which is just 1.87 dBi lower than the directivity pertaining to a theoretical power pattern being constant in the region θ≤13.4° and zero elsewhere.

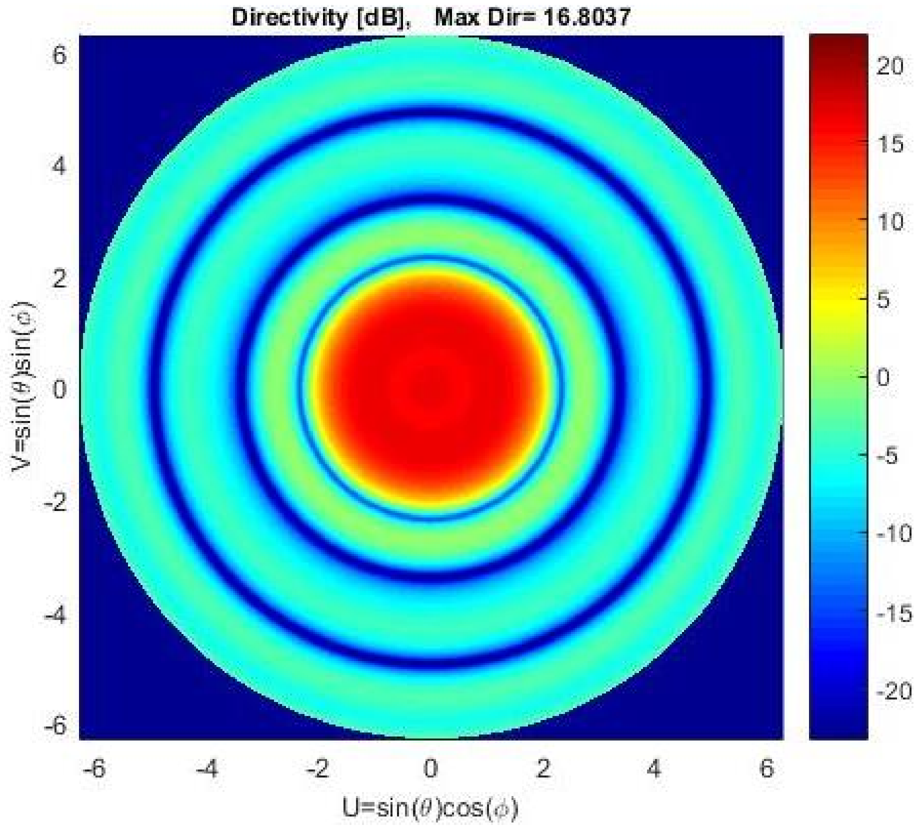


Fig. 4.5.3-12: Contour plot of the optimal square amplitude arrays factor

The problem of the optimal synthesis of shaped beams in the presence of completely-arbitrary lower and upper bounds on the power distribution has been solved by exploiting ring symmetric isophoric sparse arrays. The proposed technique allowed to considerably improve the performance achievable through previous approaches. In particular, for equal power pattern masks, the required number of arrays elements has been significantly reduced.

The approach can be used in conjunction with the techniques in [22],[24],[26] in such a way to exploit also the feeds' shape and size as a degree of freedom of the design and hence to get a further enhancement of performance.

In the previous test cases, the synthesis of the RCAS has been performed through the method in [29], and the subsequent discretization steps respectively required 1.0, 1.16 and 1.04 seconds to be performed by a calculator having an Intel Core i7 2.50 GHz CPU and a 10 GB RAM. The algorithms are developed in Matlab environmental and a verification by means the GRASP Ticra software is carried out.

4.5.4 Phase-Only reconfigurable circular continuous sources discretized into isophoric sparse ring arrays

One of the aims of this thesis work is to deal with the very important topic on the synthesis of isophoric sparse arrays which are able to be reconfigured acting only in the phase distribution. In particular, in section 3.4 of this thesis a new approach to the optimal, mask-constrained power-pattern synthesis of Circularly Symmetric Continuous Aperture Sources (CSCASs) [18] able to dynamically reconfigure their radiation behavior by just modifying their phase distribution is proposed. The achieved solutions are used as a reference in the optimal synthesis of phase-only reconfigurable Circular Ring Isophoric Sparse Arrays (CRISAs) for applications of high interest including the multibeam satellite coverage of Earth.

As a distinguishing feature, despite the strong non-linearity of the problem (which is a power-pattern, mask constrained synthesis wherein both the arrays locations and excitation phases are unknowns), the proposed solution procedure has been conceived as a Convex Programming (CP) optimization plus a couple of deterministic steps. This has been possible by virtue of the joint, optimal exploitation of the techniques respectively published in [50] (for an optimal exploration of the search space), [69] (for the optimal synthesis of CSCASs), and of the fast deterministic discretization of CSCASs into CRISAs which is one of the object of this thesis.

Being able to address the synthesis of isophoric planar arrays by exploiting only CP algorithms, the proposed approach represents a remarkable novelty with respect to the available methods. In fact, it is innovative with respect to the method published in [20] (which avoids global optimization but only applies to 1-D arrays) as well as to the one presented in [79] (which applies to 2-D arrays but resorts to global optimization).

In order to validate the full procedure, a numerical example is carried out. In particular, the only-phase continuous source presented in the section

3.4.1 is used as a reference for the synthesis of an only-phase reconfigurable isophoric sparse array, see Fig. 4.5.4-1.

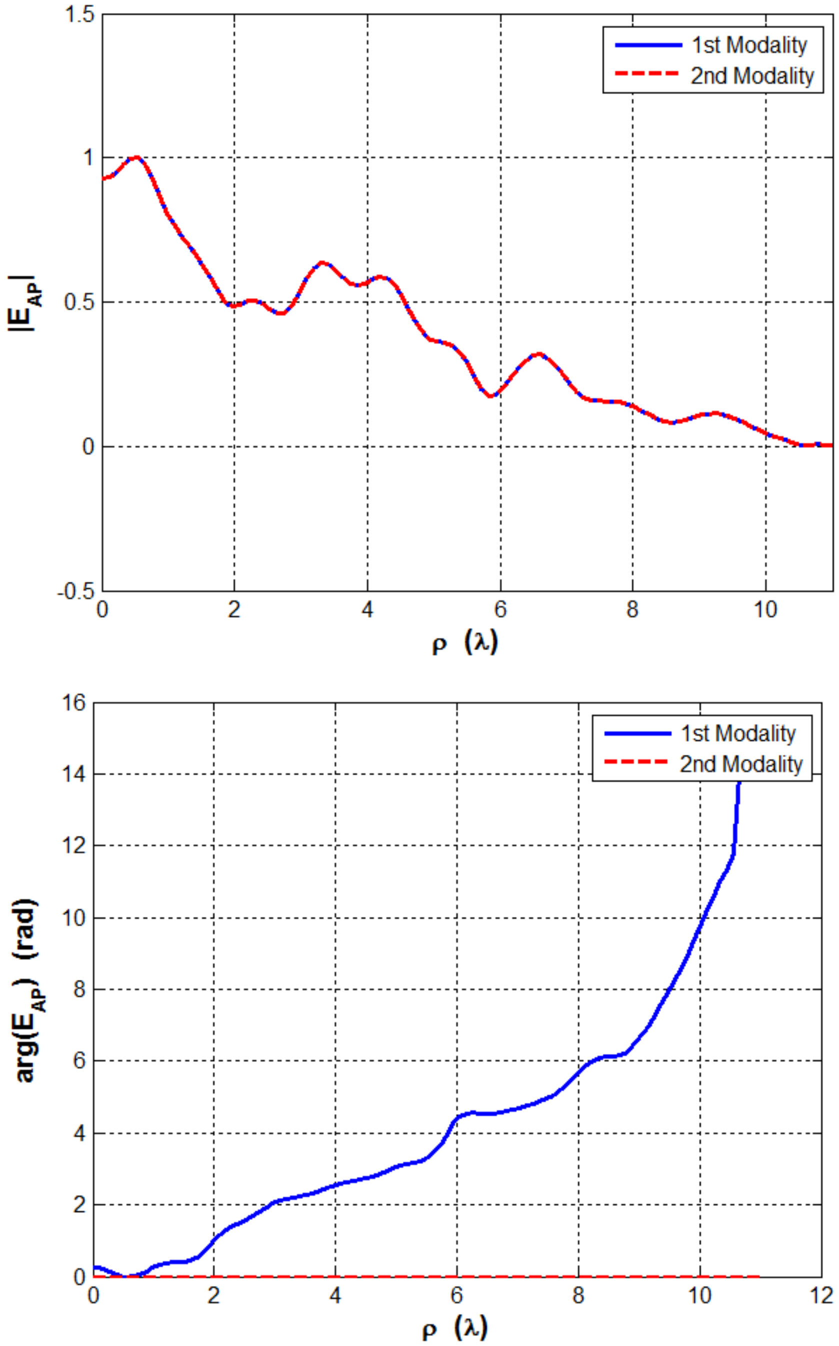


Fig. 4.5.4-1: Final phase-only reconfigurable power patterns synthesized in section 3.4.1

Applying the optimal deterministic technique for discretizing a complex source to the source pertaining to the shaped beam modality, the blue curve of Fig. 4.5.4-1, which led to the arrays layout and excitation phases shown in Fig. 4.5.4-2 and Fig. 4.5.4-3(b), respectively

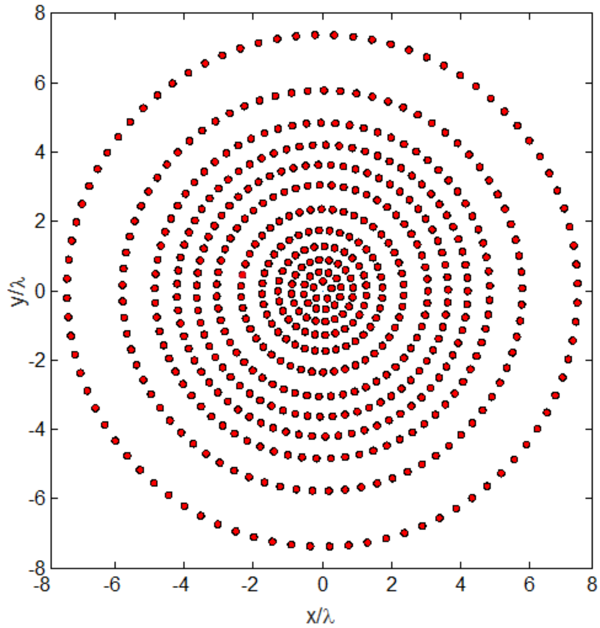
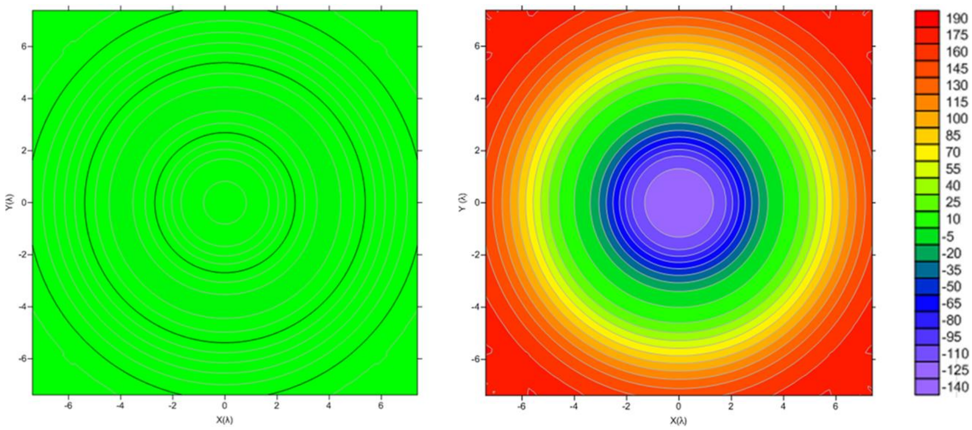


Fig. 4.5.4-2: Synthesized isophoric sparse ring arrays layout (isotropic element pattern embedded).



(a) (b)

Fig. 4.5.4-3: Excitation phase distribution (in degrees) synthesized for the pencil beam [subplot (a)] and the shaped beam [subplot (b)]

The synthesized CRISA is composed by 567 isotropic elements and hence allows a reduction of roughly the 20% of elements with respect to a fully-populated array covering the same aperture (i.e., a circular region of radius 7.5 λ) with a constant λ/2 inter-element spacing. By using a uniform excitation amplitude, this arrays is able to generate:

- the pencil beam depicted in Fig. 14, as long as all excitation phases are set to zero as shown in Fig. 13(a);

- the shaped beam depicted in Fig. 15, as long as the excitation phases of Fig. 13(b)are adopted.

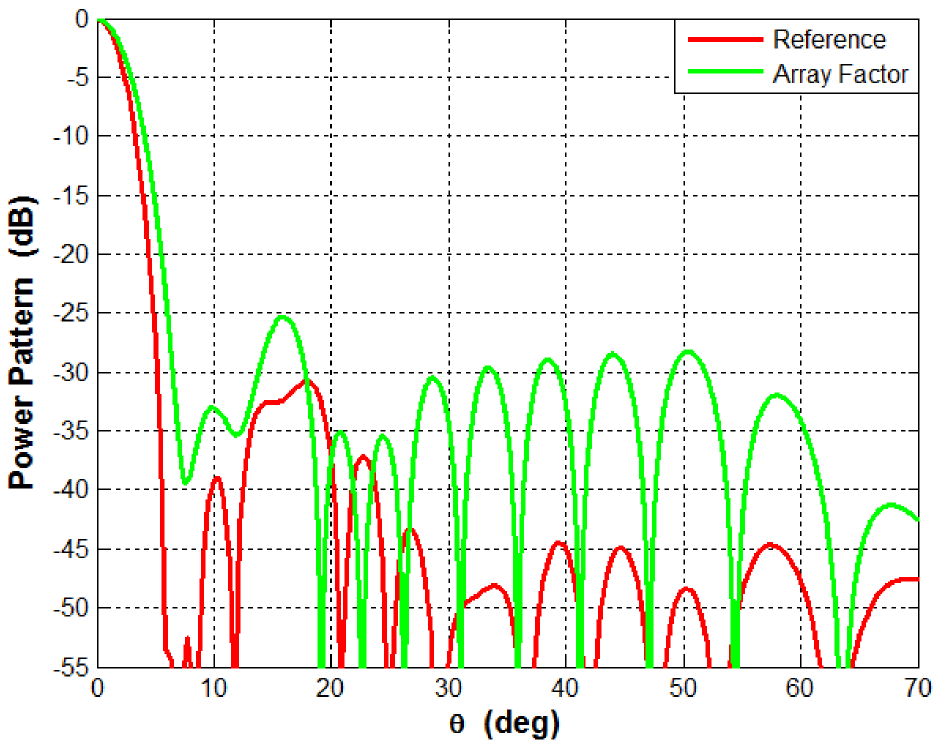


Fig. 4.5.4-4: Pencil beam radiation modality: comparison between the power patterns radiated by the reference continuous source and the synthesized isophoric arrays shown in Fig. 4.5.4-2 and excited with the phase distribution shown in Fig. 4.5.4-3(a).

Notably, in order to effectively evaluate the achieved radiation performances, in both Fig. 4.5.4-4 and Fig. 4.5.4-5 the reconfigurable array's power patterns are compared with the ones corresponding to the reference continuous source. As it can be seen, despite the adoption of isotropic feeds and the reduction of the overall number of elements with respect to a 'classical' $\lambda/2$-equispaced array, a good fitting of performances is experienced. This circumstance confirms not only the validity of the proposed approach from the theoretical point of view, but also its profitable applicability to the realization of antennas for applications of high interest.

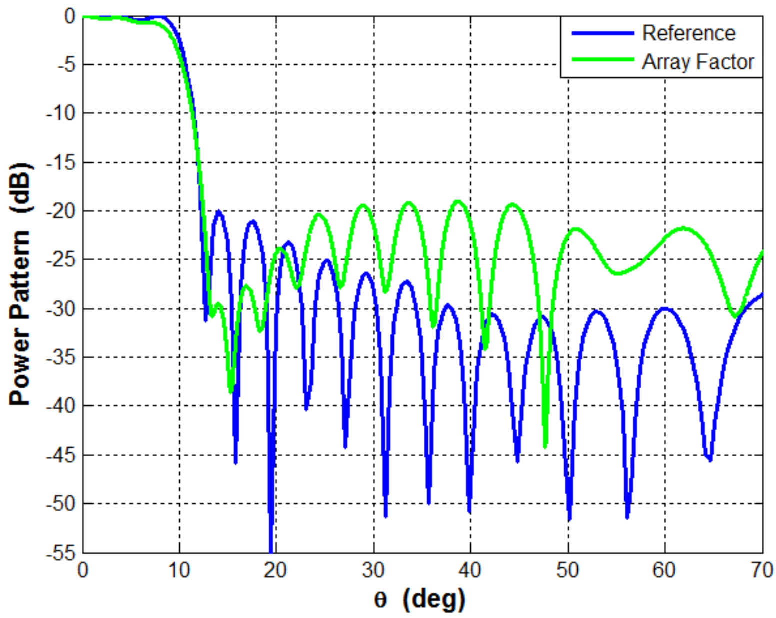


Fig. 4.5.4-5: Shaped beam radiation modality: comparison between the power patterns radiated by the reference continuous source and the synthesized isophoric arrays shown in Fig. 4.5.4-2 and excited with the phase distribution shown in Fig. 4.5.4-3(b).

A new approach to the synthesis of phase-only reconfigurable continuous circular sources and isophoric sparse ring arrays has been presented and assessed.

The proposed technique is able to deal with an arbitrary number of shaped and pencil beams, and exploits at best all the knowledge available in the separate synthesis of the different patterns. The engine of the procedure is based on convex programming optimizations and fast spectral factorizations, guaranteeing at the same time a low computational burden and the achievement of globally-optimal solutions.

The method for the synthesis of only-phase reconfigurable isophoric sparse arrays uses the two-steps procedure starting from the synthesis of the optimal reference source able to be phase-only reconfigured which is a novelty introduced in this thesis and exposed in section 3.4. The second step concerned the use of the proposed method for the optimal deterministic discretization of the reference complex source in order to define the concentric ring isophoric arrays layout.

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Conclusion and future development

The present thesis work deals with the problem to design isophoric sparse arrays arranged in concentric ring able to radiate a shaped beam where the radiations constraints are expressed by means a power mask. As described in this work, the synthesis of isophoric sparse arrays is one of the most critical synthesis problems in terms of an analytical solution.

The proposed solution is concerned with a full deterministic procedure constituted by two distinct steps. At the first, a continuous reference source is found in order to find the optimal theoretical solution. This step is carried out exploiting at the best the available methods in the literature for the deterministic design of continuous source able to radiate a pencil beam or a shaped beam. After the continuous source is found, the second step of the procedure performs the reference continuous source discretization avoiding any kind of global optimization algorithm.

The novelty exposed in this work is not related to the introduction of the two-steps procedure, which is a well-known procedure in the antenna synthesis problem, but it is relative to the deterministic discretization of the 2D complex source (typical source for shaped beam) in an isophoric sparse arrays. This aspect was an open point in the scientific community. Several approaches concerning the solution of this problem were found but none of them are based on a full deterministic procedure or they are not obtained by means a true rigorous method.

To reach the aim of this thesis a deep investigation of the literature is performed in order to exploit at the best what is just done in the antenna synthesis. In particular an investigation of the synthesis of continuous sources radiating pencil and shaped beam and reconfigurable patterns is carried out.

In addition, the generic approach for discretizing a continuous source in isophoric sparse array is assessed.

Taking advantage of all these deterministic methods, the effort produced in this thesis work was oriented to finalize the last open point in the synthesis of isophoric 2D arrays concerning the case of shaped beams. A full deterministic and rigorous method is presented and several examples are furnished providing the goodness of the proposed solution. The goodness of this method is also justified by a recent scientific publication ([95]) where a comparison between the proposed deterministic method and a hybrid approach (using deterministic and optimal optimization techniques) is described. The comparison underlines that the proposed method is able to reach the optimal solution in a fast manner respect to the hybrid approach which is based on the Simulated Annealing global optimization.

Another novelty introduced in this thesis is the capability to design a concentric ring isophoric sparse arrays able to be reconfigured acting only in changing the phase distribution on the antenna aperture. This kind of reconfigurability is suitable for the satellite communication, in fact the operator can have the opportunity to change the multibeam coverage on the Earth in according to the traffic density without modifying the hardware on the satellite and without increasing the complexity of the antenna. In the thesis a new deterministic method is introduced for the definition of a reconfigurable continuous source able to be an only phase reconfigurable. Applying at this source the deterministic and fast procedure for the synthesis of circular ring isophoric sparse arrays, it is possible to perform the complete synthesis starting from the definition of the power mask and ending with the optimal arrays layout. The synthesis is cast as a convex programming problem and can handle an arbitrary number of pencil and shaped beams.

The synthesis procedure reported in this thesis permits to define the array layout for an isophoric sparse array arrange in concentric ring able to meet the desired electrical performance. Considering a real working case, during the synthesis process additional constraints need to be considered taking in account not only the electrical requirements but also some mechanical ones. In particular, some constraints can be due to the need to accommodate the thermal control system to avoid the damage of the active components inside the array. The well know thermal control system based on the use of the heat pipes, which is used in the on board antenna, foreseen the accommodation of the system very near to the active components. This aspect can influence the location of the active elements inside the array. Others constraints can be due to the minimum size of the radiating element which can be manufactured. In fact very small radiating elements cause the increase of the manufacturing complexity with an increase of the overall antenna costs.

Considering all these aspects, future development can be done introducing some real constraints in the overall synthesis procedure in order to have an array architecture which meets at the best the operative condition in the satellite environmental.

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