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Forest Fires: Silvicultural Prevention and Mathematical Models for Predicting Fire Propagation in Southern Italy

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Abstract: In the Mediterranean basin, coniferous reforestation mainly comprises forest stands highly susceptible to fires. When silvicultural treatments have not been performed for decades after plantation, these stands often exhibit high vertical and horizontal tree density, along with a significant occurrence of lying and standing deadwood, thereby increasing the fuel load. On average, these pine forests are characterized by high values of above-ground biomass, ranging from 175 to 254 Mg ha^{-1} for the younger and the older ones, respectively. The theoretical heat energy produced per surface unit, in the case of the total combustion of the above-ground biomass, is also high, varying from 300 to 450 MJ ha^{-1} depending on the stage of stand development. In this study, we demonstrated the importance of silvicultural interventions in reducing the pyrological potential in pine reforested stands located in southern Italy, also giving attention to the water savings needed during extinction phases. In detail, we applied a preliminary mathematical reaction-diffusion model aimed at predicting the development of forest fires. The model was applied using data obtained through the estimation of the pyrological potential in terms of heat energy produced per surface unit (1 hectare) and the variation in the critical surface intensity. We verified that, when silvicultural interventions are applied, they induce a reduction of heat energy ranging between 17 and 21%, while the extinguishing water saved ranges between 600 and 1000 Mg ha^{-1}. Moreover, when the silvicultural interventions are implemented, the probability of the transition from surface fire to crown fire can be reduced by up to 31%. The most effective results on fire risk mitigation are mainly obtained when thinning aimed at reducing canopy and tree density is carried out in the younger phases of the reforested pine stands.

Keywords: Mediterranean mountain forests; forest management; tree density reduction; planted forests; *Pinus* spp.; heat energy; mathematical model; fire propagation

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1. Introduction

In the countries of Mediterranean Europe, coniferous reforestation, carried out from the early decades of the past century until the 1980s, is widespread in the forest landscapes of many inner and mountainous areas. In southern Italy, these reforestations were conducted using many species of *Pinus* spp. (*Pinus pinea* L., *Pinus pinaster* Aiton, *Pinus halepensis* Mill, *Pinus radiata* D. Don and *Pinus nigra* J.F. Arnold), with the primary aim of increasing soil protection and the stability of slopes in mountainous areas [1]. In the

Calabria region, from 1955 until the early 1980s, almost 155,000 hectares were reforested [2]. Currently, these plantations vary in age from 40 to 75 years old. The planting density was always very high, ranging from 2500 to 3250 seedlings per hectare, up to 15,000 where the plantation was carried out with sowing seeds [3].

Unfortunately, after the plantation and the first cultivation treatments, no thinning interventions were conducted in the following decades, as already detailed in other studies [4], even though such interventions are essential for ensuring the long-term resistance and resilience of these plantations, as they improve the structural and mechanical stand stability in the long term. In fact, after planting, many reforested stands were progressively abandoned. As a result, they are characterized today by poor vegetative conditions in terms of stand structure (highly dense) and very slow growth trends. Furthermore, in the absence of forest management, conifer reforestations become unstable over time, and individual trees tend to fall or uproot in adverse weather conditions [5]. These stands are highly vulnerable to natural disturbances such as strong winds, heavy snowfalls, and pest attacks, leading to a significant increase in unhealthy trees and standing and lying deadwood. This accumulation significantly increases the forest fuel load, making highly dense forest stands particularly vulnerable to fires, which are very common in Mediterranean mountainous environments [6]. As a result, a fire that affects a reforested stand correctly and regularly managed would probably be characterized by low intensity; conversely, an abandoned and never thinned artificial stand could experience potentially dangerous and extremely high-severity fires [7]. Therefore, the vulnerability of coniferous reforestation to fire can be attributed to the typical Mediterranean environment as well as to the absence of silvicultural treatments. This lack of management results in a highly dense vertical and horizontal forest structure, where interruptions of the crown continuity are absent. These two factors contribute to an increase in the fuel load, which makes these plantations even more sensitive to fire risk.

Fires are a common, albeit negative and predictable, disturbance of Mediterranean and boreal forests [8,9]. In addition, climate change has increased the occurrence of environmental conditions that produce and sustain forest fires [10]. Consequently, wildfire behavior has changed across many different regions and forest types in recent decades [10,11], with negative effects on forest ecosystems. In this new scenario, fire severity can be associated with extreme post-fire changes, including total or near-total overstory mortality and heavy losses of biodiversity [12]. Moreover, higher fire severity induces deep ground-layer consumption, leading to changes in soil properties such as organic matter content, nutrients, electrical conductivity and the level of soil hydrophobicity [13,14].

However, the implementation of sustainable forestry practices can be the key factor in reducing wildfire risk, maintaining forest health and improving its resistance and resilience to natural disturbances. As climate change and human activity increase the frequency and intensity of wildfire events that threaten forest survivability, developing sustainable forest management plans can support wildfire prevention. Forest management is a rigorous, methodical and time-tested combination of practices and technologies designed in part to reduce the severity and frequency of forest fires and mega-fires [15,16]. More in detail, forest management tools can include (i) creating fire breaks that can interrupt fire spread by removing the vegetation that fuels it; (ii) properly spacing planted trees; (iii) reducing understory vegetation which, if left on the forest floor, acts as fuel for a fire; (iv) thinning forests, which involves the removal of weak and diseased trees to give more space for healthy trees to grow, making the forest less susceptible to fire (regions with less precipitation, such as Southern Europe, benefit the most from tree removal); and (v) prescribing fires to reduce fuels.

Unfortunately, for most of the 20th century, and still today, the approach to agro-forest fires has been almost exclusively based on fire suppression. However, in an environmental context such as the Mediterranean basin, characterized by a high frequency of fires in the dry summers, mitigating the negative effects of fires on natural ecosystems at local and global scales requires an integrated approach. This approach should combine fire suppression systems with prevention strategies [17–20]. Nowadays, these issues are even more important as climate change is increasing vulnerability to forest fires, leading to higher intensity and frequency, even in an environmental and climatic context where the fire season is longer than in the past [21].

Fire prevention activities include all the silvicultural interventions (thinning, cleaning, pruning and prescribed fire) that significantly improve the forest stand composition and structure [20–22]. This approach must focus on fuel management in the most at-risk areas, with the ultimate aim of reducing fire fuels at the landscape scale [23,24]. Therefore, it is possible to decrease the probability of severe forest fire behavior through the active management of forest stand structures and surface fuel characteristics, quantifying the effectiveness of fuel treatments. For example, fire risk can be decreased by using various forest management techniques such as (i) reducing surface fuels; (ii) increasing the height of the crown insertions; and (iii) reducing the tree density, thus creating gaps between tree crowns. In fact, the usefulness of forest management activities for fire prevention and forest fuel reduction has already been highlighted in several studies [25–33]. To evaluate the suitability and effectiveness of preventive management measures (e.g., thinning) in coniferous plantations that are particularly vulnerable to fires, it is necessary to assess both the fuel amount and its potential impact on forest fires, as well as the respective heat energy that can potentially be removed through silvicultural interventions.

Since 2015, the European Forest Fire Information System (EFFIS) has been a crucial component of the European Union's Copernicus Emergency Management Services (EMS) program, enabling comprehensive data exchange on forest fires and their impacts [34]. In Italy, during 2023, evergreen broadleaf forests were the most affected by fires, with 7071 hectares burned, followed by conifers (2145 hectares), deciduous broadleaf forests (1130 hectares) and unclassified forests (569 hectares). Remarkably, from 18 to 27 July, 8448 hectares of forest ecosystems burned, accounting for nearly 78% of the total burned area for the cited year. Given these circumstances, it is essential to develop models that accurately represent the complex dynamics of fire spread using a robust physical approach. Additionally, simplifying these models is necessary for facilitating the application of advanced data analysis and numerical integration techniques.

Various mathematical models have already been proposed in the literature. According to the classification by Pastor et al. [35], these models can be categorized based on the nature of their equations, the selected variables and the physical system being modeled. The first classification scheme includes three types of models: (1) theoretical models, derived from the principles of fluid mechanics, combustion and heat transfer; (2) empirical models, based on statistical correlations from experiments or historical wildfire data; and (3) semi-empirical models, which combine general theoretical expressions with experimental data. The second classification scheme includes two types of models: (I) wildland fire spread models, which provide mechanisms for determining key physical variables related to fire perimeter advance, such as the rate of spread, fire line intensity and fuel consumption; and (II) fire front properties models, which describe the geometric characteristics of flames such as the height, length, depth and angle of inclination. The third classification scheme differentiates among surface fire models, applicable to physical systems comprising surface fuel under two meters in height, including small trees, shrubs, herbaceous vegetation and fallen logs; crown fire models, which pertain to the physical system formed by surface and aerial vegetation layers; spotting models, which describe the transport of firebrands or burning material carried by convection columns beyond the main fire perimeter; and ground fire models, which address the organic forest layers beneath the litter, encompassing fermentation and humus layers above the mineral soil.

Based on the aforementioned concepts, the main objectives of this study were to (1) evaluate the level of wildfire risk before and after silvicultural treatments aimed at reducing stand tree density, which left a semi-open stand with tree crowns well spaced apart; (2) assess the effect of the fuel treatments on the forest stands; (3) estimate the quantity of water pre- and post-intervention necessary for extinguishing fires; and (4) develop a preliminary mathematical reaction–diffusion model aimed at predicting the development of forest fires. Particularly, this study represents the first attempt to create a reaction–diffusion mathematical model that is directly linked to data obtained from silvicultural studies. This innovative model integrates empirical data on forest stand conditions and treatment effects, providing a novel approach for simulating and predicting fire dynamics in forested environments. By combining theoretical modeling with practical data, this study advances the field of wildfire management and offers a new tool for more accurate and effective fire risk assessment.

2. Materials and Methods

2.1. Study Area

The examined coniferous reforestations are located in two geographical contexts of the Calabria Region in Southern Italy. The first area (hereafter AREA A) is in the southernmost part of Calabria and Southern Europe, on the Aspromonte Massif, near Roccaforte del Greco (38°03′ N; 15°54′ E) (province of Reggio Calabria). It is situated at an altitude ranging between 900 and 1100 m a.s.l. The second area (hereafter AREA B) is located on the hilly and mountainous Tyrrhenian side above the isthmus of Catanzaro (between 38°42′ and 39°49′ N; 16°20′ and 16°17′ E) (provinces of Vibo Valentia and Catanzaro), at an altitude of 430–700 m a.s.l.

AREA A extends over 5000 hectares and is characterized by a reforested stand of *Pinus* spp., with an average age of approximately 75–80 years. AREA B covers 1200 hectares, where reforestations of *Pinus* spp. occur, with ages varying from 35 to 45 years. After planting, these stands were never subjected to silvicultural interventions, evolving naturally. In both study areas, reforestations were carried out with the aim of protecting the land from erosive phenomena caused by rainwater and winds. Due to the absence of silvicultural interventions, the stands have high tree densities and a continuous vertical structure. For each area, the hypothesized silvicultural interventions aimed at reducing the above-ground biomass to lower the fire risk involve medium-intensity thinning from below, removing, on average, 20% to 22% of the basal area. The identified study areas, among the forest stands most affected by fires in Southern Italy, are representative of the reforestations carried out in Calabria in the last century, both in terms of age and forest structure (vertical and horizontal).

For both study areas, the climate is typically Mediterranean, characterized by semi-arid conditions and dry summers, with local differences in precipitation and temperatures due to changes in altitude and topographic conditions. Specifically, in AREA A, the average annual temperature is 14.2 °C, with average temperatures of 5 °C in January and 23 °C in August. The mean annual rainfall is 1036 mm, with a minimum in July (13.7 mm) and a maximum in December (153 mm). According to the USDA (United States Department of Agriculture) Soil Classification [36], the most frequent soils are Humic Dystrudepts, which are acidic and poor in organic matter, with a high infiltration rate and moderate aggregate stability. AREA B is quite homogeneous from a climatic and geopedological point of view. The average annual temperature is 15.5 °C, with average temperatures of 6 °C in January and 22 °C in August. The mean annual rainfall is 856 mm. According to the USDA soil classification [36], the most frequent soils are Psammentic Dystrudepts, mixed, mesic, shallow, coarse-textured, and acidic soils.

Considering a time-lapse of 10 years (from 2013 to 2022), the fire regime in the Calabria Region is characterized, on average, by 723 annual fires, 9960 ha of total annual burnt area and an average fire size of 13 ha [37]. The fire season mainly refers to the period between July and September.

2.2. Data Collection and Analysis

Data collection was carried out in 26 circular plots with a radius of 20 m (1256 m^2) (16 and 10 plots in AREA A and B, respectively), located through a systematic sampling design. In each plot, the following parameters were recorded: (1) the number of trees, (2) the diameter at breast height (*DBH*) of all the trees, (3) the total height (*Ht*) and (4) the height of the crown insertion (H_{bc}) on 25% of the trees, homogeneously distributed across the different diameter classes. The plantation age was determined through historical records [1]. The fertility of each plantation, expressed in terms of dominant height, was evaluated based on the height development of the 10 trees with the largest diameters for each plot. In this study, the fertility index or Site Index (*SI*) expresses the dominant height at the current age of the stands (40 years for AREA A and 80 years for AREA B). Three Site Index levels were distinguished for AREA A (17.1, 18.9 and 22.7 m), and three Site Index levels were distinguished for AREA B (13, 15 and 17 m).

The development of the total height (*Ht*) and the height of the crown insertions (H_{bc}) was carried out by applying a single model. For AREA A, the diameter at breast height (*DBH*) and Site Index (*SI*) were considered as independent variables, while for AREA B, in addition to the DBH and SI, age (*AGE*) was also considered, since the tree age variation was higher (from 35 years to 45 years). Height–diameter relationships with a combination of these variables have been included in several modeling approaches of tree growth [38–40]. The model coefficients were defined using stepwise regression. The following models were used:

$$H_A = e^{\left[\beta_0 + \beta_1 \cdot SI - \beta_2 \cdot \frac{1}{DBH} + \beta_3 \cdot \frac{1}{DBH \cdot SI}\right]}, \quad (1)$$

$$H_B = e^{\left[\beta_4 - \beta_5 \cdot \frac{1}{AGE} - \beta_6 \cdot \frac{1}{DBH \cdot SI}\right]}, \quad (2)$$

where H_A and H_B are the trees' total height and the crown insertion height in AREA A and AREA B, respectively, *SI* is the site index, *DBH* is the diameter at breast height and *AGE* is the age of the stands. β symbols are the parameters to be estimated in each equation.

The stand volume (*V*) and above-ground biomass (*AGB*) were estimated using the equations reported for *Pinus* spp. in the National Forest Inventory protocols [41,42]. Moreover, the pyrological potential was estimated in terms of the quantity of the theoretical heat energy produced per surface unit (1 hectare). In the case of the total combustion of the above-ground biomass, the following formula was used [22]:

$$HE = ABG \cdot HV, \quad (3)$$

where *HE* = Total heat energy (MJ ha^{-1}), *ABG* = Trees' above-ground biomass (Mg ha^{-1}) and *HV* = lower heating value for *Pinus* spp., equal to 18,840 MJ Mg^{-1} [43].

The reduction in fire intensity achievable by modifying the fuel load also acts on the possibility of the passage from surface fire to crown fire, in which the fire spreads through the tree crowns. In order to evaluate this feature, we estimated the variation in the critical surface intensity before and after the thinning intervention (*CSI*, expressed in kW m^{-1}), applying the equation of Van Wagner [44], which conditions the passage from a surface fire to a crown fire upon reaching a critical intensity capable of eliminating the contents in water from the foliage, heating them up to the critical ignition value:

$$CSI = 0.001 \cdot (H_{bc})^{1.5} \cdot (460 + 25.9 \cdot FMC)^{1.5}, \quad (4)$$

where H_{bc} = hight of the crown insertion (m), *FMC* = foliar moisture content (%).

According to some authors [45], the *FMC* value can vary from 60 to 140%, but a value of 100% is generally considered acceptable [46,47].

Finally, the amount of water necessary for fire extinction in the case of a total combustion of the above-ground biomass was also determined, before and after the silvicultural interventions. For estimating the quantities of water, the following formula was used [48]:

$$W_{H2O} = \frac{HE}{\beta \cdot \Delta h_w}, \tag{5}$$

where W_{H2O} = Quantity of water necessary for the absorption of the heat energy developed in the combustion (Mg ha^{-1}), *HE* = Heat energy (MJ ha^{-1}) and β = Efficiency coefficient equal to the ratio between the quantity of water poured on the fire, which undergoes complete vaporization, and the total quantity of water supplied; this coefficient represents the yield of the extinguishing process. Practical and experimental considerations [49] led to estimating rather low values of β, probably less than 0.6–0.7. According to the literature [48], in our case, β was assumed to be 0.4, and Δh_w = Latent heat of water evaporation at 100 °C (2440 MJ Mg^{-1}).

For Equations (1) and (2), we preliminarily tested the effect of several independent variables, or their combinations, on tree height and the height of the crown insertions, using different models. We employed a stepwise procedure, where in each step, a variable was considered for the addition to or subtraction from the set of explanatory variables based on F-tests. This process allowed us to simplify the models, thereby obtaining more efficient statistics. The results of the models were considered significant at a probability level of 5%. For the analysis, the R programming language was used.

2.3. Mathematical Model Proposed

The propagation of forest fires is a complex process characterized by stochastic behavior. Nowadays, the extensive availability of geospatial data and the application of advanced analytical techniques allow for a detailed understanding of forest fires, facilitating the development of more realistic models. The data and analyses presented in the previous section provide essential information regarding the fuel present in the analyzed areas, which can be explored using mathematical models.

Numerous models have been proposed in the literature. According to the classification by Pastor et al. [35], these models can be grouped in three ways: by the nature of the equations, the chosen variables and the physical system being modeled. Particular innovations in this field are reported in Richards [50], where a detailed methodology for mitigating forest fires is presented, focusing on the development and application of predictive models. The authors highlight the importance of using accurate modeling techniques to effectively manage and control the spread of fires. Specifically, the model integrates empirical data, differential equations and finite difference solutions to simulate fire behavior under varying fuel, weather and topographical conditions.

The model is based on the integration of the following key components:

- Empirical Data: The model incorporates historical data on fire occurrences, fuel types and weather conditions. These data serve as the foundation for understanding the basic patterns and behaviors of forest fires.
- Differential Equations: These describe the physical processes involved in fire spread, including heat transfer, combustion and fluid dynamics. By solving these equations, the model can predict how a fire will spread under specific conditions.
- Finite Difference Solutions: This numerical method is employed to solve the differential equations that govern fire behavior. By discretizing the equations, the model can simulate fire spread over a grid representing the forest landscape.

In Richards [50], further details on how the model accounts for various factors influencing fire spread include:

- Fuel Characteristics: Different types of vegetation and their distribution across the landscape are considered. The model includes parameters for fuel moisture content, density and combustibility.
- Weather Conditions: Wind speed and direction, temperature and humidity are critical inputs. The model uses these variables to predict their influence on fire spread and intensity.

- Topography: The physical features of the landscape, such as slopes and elevation, are included in the model. These features affect how quickly a fire can move and in which direction.

Through the integration of these components, the model can provide fire controllers with real-time predictions of fire behavior, allowing for better planning and resource allocation during fire suppression efforts. The article concludes by emphasizing the need for continuous improvement and validation of the model through ongoing research and field data collection.

These types of models are used to implement computer programs to support stakeholders. The Prometheus [51] and FARSITE [52] models serve as sophisticated tools for simulating wildland fire behavior, each incorporating distinct methodologies and data inputs to predict fire growth and behavior under varying conditions.

Prometheus, developed for use in Canada, employs a deterministic approach grounded in Huygens' principle of wave propagation. It integrates spatial data on topography, fuel types and weather conditions to simulate fire spread using the Canadian Forest Fire Behavior Prediction System. This model creates detailed fire perimeters at user-specified time intervals, with each active vertex along the perimeter providing fire behavior outputs compatible with geographic information systems. The model structure is component-based, utilizing Microsoft COM technology to manage and retrieve data related to fuel, topography, weather and fire growth calculations, ensuring a modular and extensible framework for fire simulation.

FARSITE, on the other hand, is a comprehensive computer simulation model that includes existing fire behavior models for surface fires, crown fires, spotting, point-source fire acceleration and fuel moisture. It operates by integrating these models into a vector propagation technique that controls both the space and time resolution of fire growth over landscapes. The model produces vector fire perimeters at specified time intervals, with vertices containing information on the fire's spread rate and intensity. FARSITE's simulations illustrate the spatial consequences of incorporating various fire behavior models into a two-dimensional simulation, providing valuable insights for researchers and fire managers by comparing simulated fire behavior with observed patterns under simplified test conditions. Both models offer robust frameworks for understanding and predicting wildland fire dynamics, supporting effective fire management and research efforts.

In other articles in the literature, researchers have addressed the complex dynamics of fire spread through distinct methodologies. Particularly, the focus is on three models [53–55]. The first model, presented by Boychuk et al. [53], utilizes a stochastic lattice approach, where the fire spread, spotting and burnout processes are modeled using local transition rates that depend on covariates such as topography, fuel moisture and weather conditions. This model is unique in that it allows for spatially varying randomness, suitable for representing local wind gusts and fuel heterogeneity, distinguishing it from deterministic models that apply uniform perturbations across the entire fire area at each time step [53].

The second model, described by Greese et al. [54], introduces the concept of birth-jump processes, which are integro-differential equations that couple the spatial spread and growth of populations. This model is particularly relevant for scenarios where new firebrands (burning embers) are transported by the wind and may ignite new fires ahead of the main fire front, a process known as spotting. The birth-jump model integrates the dynamics of position-jump processes with reaction–diffusion models, providing a comprehensive framework for analyzing fire spread influenced by both local interactions and long-distance spotting [54].

The third model, presented in [55], focuses on the aerodynamics of firebrands and their transport through the atmosphere. It incorporates detailed descriptions of the drag force experienced by firebrands, terminal velocity and combustion processes. This model is valuable for understanding the physical behavior of firebrands during flight and their potential to start new fires, with particular attention to parameters such as wind profiles and the drag coefficient, which significantly affect the transport and eventual landing of firebrands [55].

In summary, these models differ in their fundamental approaches: Boychuk et al.'s model emphasizes local stochastic variations, Greese et al.'s model integrates spatial spread and growth through birth-jump processes and the Martin and Hillen model focuses on the physical dynamics of firebrand transport. Each model offers unique insights and tools for predicting and managing wildfire spread under various conditions and has recently been used and cited together [50–53] by the researchers who presented the work [56] that introduces a spatially and temporally continuous stochastic model for fire spread utilizing the wealth of available data. The state of the forest fire is represented by sub-probability densities of live trees and trees engulfed in flames, estimated using satellite- and ground-based sensor data. Fire dynamics are captured in a probability density function that accounts for variables such as the wind patterns, terrain slope and spotting phenomena, resulting in a set of integro-differential equations for these densities. The existence and uniqueness of solutions for this type of model are established using Banach's fixed-point theorem, and the long-term behavior of the model is also analyzed [56].

Stochastic models based on cellular automata can be considered specific instances of the model by Beneduci and Mascali [56], derived through spatial and/or temporal discretization. By defining a specific structure for the density function, we perform numerical simulations of fire spread under various conditions. For example, in the scenario of a forest fire approaching a river, simulations suggest that the probability density of burning trees remains non-zero beyond the river due to spotting

Other types of models may prove accurate and suitable for utilizing the aforementioned data. In this paper, we present a reaction–diffusion model, which considers the following two-dimensional energy equation [57–60]:

$$\rho C\left(\frac{\partial T}{\partial t} + \boldsymbol{w}\cdot\nabla T\right) - \nabla\cdot\left((k + 4\sigma\delta T^3)\nabla T\right) + h(T)(T - T_\infty) = \rho Q\lambda Y, \tag{6}$$

where ρ is the air density and C is its specific heat; $T(\boldsymbol{x}, t)$ is the average temperature at the position $\boldsymbol{x} = (x, y)$ at time t; the drift velocity $\boldsymbol{w}$ is the sum of the average wind velocity and of the gradient of the topography height function, $\boldsymbol{w} = \bar{\boldsymbol{w}} + \nabla h_T$; $k + 4\sigma\delta T^3$ is the turbulent diffusivity, with σ being the Stefan–Boltzmann constant and δ being the optical path length for radiation in the fuel bed; $h(T) = |T - T_\infty|^{1/3}$ is the heat transfer coefficient due to vertical convection, with T_∞ being the atmospheric temperature; and $\rho Q\lambda Y$ is the heat source due to combustion, where Q is the heat of the combustion, λ is the reaction rate and Y is the unitary concentration of solid fuel.

We postulate that the solid fuel undergoes a transformation into combustion products and heat through a single-step chemical reaction. For the sake of simplicity, the gaseous fuel phase is not considered in this model. This transformation can be described by the following reaction equation:

$$\frac{\partial Y}{\partial t} = -\lambda Y, \tag{7}$$

$$\lambda = H(T - T_{pc})A(T)e^{-\frac{E}{RT}}, \tag{8}$$

where the reaction rate λ is given by the modified Arrhenius law, R is the universal gas constant, E is the activation energy, $A(T) = \bar{A}|T|^{1/2}$ is the pre-exponential factor, T_{pc} is the phase-change temperature and $H(\theta)$ is the Heaviside function.

We introduce the following scaling approach to facilitate our analysis:

$$\boldsymbol{x} = l_0\hat{\boldsymbol{x}}, \quad t = t_0\hat{t}, \quad \boldsymbol{w} = \frac{l_0}{t_0}\hat{\boldsymbol{w}}, \ T = T_\infty(1 + \varepsilon u), \quad Y = Y_0 v, \tag{9}$$

By applying appropriate non-dimensionalization techniques, we simplify the complex variables and parameters involved in the combustion process. This involves rescaling the spatial and temporal coordinates, as well as the physical quantities such as the temperature, concentration of reactants and other relevant properties. The aim is to reduce the number of parameters and to highlight the dominant physical processes governing the reaction. This scaling approach allows us to transform the original set of equations into a more tractable form, making it easier to identify key dimensionless groups and to perform subsequent analytical or numerical analysis. The resulting scaled equations provide insights into the behaviour of the system under different conditions and enable a more systematic investigation of the combustion dynamics.

In the scaling adopted, l_0 is a characteristic value for the spatial variables, t_0 is a characteristic value for the time variable, Y_0 is a characteristic value for the initial fuel distribution and ε is the inverse of the scaled activation energy:

$$\varepsilon = \frac{RT_\infty}{E}, \tag{10}$$

In the development of the model, we make the following choices:

$$l_0 = \sqrt{\frac{kt_0}{\rho C}}, \quad t_0 = \frac{\varepsilon e^{\frac{1}{\varepsilon}}}{q\bar{A}T_\infty^{1/2}}, \tag{11}$$

where

$$q = \frac{Y_0 Q}{CT_\infty}, \tag{12}$$

is the scaled combustion heat. We obtain the scaled equations

$$\frac{\partial u}{\partial t} + \hat{\boldsymbol{w}} \cdot \hat{\nabla} u - \hat{\nabla} \cdot \left(K(u)\hat{\nabla} u\right) = f(u)v - g(u), \tag{13}$$

$$\frac{\partial v}{\partial t} = -\frac{1}{q} f(u)v, \tag{14}$$

with $\hat{\nabla}$ being the formal vector of derivatives with respect to the components of $\hat{\boldsymbol{x}}$, and

$$K(u) = 1 + \kappa(1 + \varepsilon u)^3, \tag{15}$$

$$f(u) = H\left(u - u_{pc}\right)(1 + \varepsilon u)^{\frac{1}{2}} e^{\frac{u}{1+\varepsilon u}}, \tag{16}$$

$$g(u) = \alpha |u|^{1/3} u, \tag{17}$$

with

$$\kappa = \frac{4\sigma\delta T_\infty^3}{k}, \ \alpha = \frac{\bar{h}\varepsilon^{1/3} T_\infty^{1/3} t_0}{\rho C}, \tag{18}$$

In the subsequent sections, we will simplify our notation by omitting the "hat" symbol typically used to indicate dimensionless variables. This change is intended to streamline our mathematical expressions and improve the readability of our equations.

In conclusion, the inclusion of variables such as (1) the number of trees, (2) the diameter at breast height (*DBH*) of all trees, (3) the total height (*Ht*) and (4) the height of crown insertion (H_{bc}) for 25% of the trees, uniformly distributed across different diameter classes, is fundamental for the reaction–diffusion model. These variables are critical, as they provide detailed and specific information about the forest structure, which directly influences fire behavior and dynamics. The number of trees and their diameters contribute to understanding the density and distribution of fuel within the forest stand. These data are essential for evaluating the fuel load, a key factor in determining fire intensity and spread. The diameter at breast height (*DBH*) provides insight into the size of the trees, affecting the amount of combustible material available and its potential to sustain and propagate a forest fire. The total height (*Ht*) and the height of crown insertion (H_{bc}) are crucial for assessing the vertical structure of the forest. The height of the tree crowns is particularly significant because it determines vertical fuel continuity and the potential for crown fires.

The spacing of crown heights influences how easily a fire can climb from the ground to the canopy, thus affecting the overall fire risk and behavior. By incorporating these variables, the reaction–diffusion model can more accurately simulate the impact of the fuel load and forest structure on fire behavior. This detailed characterization allows the model to better estimate fire spread and intensity, as well as the temperature thresholds required for ignition. Therefore, these variables are indispensable for refining the model's predictions and enhancing its applicability in real-world fire management scenarios.

3. Results

3.1. Silvicultural Approach

Data from 2024 indicate that, in Calabria Region, the pine reforestations range in age from around 35 to 80 years old [1], with a greater concentration in the latter age classes. The main dendrometric parameters of the plantations, before and after the hypothesized silvicultural interventions (thinning), are shown in Table 1.

Table 1. Pine plantations' dendrometric and pyrological characteristics before and after thinning interventions.

	80-Year-Old Stand			**40-Year-Old Stand**		
	Thinning			**Thinning**		
	Before	**After**	**Diff%**	**Before**	**After**	**Diff%**
Number of trees [N° ha^{-1}]	1107	757	−31.6	1510	940	−37.8
Mean Diameter [cm]	28.5	30.4	6.7	21.3	23.7	11.5
Mean Height [m]	17.3	17.5	1.2	15.4	15.8	3.1
Crown Insertion Height [m]	7.8	8.1	3.8	7.4	8.9	19.9
Basal Area [m^2 ha^{-1}]	70.6	54.8	−22.4	53.6	41.5	−22.6
Volume [m^3 ha^{-1}]	523.2	410.6	−21.5	354.2	290.2	−18.1
Above-ground biomass [Mg ha^{-1}]	254.6	199.3	−21.7	175.7	142.6	−18.8
Heat energy [MJ ha^{-1}]	453.2	354.8	−21.7	312.7	253.9	−18.8
Critical Surface Intensity [kW m^{-1}]	369.0	389.9	5.7	339.5	445.6	31.3
W_{H2O} [Mg ha^{-1}]	4643	3635	−21.7	3204	2601.3	−18.8

The pine forests were all characterized by high tree densities, with values of the basal area that, in some plots, exceed 100 m^2 ha^{-1}, despite the high mortality due to the self-thinning processes. Before thinning, for both the study areas investigated, the trees distributions in diameter classes (Figure 1) highlight a high occurrence of small trees, especially in the young pine forests.

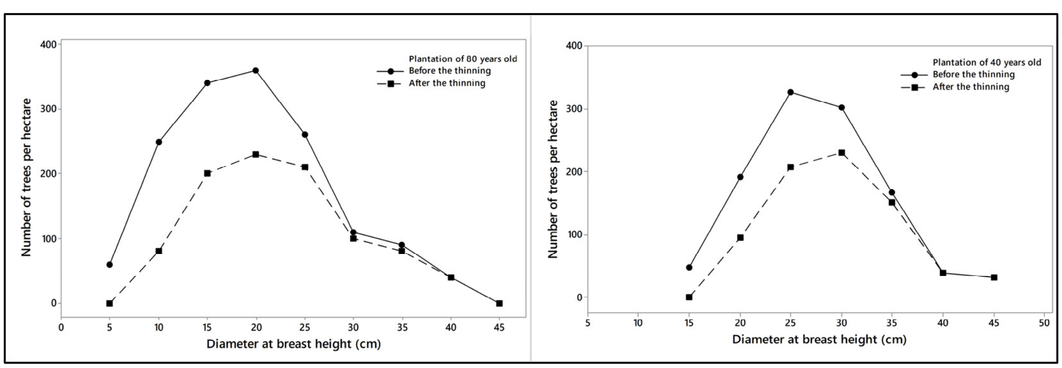

Figure 1. Distribution of the number of trees before and after the thinning interventions in the plantations of 80 and 40 years of age.

In relation to the different pedoclimatic contexts examined, the above-ground biomass values (important parameter in terms of pyrologic potential) vary, on average, from about 176 Mg ha^{-1} for the youngest stands (35–40 years) to about 255 Mg ha^{-1} for the older ones (75–80 years). Moreover, in Table 2, the trees' height curve and the crown insertion height are reported with the related values of the regression coefficients, the determination coefficients (R^2) and Root Mean Square Error (*RMSE*). Furthermore, Figure 2 shows the Site Index (*SI*) effect on tree height development for Area A.

Table 2. Model equations for total tree height (*Ht*) and crown insertion height (H_{bc}) for both older (age = 80) and younger (age = 40) plantations, with the equation parameters, standard error of parameters (S.E.), significance values (T and P), coefficient of determination (R^2) and root mean square error (RMSE).

Model Equation		β_0	β_1	β_2	β_3	R^2	**RMSE**
Equation (1) (*Ht*)	Parameter	1.807	0.065	−21.387	303.701	0.702	0.0681
80-year-old stand	S.E.	0.0157	0.008	3.483	62.483		
	T	11.522	7.892	−6.141	4.861		
	Sig.	<0.001	<0.001	<0.001	<0.001		
Equation (1) (H_{bc})	Parameter	0.662	0.105	−39.352	423.213	0.677	0.189
80-year-old stand	S.E.	0.434	0.023	9.640	172.951		
	T	1.525	4.605	−4.082	2.447		
	Sig.	0.129	<0.001	<0.001	0.16		
Equation (2) (*Ht*)	Parameter	4.721	−66.806	−101.660		0.703	0.0902
40-year-old stand	S.E.	0.072	2.958	6.802			
	T	65.854	−22.587	−14.946			
	Sig.	<0.001	<0.001	<0.001			
Equation (2) (H_{bc})	Parameter	3.630	−30.353	−277.096		0.397	0.2555
40-year-old stand	S.E.	0.203	8.375	19.260			
	T	17.883	−3.624	−14.387			
	Sig.	<0.001	<0.001	<0.001			

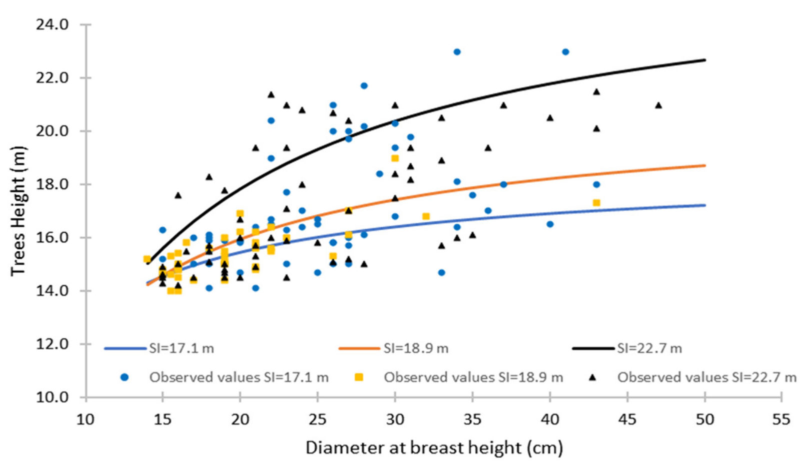

Figure 2. Tree height as a function of diameter classes, shared by fertility classes (Site Index).

Hypothesizing the fuels treatment and the thinning activities, the stand density (in terms of the number of trees per hectare) was reduced by 38% and the 32%, bringing the number of trees to 940 and 757 per hectare for the youngest and most adult stands, respectively (Table 1). Furthermore, about 22–23% of the basal area was removed for both the older and the younger plantations. As a result, there were changes in the mean diameter

(*Dm*) and mean height of the trees (*Hm*) but also for the distance between the tree crown and surface fuel (H_{bc}): these factors can partially determine the beginning and spread of the crown fire. In fact, the higher the crown base, the lower the canopy fuels amounts and the lower the chance of crown fire. The crown insertion height recorded a consistent percentage increment, especially when thinning was carried out early (Table 1). In this case, the increment was almost 20%, compared to just 4% when thinning was carried out too late, showing that thinning carried out too late seems to be less effective for this parameter, which is so crucial for avoiding crown fires. In terms of above-ground biomass (*AGB*), a reduction of 21.7% and of 18.8% was recorded for the older and the younger stands, respectively (Table 1).

The variation in heat energy (*HE*) theoretically produced by the total combustion of the standing biomass had a substantially identical trend to the biomass, being strictly connected to it (Table 1). When thinning activities are realized, the reduction in pyrological potential was about 98,000 kJ m^{-2} in the older pine forests and about 59,000 kJ m^{-2} in the younger ones. These values express the maximum amount of heat energy that can theoretically be developed if the standing biomass is completely burned. Figure 3 shows the quantities of pyrological potential before and after thinning operations. For both of the areas, in terms of heat energy (*HE*) produced in the event of a forest fire, the Wilcoxon paired-sample test shows high significance ($p < 0.01$) between before and after thinning (Figure 3).

The critical surface intensity (*CSI*) variation, linked to changes in the crown insertion height, significantly increases, especially when thinning is carried out early (up to about 31.0%). In contrast, when thinning is postponed, CSI varies by only 5.7% (Figure 4). However, even when the thinning is carried out late, the silvicultural interventions still show their effectiveness for fire containment, albeit to a lesser extent (Table 1).

Finally, the quantities of water needed (W_{H2O}) to absorb the heat energy produced (*HE*) in the event of a forest fire, before and after silvicultural interventions, vary in relation to the stands' age and to the amount of the above-ground biomass removed with thinning activities. In absolute terms, before the silvicultural interventions, almost 4650 Mg ha^{-1} and about 3200 Mg ha^{-1} of water is needed for the older and younger stands, respectively (Table 1). After the silvicultural interventions, the percentage of the water use reduction was 16.7% and 14.1% for the older and younger stands, respectively.

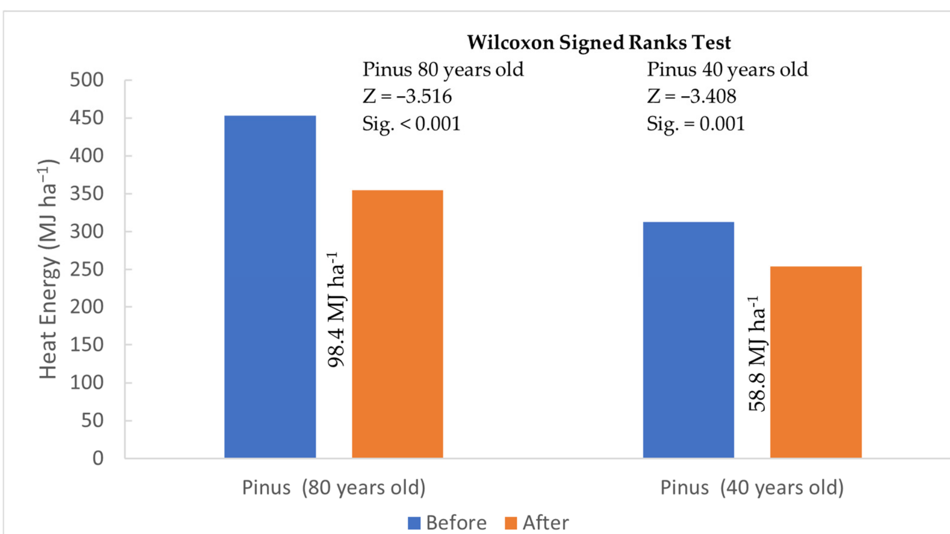

Figure 3. Heat Energy level before and after the thinning interventions.

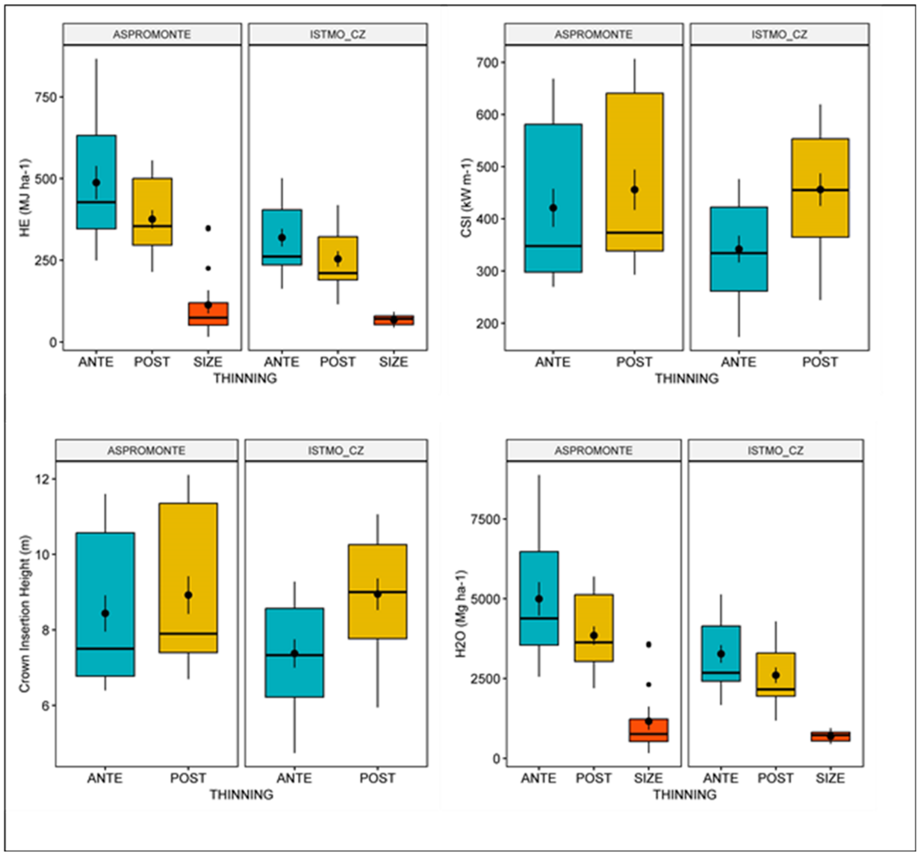

Figure 4. Heat energy (*HE*), Critical Surface Intensity (*CSI*), crown insertion height and water amount (*H2O*) before and after thinning. The midlines of the boxplots are the median, the points into the boxes refer to the mean values, the boxes show the first and third quartiles, and the points outside the boxes represent the outlier values.

3.2. Numerical Solution of the Mathematical Model and Numerical Simulations

We conduct a numerical integration of Equations (13) and (14), in a spatial domain Ω, with prescribed initial data u_0, v_0 and Neumann boundary conditions on ∂*Ω*:

$$\frac{\partial u}{\partial t} + \boldsymbol{w} \cdot \nabla u - \nabla \cdot (K(u)\nabla u) = f(u)v - g(u), \tag{19}$$

$$\frac{\partial v}{\partial t} = -\frac{1}{q} f(u)v, \qquad (\boldsymbol{x}, t) \in \Omega \times (0, t_{\text{fin}}),$$

$$u(\boldsymbol{x}, 0) = u_0(\boldsymbol{x}), \qquad v(\boldsymbol{x}, 0) = v_0(\boldsymbol{x}), \qquad \boldsymbol{x} \in \Omega,$$

$$\boldsymbol{n} \cdot (K(u)\nabla u) = 0, \qquad \boldsymbol{x} \in \partial\Omega$$

where $\boldsymbol{n}$ is the external normal to $\partial\Omega$.

We consider a rectangular grid in the domain $\Omega = [0, L_x] \times [0, L_y]$, with $(n_x + 1)(n_y + 1)$ grid points

$$\boldsymbol{x}_{i,j} = (x_i, y_j) := (ih_x, jh_y), \; h_x = \frac{L_x}{n_x}, \; h_x = \frac{L_x}{n_x},$$

with $i = 0,1, \ldots, n_x$, $j = 0,1, \ldots, n_y$. We discretize Equation (19) by means of the box-integration method, integrating it over the box [60–62].

$$I_{i,j} = \left[x_i - \tfrac{1}{2}h_x, x_i + \tfrac{1}{2}h_x\right] \times \left[y_j - \tfrac{1}{2}h_y, y_j + \tfrac{1}{2}h_y\right], \tag{20}$$

assuming $i = 1, \ldots, n_x - 1,\ j = 1, \ldots, n_y - 1$, obtaining the following equations:

$$\frac{du_{i,j}}{dt} + w_{x,i,j}\left(\frac{u_{i+1,j} - u_{i-1,j}}{2h_x}\right) + w_{y,i,j}\left(\frac{u_{i,j+1} - u_{i,j-1}}{2h_y}\right) + \frac{L(u_{i+1,j}) - 2\,L(u_{i,j}) + L(u_{i-1,j})}{h_x^2} + \frac{L(u_{i,j+1}) - 2\,L(u_{i,j}) + L(u_{i,j-1})}{h_y^2} = f(u_{i,j})v_{i,j} - g(u_{i,j}), \tag{21}$$

where $L'(u) = K(u)$ and $u_{i,j}(t) = u(\boldsymbol{x}_{i,j}, t)$. A discretized version of Equation (19) can be obtained in a similar fashion:

$$\frac{dv_{i,j}}{dt} = -\frac{1}{q} f(u_{i,j})v_{i,j}, \tag{22}$$

Equations (21) and (22) are supplemented with discretized versions of the boundary condition (19) and constitute a system of ordinary differential equations for the unknowns $u_{i,j},\ v_{i,j}$, with initial data obtained by considering (19) on the grid points. The resulting system can be time-discretized by using a Runge–Kutta method of the desired order; in our case, we chose the fourth order.

We employed the aforementioned numerical algorithm to solve our fire propagation model within a test rectangular domain, analyzing two distinct scenarios. The equations presented earlier in this section were implemented using MATLAB 2021a, which allowed us to obtain Figures 5 and 6.

In the first scenario, we assumed a flat topography combined with a constant wind direction and random fuel. This setup allows us to isolate and understand the effects of wind on fire spread without the added complexity of varying terrain. The simulation results for this case are illustrated in Figure 5. These figures provide visual insights into the fire's progression over time, highlighting how the constant wind influences the spread rate and pattern of the fire front.

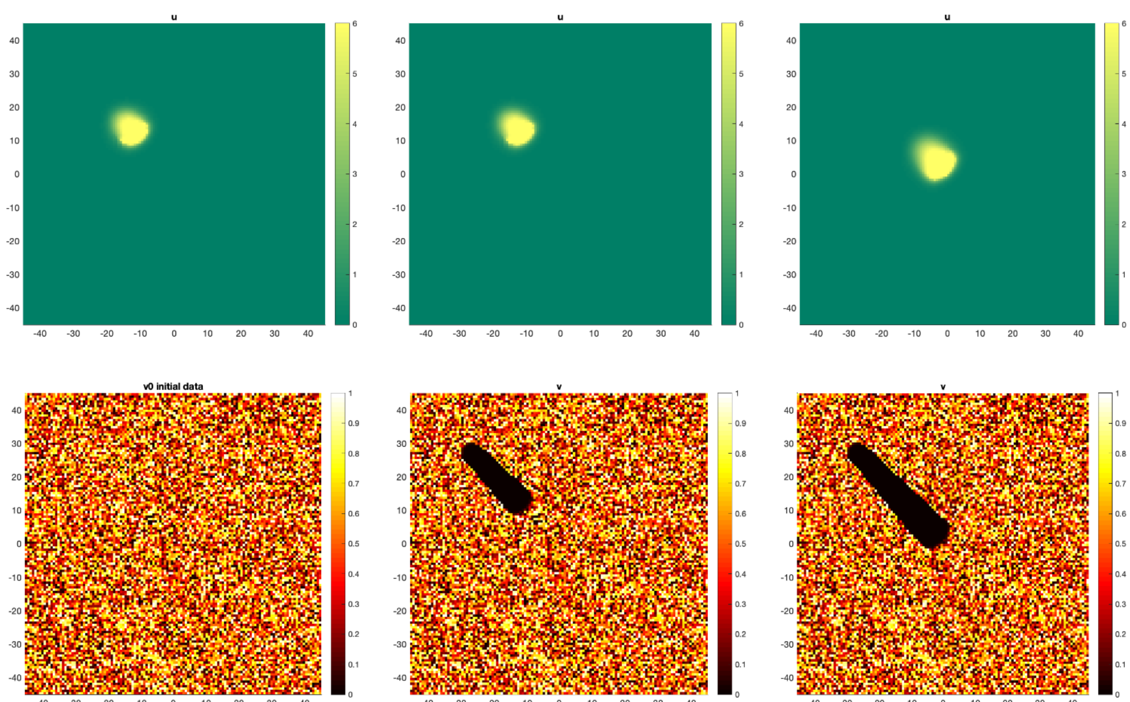

Figure 5. Numerical experiment for the first scenario, with an initial random distribution of fuel and a single initial fire (spot with higher heat). The first line shows the nondimensional heat distribution at nondimensional times t = 0, t = 6 and t = 10, and the second line shows the nondimensional fuel distribution and burden area at different time steps.

In the second scenario (Figure 6), we planned to introduce more complex conditions, such as varying terrain and constant wind directions, to examine how these factors interact and affect the fire propagation dynamics. By comparing the results from these different scenarios, we can better understand the critical variables that drive fire behavior in diverse environmental conditions. This comprehensive approach ensures that our model is robust and capable of simulating real-world fire spread accurately.

These simulations are crucial for validating the effectiveness of our model and demonstrating its applicability to different fire scenarios. The insights gained from these results can inform fire management strategies and improve predictive capabilities, ultimately aiding in better preparation and responses to wildfire events.

In the second scenario, we considered a test topography described by a regular function, as in San Martín et al. [63], shown in Figure 7 and obtained by the following equation:

$$h(x,y) = 4\left(\xi(x+0.5, y+0.5, 0.6) + \xi(x-0.9, y-0.9, 0.9)\right),$$

$$\xi(x,y,\gamma) = \exp\left(-\frac{1}{\gamma}(x^2 + y^2)\right), \tag{23}$$

In both cases, we chose the values reported in Table 3. It is important to note that we tackled the fully nonlinear system in our numerical simulations, rather than resorting to its linearized version, as done by San Martín et al. [63]. This approach allows us to capture the intricate dynamics and interactions inherent in fire propagation more accurately.

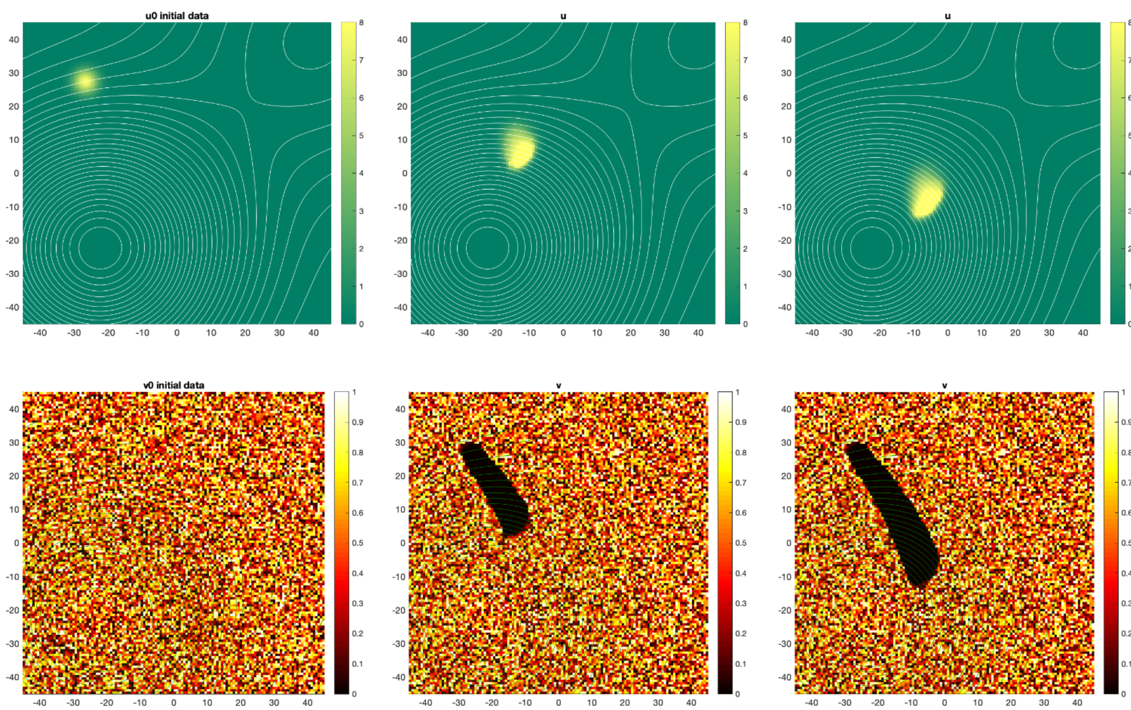

Figure 6. Numerical experiment for the second scenario, with an initial random distribution of fuel and a single initial fire (spot with higher heat). The first line shows the nondimensional heat distribution at nondimensional times t = 0, t = 6 and t = 10, and the second line shows the nondimensional fuel distribution and burden area at different time steps.

Table 3. Values used in the numerical simulations.

Ambient Temperature	T_∞	300	K
Activation Energy	E	83.68	kJ/mol
Heat of Combustion	Q	15,900	kcal/kg
Pre-exponential coefficient	$\bar{A}T_\infty^{1/2}$	10^9	s^{-1}
Density	ρ	100	kg/m^3
Specific heat	C	1	kJ kg^{-1} K^{-1}

By solving the fully nonlinear system, we account for the complexities and nonlinearities often present in real-world fire scenarios. This includes the nonlinear effects of heat transfer, combustion and the influence of environmental factors such as wind and terrain. In contrast, linearizing the system can sometimes oversimplify these interactions, potentially leading to less accurate or insightful results.

Our choice to work with the full nonlinear equations underscores the robustness and fidelity of our model. It enables a more comprehensive understanding of fire dynamics, providing results that are more reflective of actual fire behavior. This level of detail is crucial for developing effective fire management and mitigation strategies, as it ensures that our predictions and analyses are grounded in a realistic representation of the physical phenomena involved.

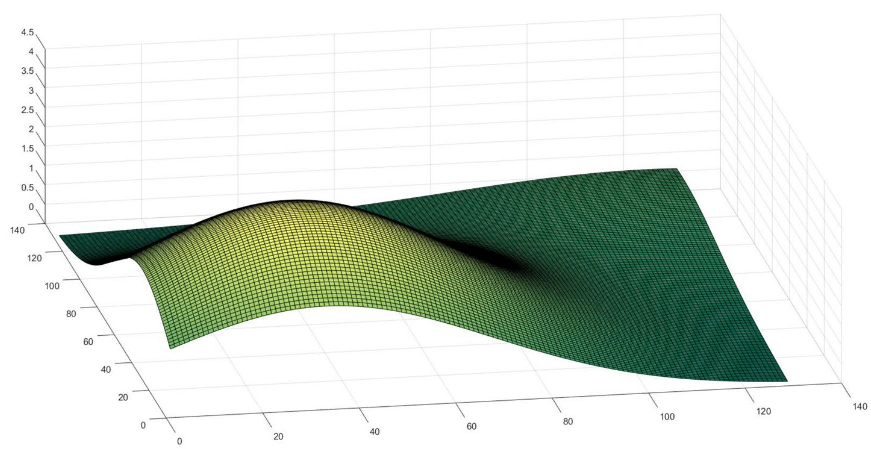

Figure 7. Test topography described by a regular function and used for the simulations in the second scenario.

4. Discussion

The reforestations carried out since the second half of the last century in many inner areas of the Apennines have demonstrated their crucial role in enhancing the protective function of forests in Mediterranean mountainous environments as well as promoting productive forest functions in the long term. However, decades after these reforestations, the absence of silvicultural interventions and thinning operations has made these stands highly vulnerable to natural and anthropic disturbances, such as forest fires. These forests are now structurally too dense, both vertically and horizontally, making them poorly resistant to strong winds, heavy snowfalls and tree crown fires. It is therefore essential to make these forest stands more resilient and resistant to maintain and promote their functions, including social and landscape roles.

To maximize the ecological potential of these forests, silvicultural interventions and thinning operations should be systematically implemented through careful forest planning and management, rather than being sporadic or occasional. This approach is crucial for effectively increasing the resistance of these stands to natural disturbances and forest fires.

The results of this study demonstrate how forest management, through appropriate silvicultural interventions, can enhance the resilience and resistance of these plantations against negative natural and anthropogenic disturbances, which are increasingly common due to global climate change.

The socio-economic changes that have occurred in the Mediterranean basin over the last 70 years (i.e., the abandonment of traditional forest practices and uses), along with climate change and other factors, have also led to a higher risk of large forest fires [64]. Many authors [65,66] have already demonstrated how these plantations, especially those established with coniferous trees, are characterized by low resilience to the new fire regimes that are now more common than in the past. The lower resilience and higher vulnerability of these unmanaged stands are exacerbated by the absence of management practices and the extreme events induced by climate change (e.g., heatwaves and droughts) [67].

In untreated forest plantations, as observed in this study, the number of trees per hectare is often too high in comparison with the stand's age, with a high percentage of trees primarily associated with lower diameter classes. Some of these trees become dead downed trees due to mortality induced by strong winds or heavy snowfalls. The large amounts of dead downed trees and logs, along with the high presence of standing dead trees suppressed due to the high tree density and competition, significantly increase the risk of fire ignition and spread [68]. These forest structural features, commonly observable in many Mediterranean contexts, can be considered representative of the tree plantations established in Italy in the last century and then left untreated for decades [5]. Therefore, to mitigate the risk of "larger fires", it is essential to balance efforts between short-term fire suppression and long-term forest planning and silvicultural prevention, both aimed at preventing forest fires and large-scale wildfires, particularly in the context of ongoing climate change [64,69].

The results obtained in this study show the variations in different parameters (*AGB*, *HE*, *CSI*, W_{H2O}) following the simulation of silvicultural thinning at 35–40 years and at 75–80 years. Our findings are noteworthy, as they indicate that reducing tree density through thinning not only decreases the amount of potentially flammable above-ground biomass but also alters the critical surface intensity (*CSI*). This change also increases the crown insertion height, especially in younger stands. Thus, silvicultural interventions carried out in the first decades after plantation can be an effective preventive silviculture technique, significantly reducing the occurrence and severity of forest fires. Conversely, thinning interventions conducted later are less effective in reducing the occurrence of forest fires. Additionally, the observed increase in *CSI* values (about 31% in younger stands) highlights the greater effectiveness of early thinning compared to later ones in reducing the likelihood of a transition from surface fire to crown fire [70]. This result is significant, as we can affirm that the increase in *CSI* is useful for limiting catastrophic fires [71] due to reduced fire spread, which also positively affects the stand's structural stability, making them more resistant to other natural disturbances [6].

The increased resistance to stand flammability following thinning is also favored by the increase in the soil moisture content [72], which is particularly important in Mediterranean environments for both reducing the fire risk and mitigating the effects of climate change [73,74]. By reducing the amount of fuel through silvicultural prevention measures, a greater isolation of the fuel is achieved, both vertically—reducing the risk of surface fire reaching the crowns—and horizontally—preventing fire from spreading over large areas. Moreover, thinning also promotes forest accessibility, thus facilitating fire extinguishing operations [75], with consequent cost reductions [76], less forest stand damage and quicker forest regeneration after fires [77,78]. Among the various thinning methods, thinning from below is considered the most suitable for fire prevention compared to selective thinning from above [22,29,79]. Additionally, when the fuel load is reduced and the spatial distribution of living and dead trees is well managed, the implementation of prescribed burning could be considered, integrating the various silvicultural prevention activities [22].

Nevertheless, after thinning, it is possible that wind speeds, which are influenced by stand density, in turn affect the fire intensity and may increase fire spread in more open stands, with consequent increases in damage severity [20]. However, Harrington [7] showed that in a dense Douglas-fir stand subjected to thinning, the fire intensity would not change significantly due to silvicultural treatments until the tree-top wind speeds exceed 20 mph (about 32 km/h). Therefore, this expected increase in fire intensity should not necessarily lead to an increased potential for crown fire. Instead, reductions in fuel quantities and increases in crown base heights result in a low probability of fire spreading, greatly reducing the chance of crown fire initiation. However, with the possible development of herbaceous and shrubby layers, the likelihood of crown fire spread could remain unchanged. In such cases, applying prescribed fire to further reduce the surface fuel may be very useful. In a *Pinus halepensis* reforestation, Iovino et al. [72] demonstrated the effectiveness of thinning combined with prescribed fire, highlighting that if carried out before thinning, prescribed fire acts mainly on fine fuels and deadwood, whereas if carried out after thinning, it can reduce the coarse woody debris resulting from cutting activities [72].

Finally, in a forest stand managed with fire prevention criteria, the amount of water that could be saved in case of a fire is significant, decreasing by about 20% in both examined situations (Table 1). Additionally, the reduction in heat energy allows for the decreased use of aircraft with financial benefits.

For silvicultural prevention purposes, it is also important to consider the climatic characteristics of the Mediterranean region relative to the fire regimes. In these landscapes, fire regimes tend to be increasingly "climate-driven" compared to the preindustrial past, when fire regimes were "fuel-limited" [80]. This shift appears to be even more pronounced in economically developed areas compared to less developed ones [81].

Given the current abandonment of many pine reforested stands and their low resistance and resilience to fire, an issue arises regarding how to manage these plantations. In recent years, strategies for the renaturalization of forest plantations [82,83] have revitalized the ecological role and value of these stands [84,85]. We believe that renaturalization processes should focus on establishing and promoting native broadleaf species to ultimately create mixed forest stands typical of these Mediterranean inner environments. This management approach could also lead to stands that, compared to pure pine forests, are characterized by lower flammability. Therefore, a forest management approach oriented towards the renaturalization of these reforestations can be a valid tool for mitigating the effects of climate change on forests [81]. In the short term, this will lead to a gradual increase in biodiversity, while in the long term, it will promote the occurrence of mixed stands where broadleaf trees of various species will also be present, depending on soil conditions and local geo-climatic factors. This approach will enhance forest resilience to different types of disturbances, particularly wildfires, and improve soil conservation, making the overall forest landscape more complex [1]. These management approaches will also contribute to enhancing the provision of several other ecosystem services.

Additionally, as demonstrated by Alì et al. [60] and Reisch et al. [86], implementing reaction–diffusion mathematical models can be useful for predictive purposes, assisting stakeholders in wildfire prevention and suppression. These models, which incorporate data obtained from the previously conducted and presented studies, provide realistic potential scenarios, thus helping reduce response times. The models utilize a range of inputs, including real-time meteorological data, fuel load information and topographical features, to accurately simulate fire spread dynamics. By integrating these diverse data sources, the models can predict the rate and direction of fire propagation, allowing for more informed decision-making in emergency situations.

Mathematically, the proposed innovations offer several advantages over other models in the literature. They enable better numerical implementation and convergence, ensuring that the models are both reliable and efficient. The improved algorithms adhere to specific physical laws, identifying preferential pathways of propagation influenced by factors such as the topographic slope, the wind direction and the calorific value of the fuel present in the area, as shown in Figures 5 and 6. Moreover, these advancements facilitate the creation of more detailed and accurate fire behavior predictions. The incorporation of physical parameters such as the moisture content, fuel type and atmospheric conditions enables the models to adapt to various environmental scenarios. This adaptability is crucial for generating precise forecasts under different conditions, thereby enhancing the overall effectiveness of wildfire management strategies.

In summary, the development and application of advanced reaction–diffusion models represent a significant step forward in wildfire management. By leveraging comprehensive data integration and improved numerical techniques, these models provide essential tools for predicting fire behavior, optimizing resource allocation and ultimately reducing the impact of wildfires on communities and ecosystems. In fact, the reaction–diffusion PDE model demonstrates exceptional capabilities in the real-time monitoring of fire simulations, providing a detailed and dynamic representation of fire evolution. This advanced simulation ability is notably superior to existing techniques, such as cellular automata, due to its enhanced precision and realism. Unlike cellular automata, which may offer more abstract and less detailed representations of propagation phenomena, the PDE-based approach allows for a more sophisticated and accurate modeling of fire behavior. This superiority is particularly evident in the model's ability to swiftly adapt to changes in context and environmental conditions, thus providing a more reliable and timely monitoring and forecasting tool. The study confirms that, for high-fidelity fire simulations and real-time monitoring, the reaction–diffusion model represents a significant advancement over traditional methodologies.

The overall strength of the article lies in its substantial contribution to the field of forest fire management in the Mediterranean region, particularly through its innovative approach to modeling and data collection methods. To enhance this contribution, it is important to elaborate on the potential practical applications of the proposed mathematical model and how it can advance the field. The mathematical reaction–diffusion model presented in the article, when combined with detailed silvicultural data, such as the number of trees, diameter at breast height (*DBH*), total height (*Ht*) and height of crown insertion (H_{bc}), provides a robust framework for simulating and predicting fire behavior. This model's accuracy is significantly enhanced by incorporating these parameters, which offer a comprehensive understanding of the forest structure and fuel characteristics. Moreover, integrating this model with real-time data collection methods, such as those obtained from drones, can further advance its practical utility. Drones equipped with sensors can continuously monitor and collect updated information on forest conditions, including the tree density, crown height and fuel load. These real-time data can be fed into the reaction–diffusion model to adjust predictions dynamically and reflect current forest conditions accurately.

The establishment of a control center that integrates the mathematical model with drone data can greatly enhance fire management practices. Such a center would facilitate real-time monitoring and analysis, allowing for timely and informed decision-making regarding fire risk and management strategies. It could also enable rapid adjustments to fire suppression efforts based on up-to-date information, improving the overall effectiveness of fire management operations. In summary, the integration of the mathematical model with detailed silvicultural data and real-time drone monitoring represents a significant advancement in forest fire management. It offers a sophisticated tool for predicting and managing fire behavior, ultimately leading to more effective and responsive strategies for protecting and managing forest ecosystems in the Mediterranean region. For future developments related to the mathematical model, the researchers aim to improve it by coupling the model presented in this paper with approaches based on genetic algorithms [87] and a finite element modeling approach [88,89].

5. Conclusions

In the Mediterranean environment, pine reforestations are among the forest stands most affected by fires. Due to the absence of silvicultural treatments, these stands are characterized by an expansion of fuel loads, which facilitates the spread of fires. This study underscores the importance of silvicultural interventions, such as bottom-up thinning, in mitigating fire risk. This intervention removes trees in the smaller-diameter classes, which, in the event of a forest fire (with medium and high severity), burn first and contribute to the propagation of the fire. Additionally, silvicultural interventions disrupt the vertical and horizontal continuity of the forest fuel, resulting in positive effects on the overall efficiency of forest stands and greater resilience after forest fires.

The study highlighted the reduction in heat energy during total combustion and the decreased water consumption needed for extinguishing fires. It also emphasized the significant reduction in the transition from surface fires to canopy fires, as evidenced by the increase in critical surface intensity (CSI). On the other hand, the decrease in heat energy allows for a reduction in the use of aircraft, leading to financial benefits. However, the increase in the CSI and the significant increase in the crown base height primarily occur if thinning interventions are carried out within the first few decades after planting. Forest fires that exceed the critical value surpass the extinguishing capacity of all available resources. This indicates that fire suppression is most successful when prevention measures, implemented through effective forest management, have been applied. Consequently, it is necessary to integrate fire suppression models with appropriate prevention initiatives.

Additionally, the proposed reaction–diffusion mathematical model, although presented in a preliminary phase in this study, holds significant potential as a valuable tool for integrating data obtained from forest population analysis with meteorological station data and topographic information. The seamless integration of these diverse data sources can provide robust support to stakeholders involved in wildfire prevention and land protection efforts. As demonstrated, these models offer realistic fire spread scenarios by incorporating diverse data sources, aiding in wildfire prevention and responses. The model's innovations offer improved numerical implementation and convergence, adhering to physical laws that account for topography, wind directions and fuel properties. This results in detailed and accurate fire behavior predictions, enhancing wildfire management and resource allocation.

Finally, in accordance with the European Commission report on land-based wildfire prevention [90], we believe that mitigating wildfire risks also requires that "people need to be informed and educated about wildfires so that through their actions they do not increase fire risks but, on the contrary, actively support the mitigation of wildfires. This requires investments in education, human resources, planning tools and equipment by the competent authorities at all levels (local, regional, national and European)".

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