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Geometric References of Roman Mosaics in North Africa

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Article abstract:

This paper deals with the geometric analysis of the Roman mosaics found in the domūs of the archaeological areas of Volubilis, in Morocco, and Bulla Regia, in Tunisia. It analyzes the expressions of mosaic art which in the Roman Empire, between the second and fourth centuries A.D., attained a particular geometric decorative refinement. The tessellation of the mosaics of Volubilis and Bulla Regia is just an example of a cultural universe that for centuries has been a testimony to the lifestyles and myths of the Roman Empire and that today, if appropriately exploited, could contribute to enhance the knowledge and the identity of the Mediterranean heritage. The research proposes a reading of the mosaic apparatus through the study of the constitutive geometric elements. This leads to an analysis useful for the classification of figures and symmetry patterns, and understanding of their meanings. The methodology adopted is that of disassembly and reassembly of areas of mosaic tiling, creating a dialogue between the two-dimensional geometric elements of the flooring and the architectural context in which they are inserted.

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Geometric References of Roman Mosaics in North Africa

Context

The geometric study of the Roman mosaics at Volubilis and Bulla Regia is an in-depth examination of a single aspect of a broader research on the characteristics of architectural culture in the Mediterranean area. In particular, this paper deals with North African examples examined in their architectural, historical and cultural context, in which the geometrical aspect is returned to its rightful function. The study, begun almost ten years ago, identifies and represents formal architectural and decorative matrices, analyzes principles of occupancy and housing qualities.

The research on the North African mosaics allows us to understand the idea of living at the time of the Roman empire. It describes the conviviality, the mental speculation of domestic spaces of the classical era. The research prompts reflection on the refined culture of a society with a complex identity, which relied on a structured symbolism filled with multiple cross-fertilisations. Specifically, the mosaic art of the African Roman domus individuates a repertoire of geometric and figurative forms in which graphics and symbology are interwoven in a dynamic symmetry. The figurative compositions present in the geometric carpets are filled with references to Greek and Roman mythological tales. Myth therefore becomes a strategy for glorifying the ideal of beauty, harmony, and balance.

The Bardo Museum in Carthage, the most important archaeological museum in Tunisia, houses the richest collection of Roman mosaics in the world and offers an overall panorama of great value. The large mosaic pavements coming from all regions of Tunisia represent a fundamental artistic and figurative testimony because they narrate daily life in the domus. Beyond the artistic value of the mosaic work itself, they offer a reliable representation of the places where these mosaics are located (Fig. 1).

Fig. 1 Domus Julii, V century A.D., floor mosaic, Bardo Museum, Carthage

The study of the floor decorations found in the houses of Volubilis, in Morocco,[1] and Bulla Regia, in Tunisia,[2] further investigates, both from the point of view of the images as well as from that of the

[1] Volubilis, part of the ancient Roman region of Mauretania, is the most important archeological site in Marocco. Situated 27 km north of Meknès, it is included in the list of UNESCO World Heritage Sites.

[2] Bulla Regia was part of the territory conquered for Roma in 203 B.C. by Scipione the African. In 156 B.C. it became the capital of Numidia. It is an archaeological site famous for the Hadrianic-era semi-subterranean

geometric decoration, the floor scans of some of the surveys performed in Tunisia and Morocco in the years between 2010 and 2015 (Fig. 2).

Fig. 2 View of some houses in Bulla Regia, Tunisia

The data acquired thanks to the surveys were compared with the archaeological sources and, where there were gaps or alterations, we proceeded with a redesign that reinstated the missing parts so as to be able to read the reconstructed geometric text. On these bases, the objective of the research in question was defined: that of implementing the knowledge about and the identity of the Mediterranean heritage, creating connections and comparisons for a documentation on the mosaics of the Mediterranean area, implementable for places, themes and subjects (Fig. 3).

Fig. 3 Volubilis, Morocco, *House of the Procession of Venus*, late 2nd or early 3rd century A.D., floor mosaic. Photographic documentation and redesign of the mosaic apparatus

The geometric study that follows focuses on a few of the mosaics forming part of the decoration of several of the most significant domūs. The mosaic production is datable, for both sites, between the 2nd and the 4th centuries A.D. The mosaics are executed in opus tessellatum. The composition of the ornamental motif often leads to a natural emphasis on the central medallion surrounded by geometric or floral decorations (Fig. 4).

Fig. 4 Bulla Regia, Tunisia, *House of the Clover*: from survey to the geometric reading of the mosaics

These mosaics can be considered excellent examples of the African decorative tradition, for their remarkable variety of geometric as well as figurative themes as well as for the brilliance and variety of colors. The compositions are developed according to a recurrent pattern: starting from a frame that uniformly outlines the perimeter of the room, a figurative carpet is defined which, from its outer edge, continues toward the center of the room and leads our gaze to the central parts of the mosaic floor. Here, generally, there is a figurative scene or emblemata that determines the name or function of the room. For this aspect, mythology has provided most of the repertoires: Bacchus, Neptune, Ariadne, centaurs, the four seasons, Orpheus, and Hercules are represented in medallions of various shapes (square, rectangle, hexagon and octagon). The purely material aspect of the mosaic cannot be separated from this narrative or symbolic aspect, since the use of mosaics in architecture does not have just a decorative function, but stems from the need to make structures impermeable to water.

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housing complex, built as a protection from the heat and the effects of the sun. Many of its mosaic floors have been left in place; others are displayed at the Bardo Museum in Carthage.

The carpet of small *tesserae* on floors and walls therefore originates from the technical need for insulation against moisture.

Methodology

The methodology used for this research was that of starting with a survey of the state of the art and with the acquisition of metric data. We then went on to consider the original architectural appearance in defining the form and the layout of the domus. Therefore, an ideal reconstruction of the original geometric configuration was proposed. Finally, the mosaic design was analyzed in itself, by graphically defining it in a geometric and texturized visualization. In this context, the graphical analyses were carried out by separating the constituent elements (frames, connecting bands, border, field and, if present, emblemata), and by geometric analysis. The content of the mosaics, in addition to the geometries, also includes mythologically-inspired depictions which contribute in enhancing the artistic value of the mosaic carpet.

The framing elements and the connecting bands have been disassembled and analyzed in search of recurring motifs, geometric matrices and proportional relationships. The identification of elementary units allowed, where possible, classification within the seven possible typologies of frieze patterns. The fields inside the frames and the borders often have regular geometric motifs of various typologies. Graphic analyses therefore focused on the identification of decorative modules, regulatory paths, geometric matrices and, where present, laws of aggregation that define the processes for periodical tiling of the plane. The identification of elementary cells and of the intrinsic laws of repetition and multiplication reveals compositional models that adopt the canonical symmetry movements: translation, reflection, rotation and glide-reflection. Starting from the analysis of the frames, which usually adopt linear developments of symmetry, the research focuses on the larger and more complex surfaces of the inner fields and the margins, in an attempt to identify and classify the plane crystallographic groups that define the Roman mosaics of Volubilis and Bulla Regia. For this purpose, modules, symmetry axes and vectors, modes of aggregation of elementary cells and any exception to the rules of periodic tessellation of the plane are highlighted.

The Speculation of Geometric Thought

The geometric interpretation of the mosaic allows contemporary observers to appreciate the profound knowledge of Roman artists and craftsmen as to specific geometric shapes, decorative elements, colors, where:

> … every single part works together to create the distinctive traits of this visual and artistic work, which over the centuries, leaves a visible trail of man's thoughts and traditions represented in the idealized symbols of everyday life. The valuable cultural patrimony mosaics represent is most of the figurative art from the ancient world and should be appreciated for its extraordinary charm, composition and beauty of artistic technique that, in Italy, reached its peak in the Roman era with the Sicilian examples in the villa of 'Piazza Armerina' but especially in the late Roman and Byzantine eras with the Ravennate mosaic art (Cassano and Piermattei 2007: 11).

The elements that structure the mosaic space arise systematically from an ordering thought that, in responding to a rigid geometric-mathematical scheme, demonstrates at the same time a specific peculiarity and original thought. Within a similar geometric pattern, multiple figurative expressions can be expressed, the result of personal and symbolic interpretations of the same geometric structure. Geometry thus surpasses its particular aesthetic dimension to become a middle ground between mathematics and thought, between memory and interpretation, between measurement and description, the principal instrument for analyzing the intrinsic complexity of the earthly world and the celestial sphere, the key to reading and representing what the Greeks called *alogos*: the Inexpressible.

The architecture of the domus is filled with imaginary meanings that, in this context, recall Aristotle's thought: "The soul never thinks without an image" (*De Anima* III, 7, 431). There is no form of thought that does not refer to a figure. Moreover, the Greeks have taught us that number is form[3]. Geometry is configured, therefore, as a "rational image" that shuns the deceptions of emotion. In soliciting a perception with diminished interpretative filters, it transfers the content to consciousness and reveals what lies in the folds of reality. Geometry, at the same time, is also an effective symbolic expression able to convey the primordial concepts of the Cosmos. It thus becomes a tool for the revelation of the harmony and eternal laws of nature, but also a means which translates the immateriality of the forms of the Spirit into visible traces.

Ornament and Symmetry: The State of the Art

A well-received attempt to catalogy ornamental design for Western designers was made by Owen Jones (Jones 1856). His book *The Grammar of Ornament* collected almost 300 drawings of frames,

[3] *Eidos*: Transliteration from Greek εἶδος "aspect, form". Philosophic term with which Plato indicates idea and Aristotle form. Used by Husserl to define essence the object of intuition. (*Dizionario di Filosofia Treccani*, 2009).

friezes, mosaics and decorations from all over the world, classified according to the culture and the historical period of origin. A different approach was taken by Franz Meyer (1894), who published an extensive review of drawings, grouped and analyzed according to formal characteristics: geometrical motives, natural forms, artificial objects, bands, free ornaments, etc. Along the lines of a classification based on formal characteristics, the work of Archibald H. Christie is also included, whose *Pattern Design* of 1910 (Christie 1969) went beyond the simple analysis of form and represented a first attempt to classify ornamental motifs according to their geometric properties. The decisive turning point in the field of the classification of friezes and mosaics came with the studies of Russian mathematician and mineralogist Evgraf Stepanovic Fedorov (1885; 1891), who determined 230 three-dimensional crystallographic groups and pointed out that some structures – in particular, crystals – had the same properties as many decorations disseminated throughout the Mediterranean area. He was the first to find a complete classification of plane symmetry crystallographic groups, identifying seventeen. These studies opened up new perspectives in the analysis of decorative models and led to a classification of patterns according to their symmetry characteristics.

In 1924, probably unaware of Fedorov's earlier studies, the Hungarian mathematician György Pólya and the Swiss mineralogist Paul Niggli published an essay entitled "Über die Analogie der Kristallsymmetrie in der Ebene" (Niggli and Pólya 1924), which contained illustrations of the seventeen plane symmetry groups and made a decisive contribution to the classification processes. The essay was to become a milestone for studies on the topic. In 1933, George D. Birkhoff, in *Aesthetic Measure* (1933) defined and illustrated four symmetry operations, highlighting its presence in many ornamental patterns. Also written in the 1930s was the vast contribution of the physicist H. J. Woods. His studies on the ornamentation of woven fabrics were published in several volumes (1935a; 1935b; 1935c; 1936). He proposed the first complete attempt to classify two-color one and two-dimensional patterns. His work was neglected in the following decades, yet today he is considered a point of reference for studies on the geometric characteristics of tessellations (Crowe and Washburn 1988: 5).

In 1952, Hermann Weyl, a German mathematician, physicist and philosopher, published a study on the concept of symmetry in art, in botany and in other pure sciences (1952). He started from the classical concept of symmetry as a harmony of proportions, then went on to consider it in its variegated manifestations: bilateral, translational, rotational, ornamental and crystallographic. A few decades later, in the early 1980s, Donald W. Crowe presented a flowchart for identifying the seventeen plane crystallographic groups, an interesting proposal that was further explored in a subsequent study conducted in collaboration with Dorothy K. Washburn (Crowe and Washburn

1987). In 1988 the two scholars proposed a methodology of analysis of patterns in which symmetry is used for the interpretation of the drawings of different cultures (Crowe and Washburn 1988). It would become a reference text for mathematicians, art historians, anthropologists, archaeologists and designers.

In 1987, Branko Grünbaum and G. C. Shephard published an exhaustive compendium that was to become a reference point for studies on patterns and regular tilings. Also important are the studies by István Hargittai, a Hungarian chemist who tackled the topic from the point of view of molecular modeling but also with evident attention to its applications in the artistic field (Hargittai 1986; 1989).

Research carried out at the University of Leeds at the end of the last century is of particular interest. The studies were oriented towards historical and cultural aspects (Hann 1992), towards the possibility of variation in patterns (Hann and Lin 1995) and towards the analysis of the geometries of woven fabrics (Hann and Scivier 2000a; 2000b). There was a fruitful collaboration between the scholars M. A. Hann and G. M. Thomson who, in the early 1990s, presented a flow chart for the classification of mosaics that stands out for its extreme simplicity and clarity (Hann and Thomson 1992). In a more recent publication, they instead proposed an exhaustive technical compendium on the theme of tessellation, both on the plane and in three-dimensional space (Hann and Thomson 2007).

The Concept of Symmetry

An important contribution to the concept of symmetry and its applications in the field of art is provided by Hermann Weyl. In his book *Symmetry*, published for the first time in 1952, he states:

> symmetry is a vast subject, significant in art and nature. Mathematics lies at its root, and it would be hard to find a better one on which to demonstrate the working of the mathematical intellect (Weyl 2016: 145).

Symmetry, therefore, is an element of connection between apparently irreconcilable disciplines: a point of contact between mathematical rigor and free creative expression (Gombrich 1979). But what is meant by symmetry? As we know, the term symmetry derives from the Greek σύν (syn) “with” and μέτρον (metron) “measure”. In classical Greece it corresponded to today’s concept of “proportion”. Hermann Weyl writes:

> In the one sense symmetric means something like well-proportioned, well-balanced, and symmetry denotes that sort of concordance of several parts by which they integrate into a whole. Beauty is bound up with symmetry (Weyl 2016: 3).

It is the concept that we are used to applying to classical and Renaissance art. This meaning has, however, changed over the centuries.

For the purposes of our study the modern definition of symmetry that was introduced towards the end of the eighteenth century will be particularly relevant. It will be understood as "immunity to a possible change". If we consider this meaning we can say – as Hermann Weyl states – that "a thing is symmetrical if there is something you can do to it so that after you have finished doing it, it looks the same as it did before" (as quoted in Feynman 1967: 84). More simply, we could say that the symmetry of a geometric figure is a transformation that leaves the figure unchanged. Today, in common usage, the term symmetry is understood in a more limited sense. It coincides with "bilateral symmetry". In fact, this only considers one transformation, that is, a reflection with respect to an axis. In reality, the symmetrical movements on the plane that allow obtaining a shape identical to the original one (that is, isometries), are four (Fig. 5):

- *Translation:* Sliding a shape along a straight line.
- *Reflection:* Overturning a shape on a line of symmetry.
- *Rotation* (around a center): In this case, the rotations taken into consideration are four: 2-fold (180°); 3-fold (120°); 4-fold (90°); 6-fold (60°).
- *Glide-reflection:* Combination of a reflection and a translation along the same line of reflection.

Fig. 5 The four symmetrical movements of the plane (isometries): translation; reflection, glide-reflection; rotation

By applying, on the plane, one or more symmetry operations to a shape, it is possible to obtain different types of friezes and mosaics. All periodic decorations will therefore be traceable to one or more geometric transformations: mutations that respond to precise mathematical laws but which offer unlimited expressive potential. Examples are found in Byzantine motifs, in Persian decorations, but especially in the Moorish mosaics of the Alhambra, in Granada where all the possible typologies of tessellation of the plane are found.

Identification of Unit Cell and Fundamental Region

Periodic coverings of the plane are characterized by a reticular structure. This consists of units of design – which we will call unit cells – of identical shape, size and content. The translation of these cells generates the covering of the plane with periodic decorative motifs. The meshes of a reticulation grid can be of various geometric shapes: parallelogram, rectangle, rhombus and square (Hann and Thomas 2007: 13, 14). It is not necessary to use an entire unit cell to achieve the complete coverage of a plane. It is sufficient to take a small portion of it, that is, the smallest design unit to which one or more symmetry movements can be applied. We will call this portion the fundamental region. Therefore, to identify the class of a periodic plane decoration it is necessary to identify the fundamental region, the unit cell and the symmetrical transformations that intervene in the process of periodic coverage of the plane. If we apply one or more symmetry operations to a unit cell it is possible to obtain two different types of periodic developments.

Border patterns (frieze patterns)

If the development takes place in only one direction, it is possible to generate up to a maximum of seven different types of border patterns. Each type is identified by means of an alphanumeric convention now accepted internationally. All the friezes produced, from antiquity to the present day, can be attributed to these seven categories.

All-over patterns (field patterns)

If the development takes place in at least two independent and non-parallel directions, the complete coverage of the plane is generated without gaps or overlaps. The application of one or more symmetry movements can generate up to a maximum of seventeen different types of all-over patterns, which in mathematical terms are defined plane crystallographic groups.

Classification of Border Patterns

The class of a frieze (border patterns) is expressed with a four-digit alphanumeric code (*pxyz*), which indicates the presence or absence and the possible characteristics of the various types of isometries[4] (Hann and Thomas 2007: 10, 11).

The identification of the class of a frieze can be deduced following a flow chart (Fig. 6). In this chart the type and the number of symmetry operations that intervene in the definition of the frieze are shown.

Fig. 6 Flow chart for classification of border patterns (friezes)

The House of the Procession of Venus

We will now consider two friezes found on the sides of the floor mosaic of a room in House of the Procession of Venus in Volubilis (Fig. 7). We will start from the geometric analysis and the

[4] The letter (*p*) is present in all classes. The position (*x*) indicates the symmetry movements with respect to axes perpendicular to the longitudinal direction of the frieze: (*m*) if there is a vertical reflection; (*1*) if absent. The third position (*y*) indicates the symmetry movements with axes parallel to the longitudinal edges of the frieze: (*m*) indicates a longitudinal reflection; (*a*) a glide-reflection; (*1*) indicates that neither is present. The fourth symbol (*z*) expresses the presence or absence of a 180 ° rotation (2-fold rotation): (*2*) if the rotation is present; (*1*) if it is absent (Hann and Thomas 2007: 10, 11).

identification of the fundamental region. It undergoes a 180° (2-fold) rotation. A horizontal reflection or a vertical reflection can be applied to the figure obtained. Thus, we have constructed a larger cell (unit cell) with double bilateral symmetry that we will use to obtain the frieze. More translations can be applied to it along the longitudinal axis of the frieze, or more reflections. Following the flow chart, we will verify that the frieze belongs to the class *pmm2*.

Fig. 7 Volubilis, Morocco, *House of the Procession of Venus*. Floor mosaic, late 2nd or early 3rd century A.D. Left: redrawing of mosaic. Right: analysis of the lateral friezes

Classification of All-over Patterns

Field patterns are generated by translations on the plane in at least two independent non-parallel directions. In addition to translational operations, other symmetry movements may also be present. Their combination will determine the classification in one of the seventeen plane crystallographic groups. There are various ways of codifying the mosaics. The international conventions in use today have surpassed the nomenclature suggested by Pólya in 1924. The one currently most widely used was proposed in the *International Tables of X-Ray Crystallography* (Henry and Lonsdale 1952). It expresses a four-digit alphanumeric code (*pxyz* or *cxyz*) indicating the type of unit cell, the highest order of rotation, and the presence of axes of symmetry (reflection and/or glide-reflection) along two fixed directions.[5] As with friezes, a flow chart can also be used for the classification of mosaics, but the variables that intervene are greater. Responding to the questions that the chart proposes, it will be possible to identify the code of the plane crystallographic group to which a mosaic belongs (Fig. 8). In the study presented in this paper, we have applied this classification process to several mosaics in the Mediterranean area, in particular, North Africa.

Fig. 8 Flow chart for classification of all-over patterns (field patterns). Redrawing of diagram developed by Hann and Thomson (1992)

The House of the Hunt

[5] The first letter (*p* or *c*) indicates whether the unit cell is primitive or centered. The primitive cell characterizes fifteen of the seventeen crystallographic groups and can generate complete coverage of the plane through translation operations alone. The centered cells belong to the two remaining classes and are characterized by a rhomboidal lattice. The second position indicates the highest order of rotation present (*2*, *3*, *4*, *6*). In the absence of any type of rotation symmetry, the value will be equal to (*1*). The third and fourth positions indicate the presence or absence of axes of reflection (*m*) or glide-reflection (*g*) along very precise directions (Hann and Thomas 2007: 15-16).

The first study was carried out on the mosaics of the House of the Hunt in Bulla Regia, Tunisia. After a first analysis of the floor mosaics, a section of a wing of the portico was examined (Fig. 9). First of all, the unit cell of the mosaic was identified. It is square but positioned on its diagonals and has two bilateral symmetries with respect to the two axes that pass through the vertices. The unit cell can be further divided into four parts along its median axes. The smallest pattern obtained will be called the fundamental region. Therefore the unit cell can be generated by the fundamental region, applying to it a few symmetry movements on the plane: 180° (2-fold) rotations and glide-reflection operations. Translating the unit cell along the orthogonal directions to its edges and along its diagonals, we will obtain a periodic covering of the plane. The same result can be achieved with 180° (2-fold) rotations around the centers shown in Fig. 9. If we apply glide-reflection movements we obtain, instead, the coverage only of the modules contiguous to the sides of the unit cell. Following the flowchart, we can define the class to which the mosaic belongs, among the different plane crystallographic groups: *p2gg*.

Fig. 9 Bulla Regia, Tunisia, *House of the Hunt*. Floor mosaic, 4th century A.D. Analysis of flooring of the portico

The House of the Fish

This methodology has also been applied to other Bulla Regia mosaics in the House of the Fish. The mosaics of the rooms indicated with the letters *a* and *b* have a layout with similar symmetry. The only difference is that in room *b* the unit cell is rotated by 45° (Fig. 10).

Fig. 10 Bulla Regia, Tunisia, *House of the Fish*. Floor mosaic, 4th century A.D. Mosaics analysis

In both cases the fundamental region is a right triangle and occupies 1/8 of the unit cell. The latter is generated by 90° rotations and reflections of the same fundamental region with respect to four axes inclined 45°. The periodic tiling of a plane is obtained through translations of the unit cell along the four axes identified in the illustration (Figs. 11, 12). But the same result can be achieved through 180° (2-fold) and 90° (4-fold) rotations, or also thanks to combined movements of reflection and glide-reflection. Following the indications of the flow chart it is determined that for both mosaics their class is: *p4mm*.

Fig. 11 Bulla Regia, Tunisia, *House of the Fish*. Floor mosaic, 4th century A.D. Mosaics analysis of the room *a*

Fig. 12 Bulla Regia, Tunisia, *House of the Fish*. Floor mosaic, 4th century A.D. Mosaics analysis of the room *b*

The House of the Procession of Venus

Let's go back to the *House of the Procession of Venus* in Volubilis (Morocco), in particular to the room represented in Fig. 7. The floor mosaic portion placed above the central emblem has a hexagon pattern with central squares positioned on their diagonals. A detail of it is shown in Fig. 13. The unit cell identified, however, has the shape of a parallelogram. The fundamental region is a parallelogram too but it occupies half of the unit cell. The coverage of the plane can be obtained by translating the latter along four directions – shown in Fig. 13 – or with 180° rotations around its midpoints and its vertices.

Fig. 13 Volubilis, Morocco, *House of the Procession of Venus*. Floor mosaic, late 2nd or early 3rd century A.D. Analysis of a portion of flooring

The particularity of this mosaic is given by the introduction of a motif that interrupts the periodicity of the composition. It can be assimilated to a sort of frieze that breaks the serial repetition and seems to transform a row of hexagons into prisms. It is a way of introducing a hint of three-dimensional perception into a flat mosaic, but it is probably also a technique that makes it possible to compensate two portions of the mosaic, placed on two opposite sides, which otherwise could not have been connected in a coherent way. The class to which the periodic portion of the mosaic belongs is *p2*. The class to which the frieze belongs is *pm11*.

Non-Periodic Tiling of the Plane

The House of the Procession of Venus

In another room of the *House of the Procession of Venus* we find two elements of particular interest. The first is a portion of a mosaic with a square fundamental region. The unit cell, also square, is obtained through a double reflection with respect to two orthogonal axes. The mosaic is classified as *p2mm* (Fig. 14). The second element, more unique, is given by two symmetrical side bands with a decorative motif that represents a series of side-by-side cubes constructed of squares and parallelograms (Fig. 15). Unlike the mosaics previously analyzed, this one is not given by a periodic repetition of a basic cell. If we consider the group consisting of three cubes as a recurring element, it does not cover the entire surface. It will therefore be necessary to introduce other figures, in addition to the square and the parallelogram, to cover the residual spaces. These figures, highlighted in blue, are: a rectangle and two isosceles triangles.

Fig. 14 Volubilis, Marocco, *House of the Procession of Venus*. Floor mosaic, late 2nd or early 3rd century A.D. Analysis of a portion of flooring

Fig. 15 Volubilis, Morocco, *House of the Procession of Venus*. Floor mosaic, late 2nd or early 3rd century A.D. Left: redrawing of mosaic. Right: analysis of the side bands

This procedure recalls the quasi-periodic tiling of the plane that Roger Penrose discovered in 1973 (Emmer 2006: 219-221). It does not adopt a periodically repeated lattice unit. Penrose uses various different, yet compatible tiles. Arranging them according to precise laws of aggregation determines the coverage of the entire surface. Obviously, the design of the House of the Procession of Venus does not reproduce a quasi-periodic pattern. However, it may represent an attempt, though immature, to address the topic of the coverage of a plane without the aid of periodic symmetries. Also in this case, as Penrose did, it is necessary to use tesserae with different but compatible shapes. We can consider this solution as an anticipation, still unresolved, of what approximately eighteen centuries later would be more precisely analyzed and theorized in mathematical terms.

Conclusion

The research illustrated here shows that every single component of the mosaic decorations contributes to create a distinctive character, part of a visual and artistic process. The method of decomposing and recomposing elements and repeated aggregations allows a dialogue between two-dimensional data, consisting of floor coverings, and relevant data, related to the context and environments. The architectural analysis of a monumental structure, supplemented by a geometric analysis, gives a strongly didactic value to geometric design and contextualizes architecture in its cultural tradition.

The search for the geometric rules of the project, investigated through “numbers”, has deep roots in Greek and Roman culture, a classical culture that pervades the whole West. Geometric abstraction is an instrument of control and verification that satisfies our desire for beauty. The remarkable cultural heritage represented by the mosaics, if effectively analyzed and disseminated, can become a geometric and mathematical game of Antiquity’s figurative art, capable of triggering processes of valorization. It contains within itself geometry, mathematics, art and myth, and reveals graphic expressions of an ancient world that has found eternal form thanks to the knowledge and effectiveness of an artistic technique spread throughout the Mediterranean basin.

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All photographs and illustrations are by the authors.

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Fig 1

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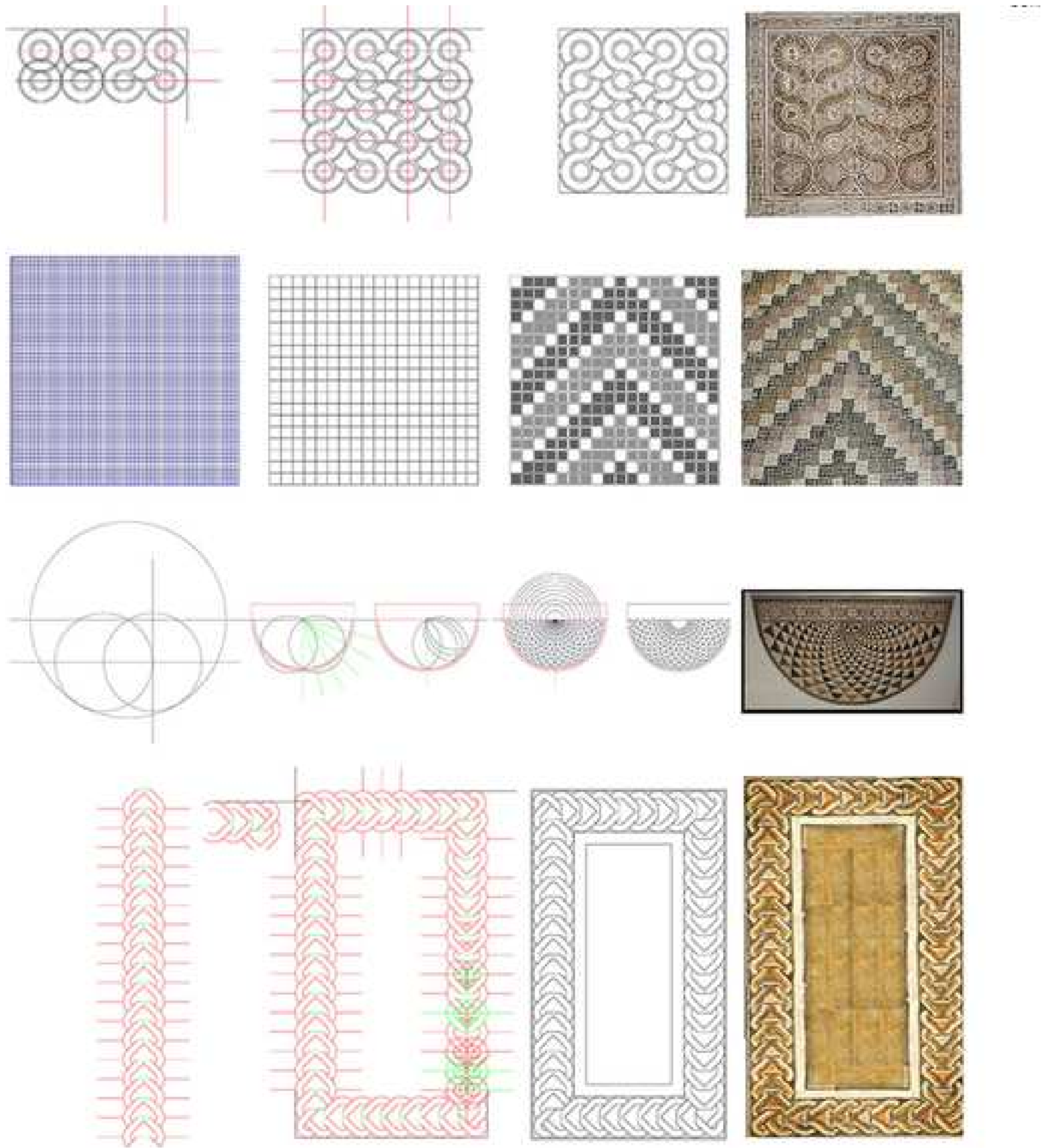
Fig 2

Fig 3

Fig 4



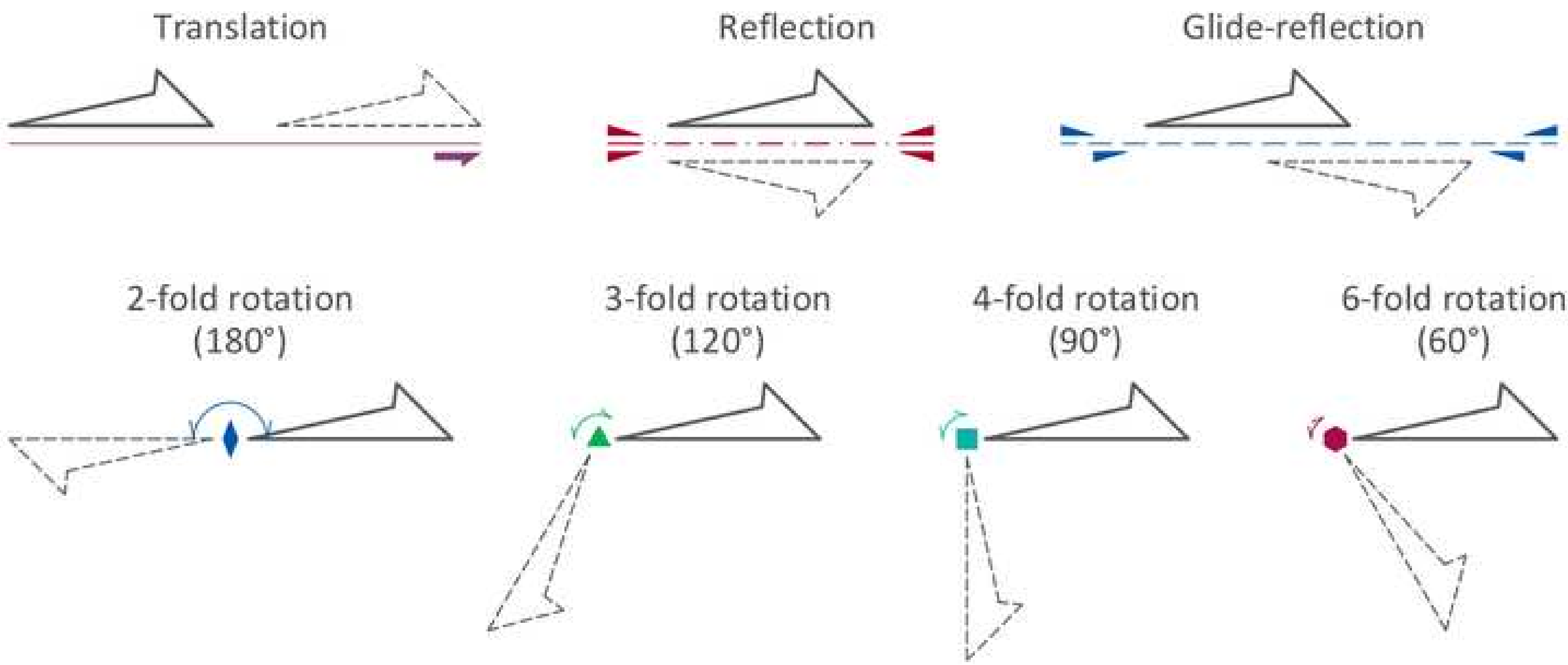

Translation
Reflection
Glide-reflection
2-fold rotation
(180°)
3-fold rotation
(120°)
4-fold rotation
(90°)
6-fold rotation
(60°)

Fig 6

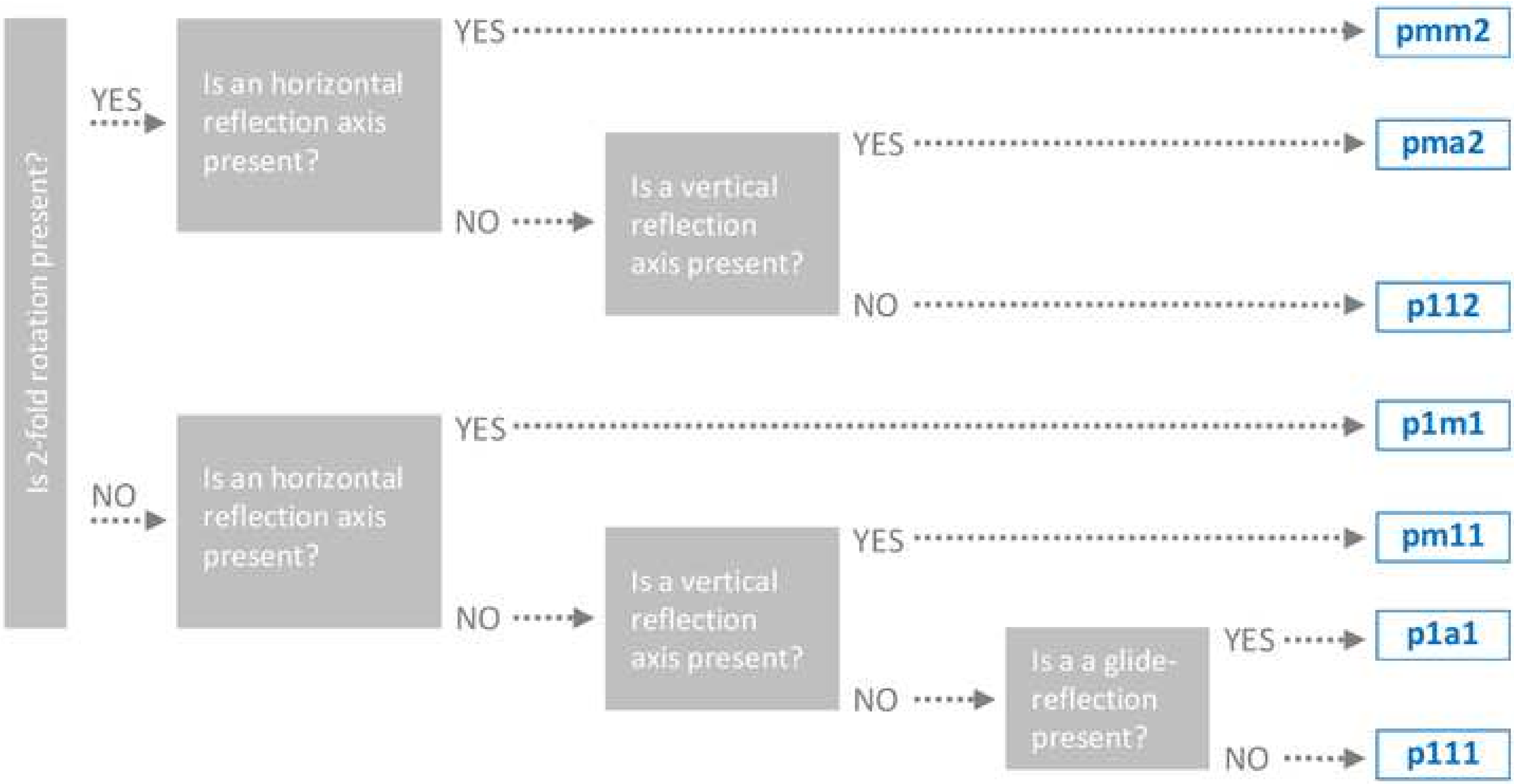

Is 2-fold rotation present?
YES
Is an horizontal reflection axis present?
YES
pmm2
NO
Is a vertical reflection axis present?
YES
pma2
NO
p112
NO
Is an horizontal reflection axis present?
YES
p1m1
NO
Is a vertical reflection axis present?
YES
pm11
NO
Is a a glide-reflection present?
YES
p1a1
NO
p111

Fig 7

Click here to access/download;Figure;Fig 07.tif

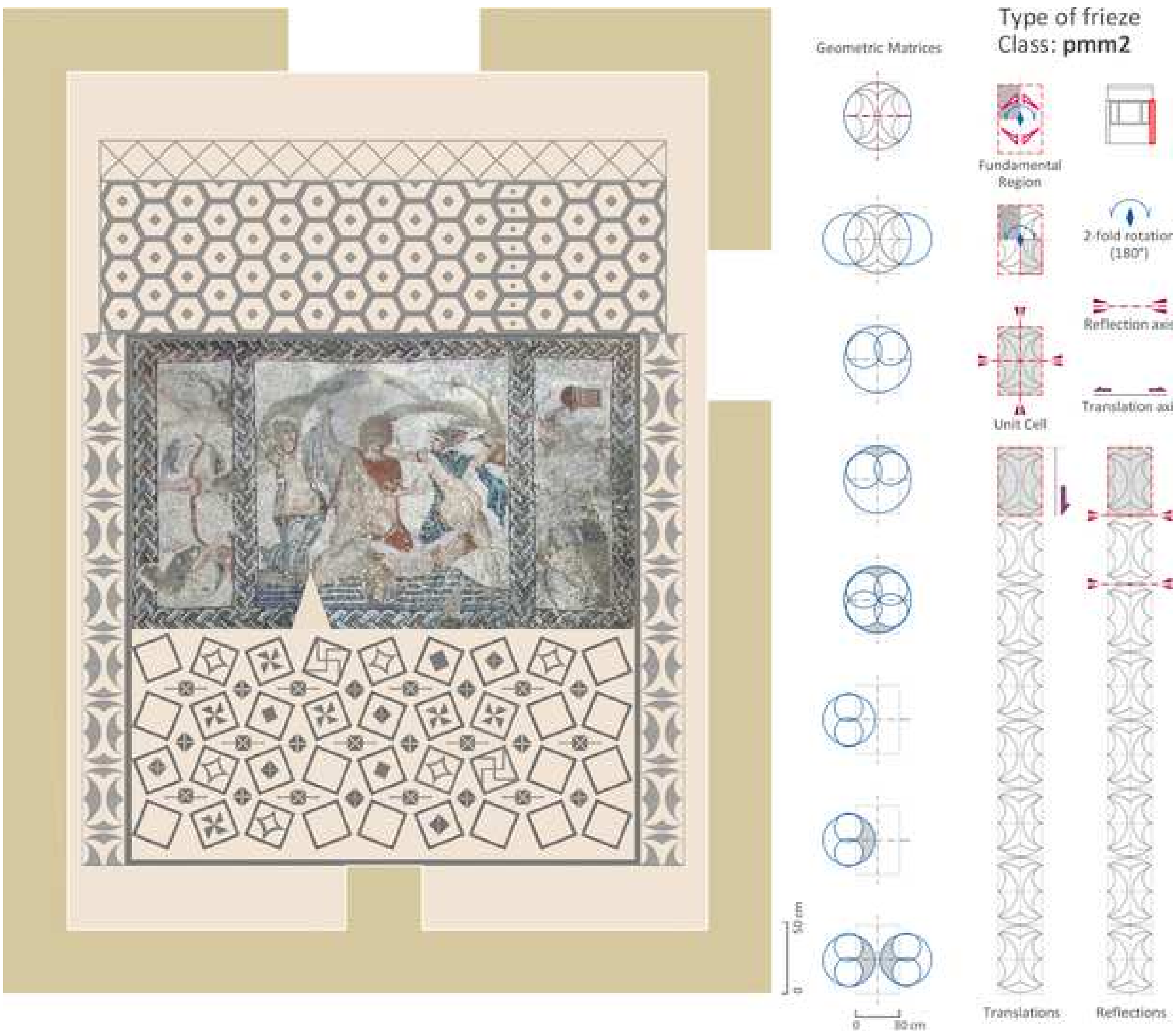


Fig 8

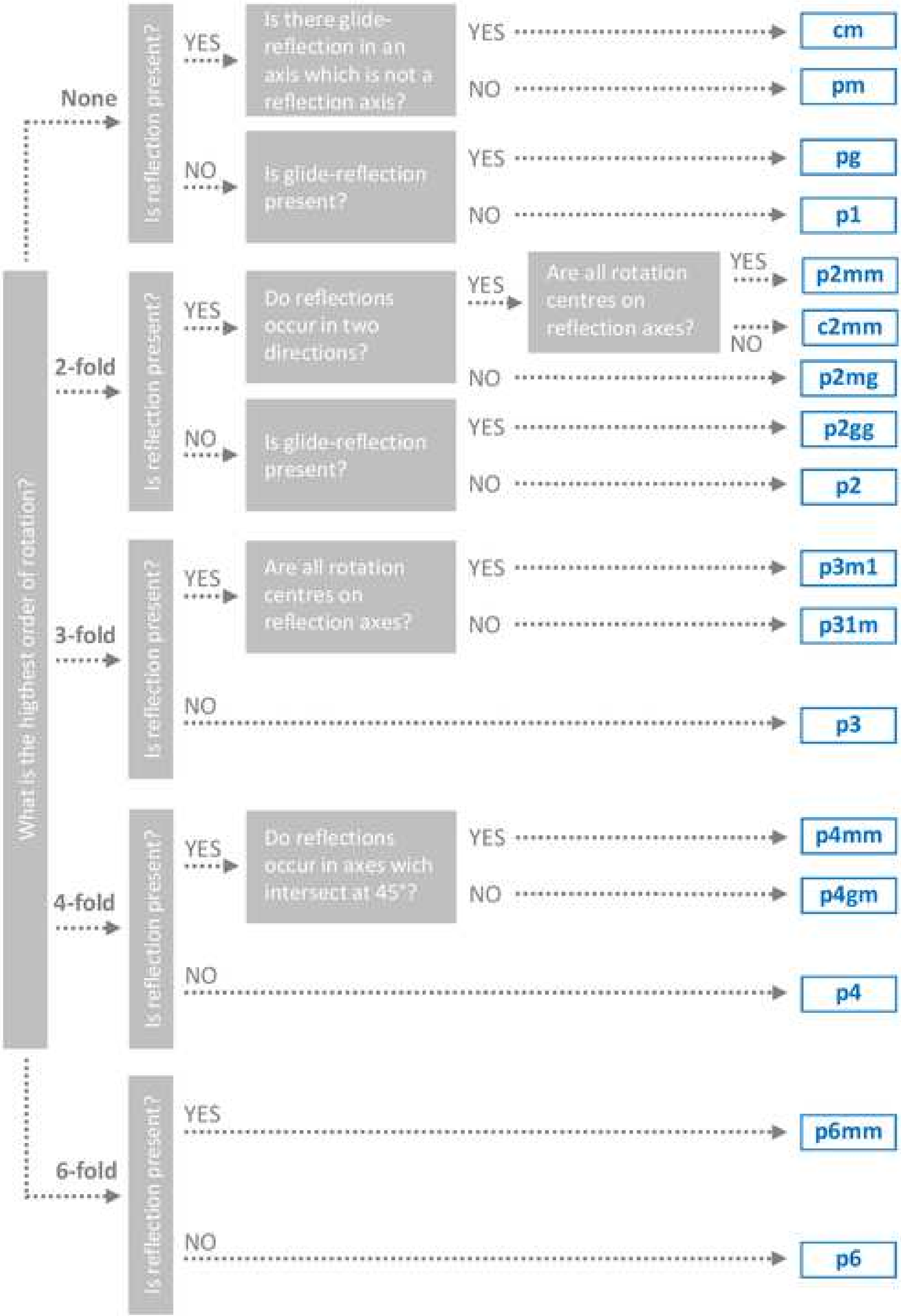
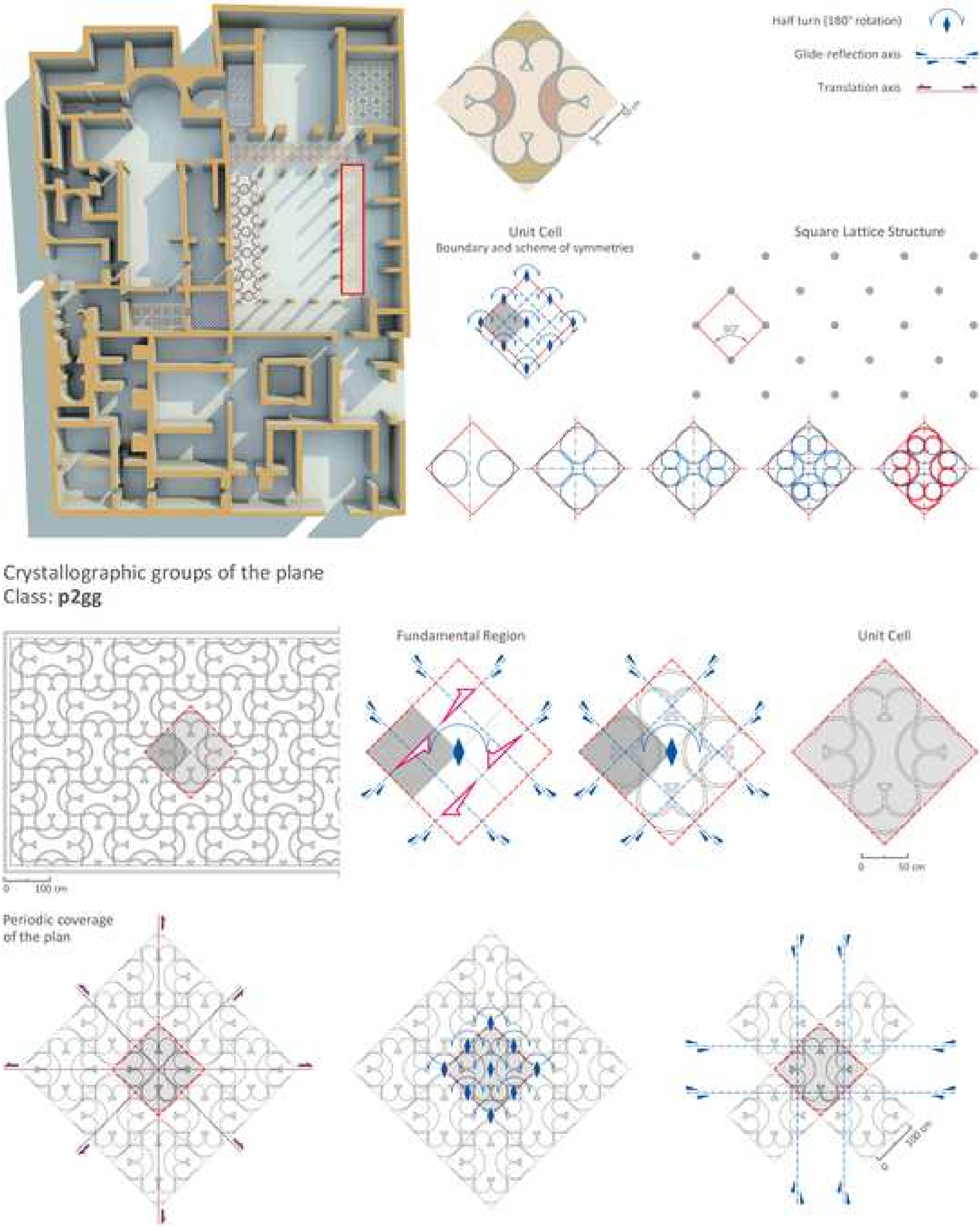
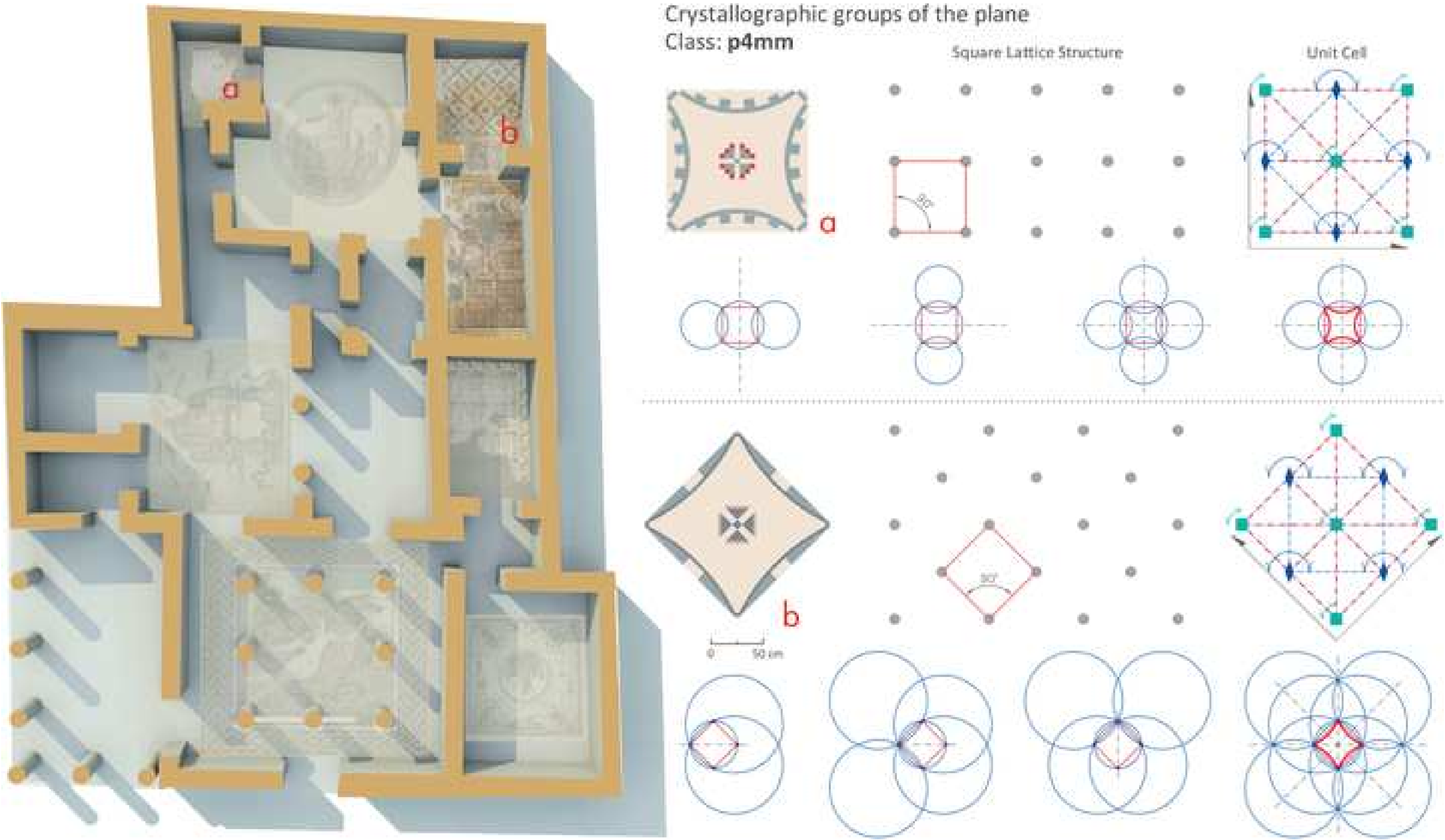

What is the highest order of rotation?
None
2-fold
3-fold
4-fold
6-fold
Is reflection present?
YES
NO
Is there glide-reflection in an axis which is not a reflection axis?
Is glide-reflection present?
Do reflections occur in two directions?
Are all rotation centres on reflection axes?
Do reflections occur in axes wich intersect at 45°?
cm
pm
pg
p1
p2mm
c2mm
p2mg
p2gg
p2
p3m1
p31m
p3
p4mm
p4gm
p4
p6mm
p6

Fig 9 Click here to access/download;Figure;Fig 09.tif





a
b
Crystallographic groups of the plane
Class: **p4mm**
Square Lattice Structure
Unit Cell
a
90°
b
90°
0
50 cm

Fig 11

Click here to access/download;Figure;Fig 11.tif

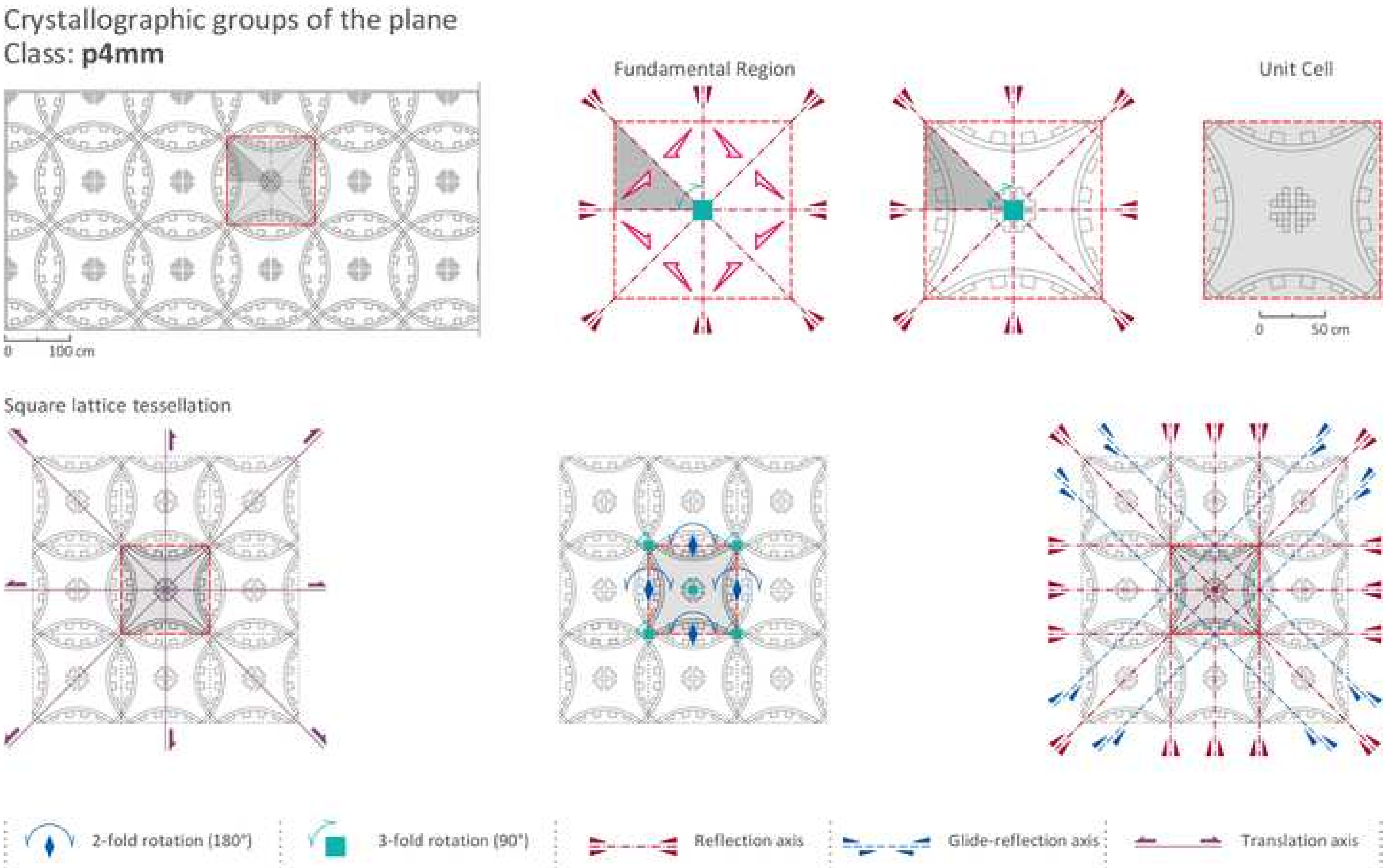


Fig 12

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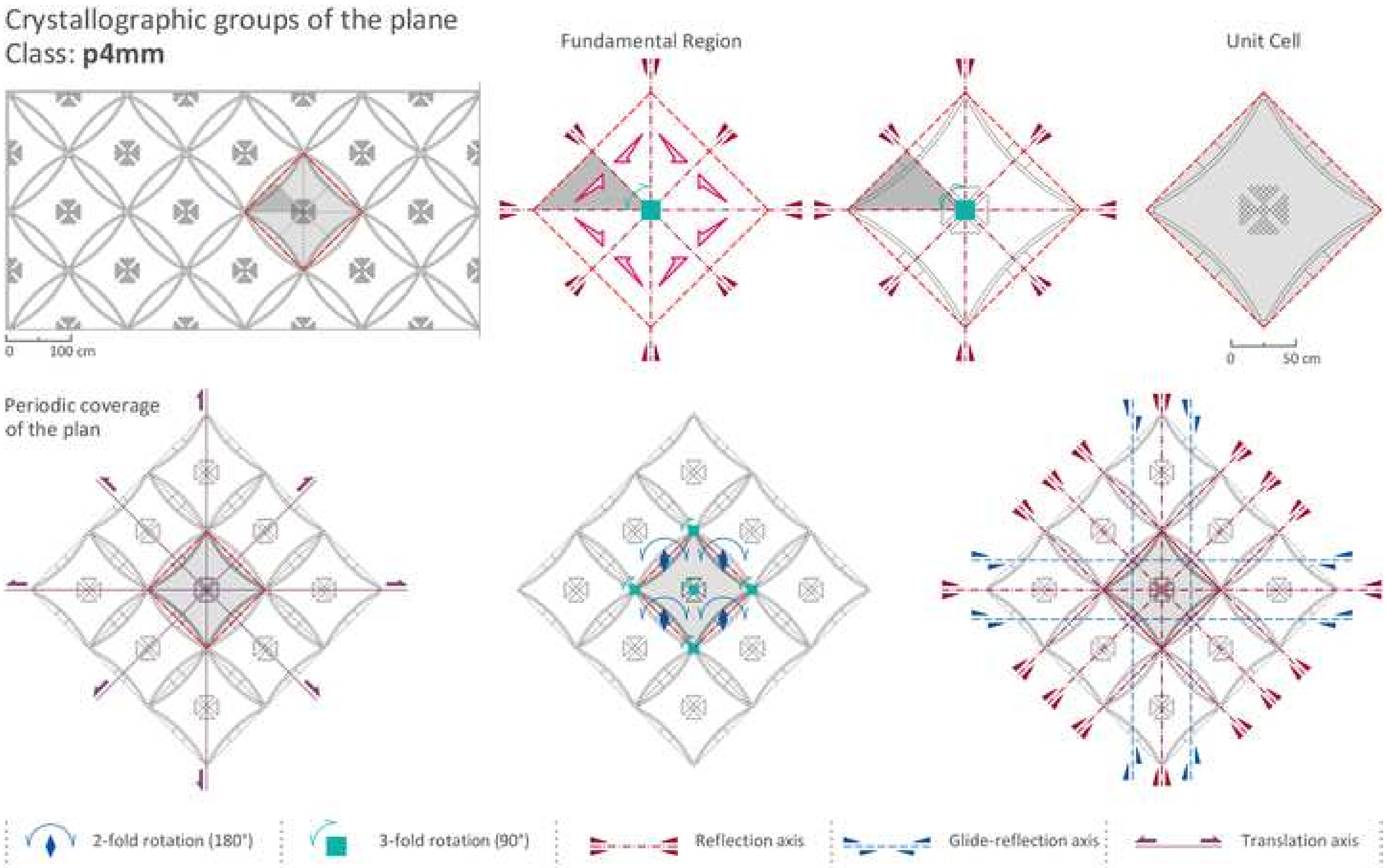

Fig 13

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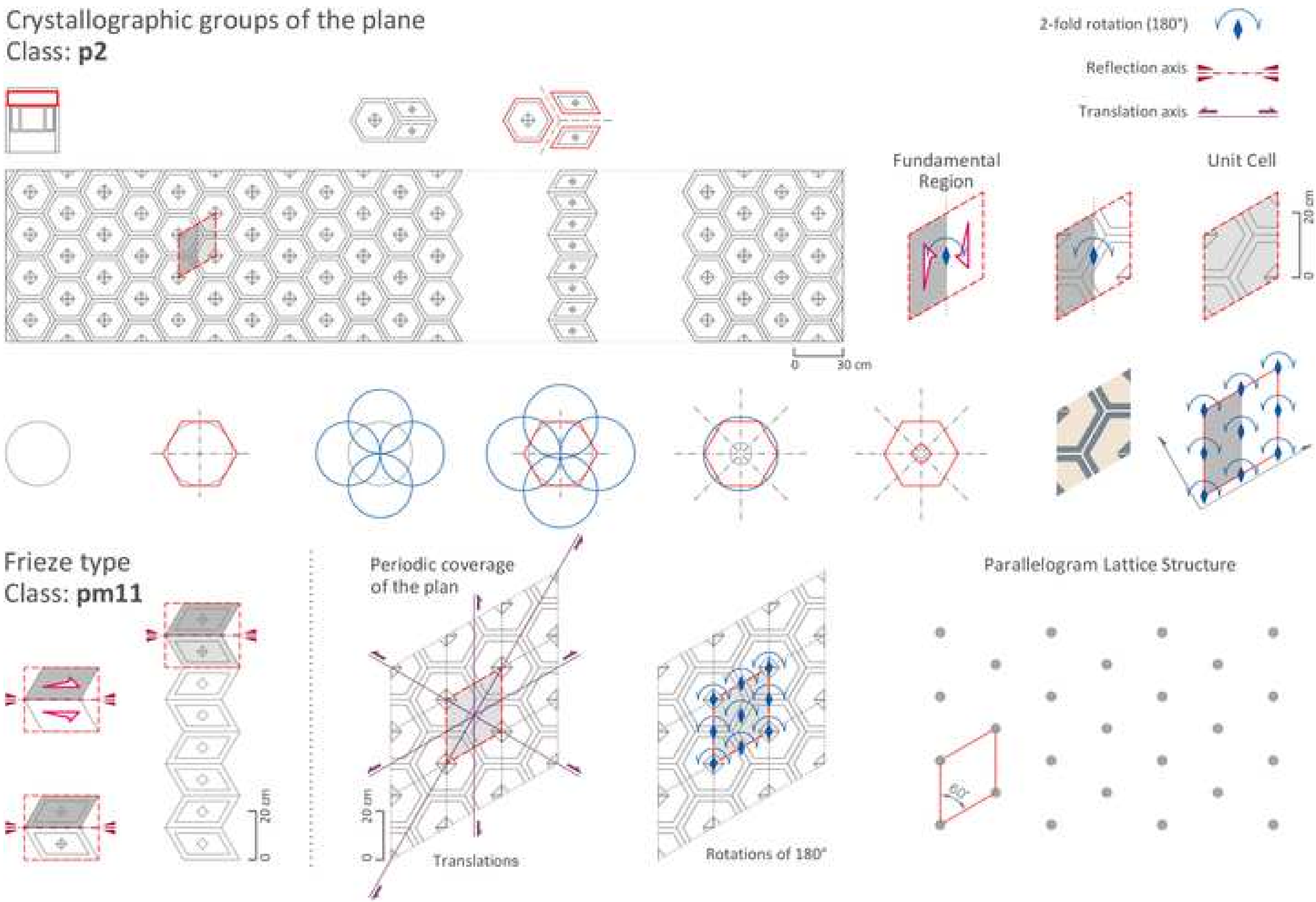

Fig 14

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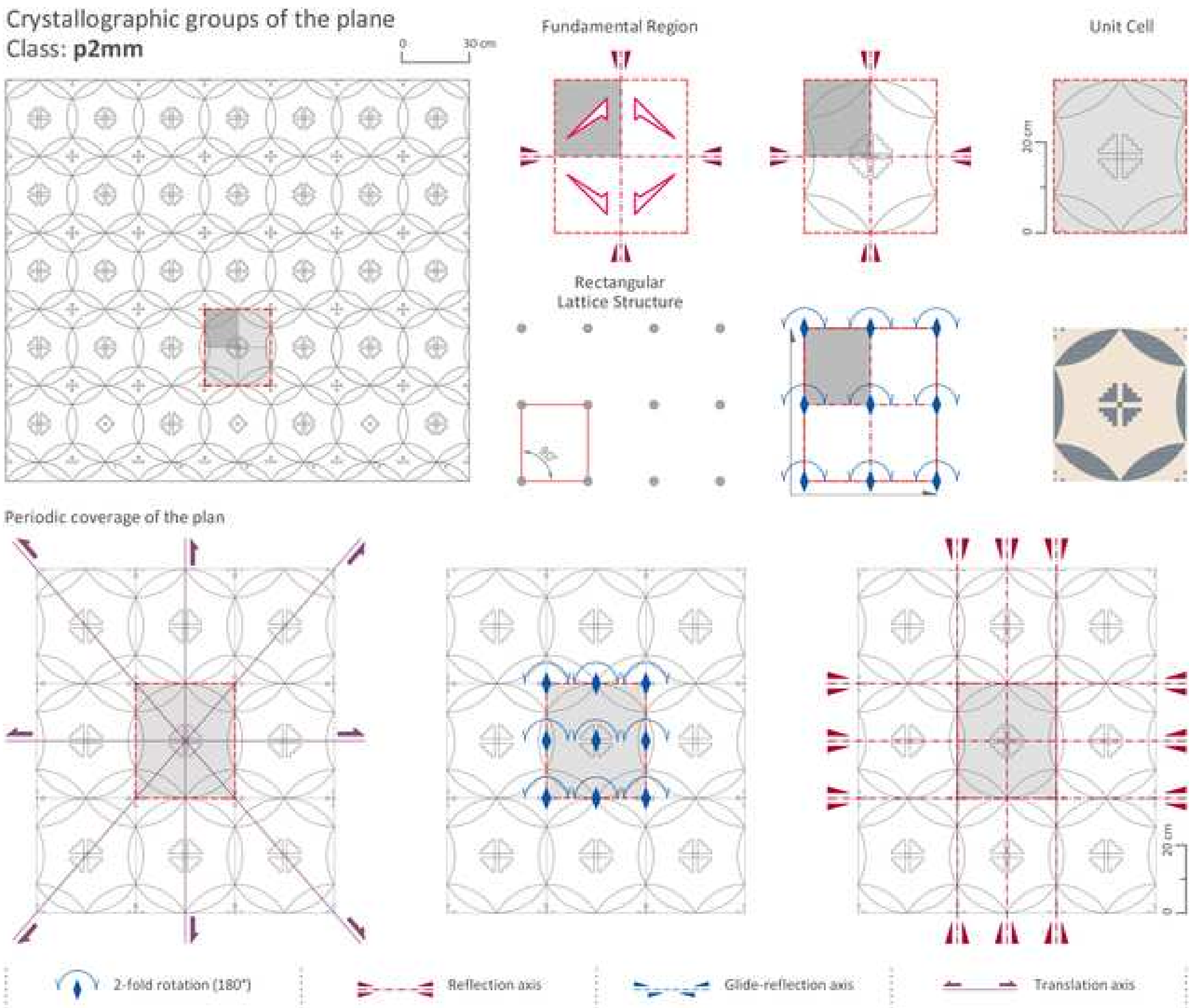
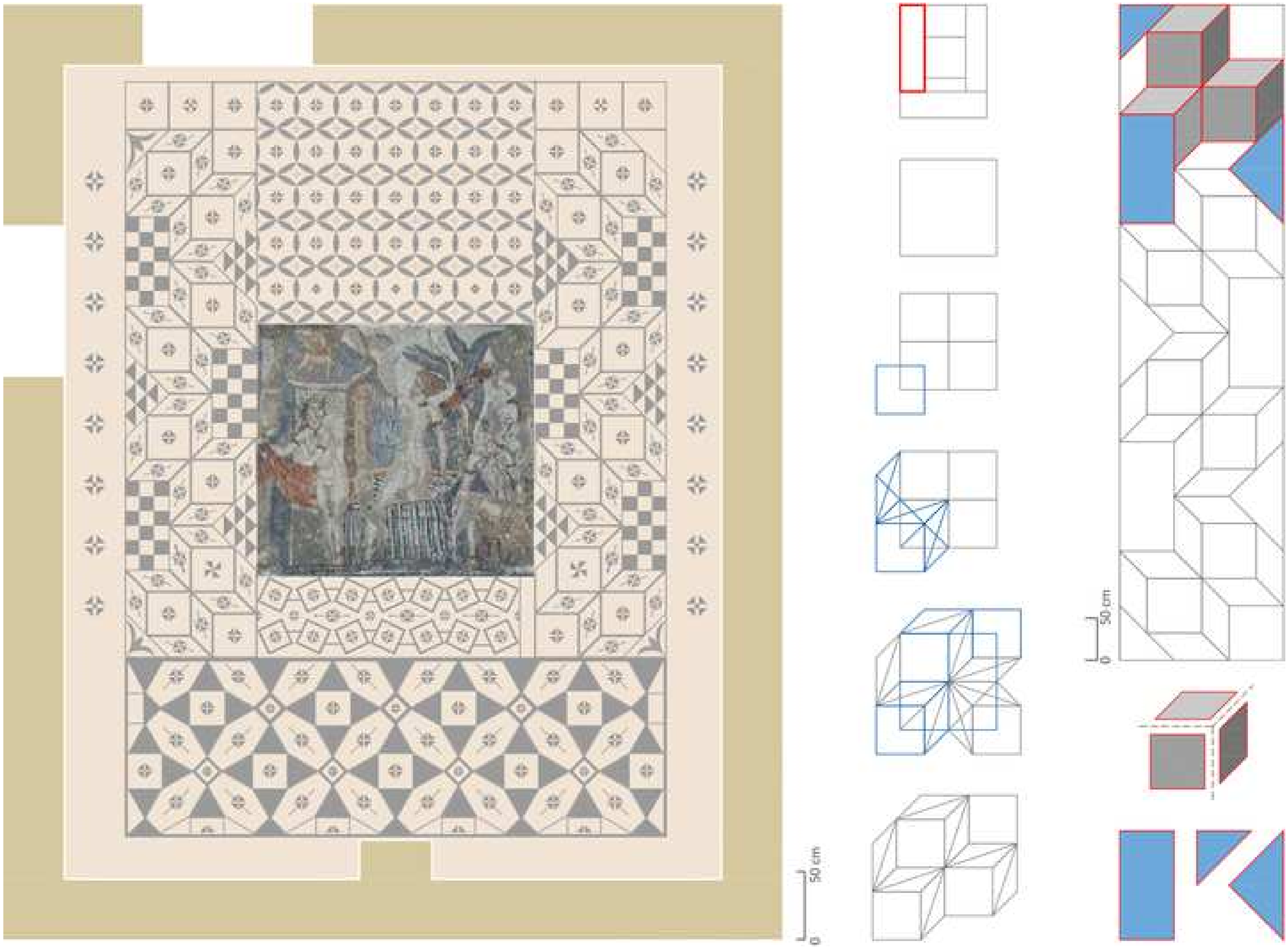


Fig 15 Click here to access/download;Figure;Fig 15.tif



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