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A Parallel Compensation Technique to Improve the Convergence of Iterative Harmonic Analysis

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***Abstract* —This paper was presented at the conference "ICHPS VI" (Bologna - Italy, 1994), but it is not available to the scientific community because the conference proceedings were never published. The recent interest in Iterative Harmonic Analysis (IHA) approaches has suggested proposing again the content of the missing paper, given its ability to overcome the convergence difficulties, which historically have limited the utilization of IHA.**

The paper focuses on the convergence difficulties of the iterative approaches based on sequential substitutions Gauss-Seidel type for updating converter injected currents and supply harmonic voltages for harmonic analyses in power systems. Explicit reference is made to the Iterative Harmonic Analysis (IHA) in its classical formulation, even if the results obtained apply to a wider set of cases. Two proposed series compensation techniques to improve IHA convergence are described and their applicability limits are analyzed. To overcome the limits, a parallel compensation technique is presented. A critical case-study, characterized by the presence of AC system shunt resonances which result in the divergence of IHA, is solved and the results obtained by different compensation impedance values are analyzed and discussed. The paper shows that the parallel compensation technique succeeds in overcoming both the convergence problems of IHA and the intrinsic applicability limits of series compensation techniques, also in presence of AC systems with critical shunt resonances.

***Index Terms* — Iterative Harmonic Analysis, Frequency Domain Models, Convergence Problems, Power Electronic Converters.**

PREMISE

To predict the harmonic distortion in practical power systems, different methods using frequency, time and hybrid time-frequency domain have been proposed. The conceptual and analytical details of these methods are concisely detailed in [1]. Nowadays, the scenario of harmonic distortion is becoming more and more complex due to the wide diffusion of nonlinear components and the fact that these components are often able to also generate interharmonics and high frequency components. Therefore, it is necessary to take advantage of all the available analytical tools considering for each merits and drawbacks to perform predictive analysis of distortion penetration.

The methods based on separating the whole system into sub-systems to be solved in different stages are useful for their ability to manage complex situations, sometimes representing each sub-system in its natural frame of reference and with customized levels of details and models. Such methods are very attractive but often require the utilization of an iterative process of the sequential substitution Gauss-Seidel type to obtain the final steady-state solution of the whole system. In [1], these methods are classified as Iterative Harmonic Analysis (IHA) and Hybrid Methods (HM), evidencing that, unfortunately, their merits are negated by possible slow convergence characteristics and narrow stability margins around the solution point that limit their applicability.

The interest for Iterative approaches is confirmed by recent literature [2-6]. In particular, in [2] an Iterative approach has been successfully applied to solve a complex system constituted by a HV/MV test system, combining the HV test system previously proposed by CIGRE/CIRED JWG C4/B4.38 with the MV IEEE benchmark test system previously proposed by the IEEE PES TF on "Harmonic Modeling, Simulation and Assessment". The analysis was conducted by means of an IHA, in order to manage the large number of MV-HV components. The convergence toward the solution was obtained utilizing a very efficient compensation technique of the physical system that was proposed by the authors of the present paper at the conference "ICHPS VI" (Bologna- Italy, 1994) in a paper not available to the scientific community because the conference proceedings were never published. Therefore, it was decided to repropose the paper, which is missing despite its intrinsic value for the practical applicability of IHA. In what follows, the paper is reproduced in its original structure and content, with minor improvements and modifications. Explicit reference is made to the Iterative Harmonic Analysis as it was originally proposed in [7, 8] and, then, subjected to further research and improvements in [9-11].

I. INTRODUCTION

Among frequency domain methods, the Iterative Harmonic Analysis (IHA) method [1, 9, 11] has particular relevance. IHA is able to take into account both the interactions among converters and between converters and the power supply system, also in presence of voltage or impedance imbalances. At the moment, it is one of the most attractive methods because it is comprehensive, and it also has a high computational efficiency; it is based on an iterative process of sequential substitutions Gauss-Seidel type for updating converter injected currents and supply harmonic voltages.

[1] Deceased on June, 16, 2019.

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Unfortunately, in some cases, particularly in the presence of low short circuit ratios (SCR) and supply system resonances, the IHA method fails to converge, thus affecting its general applicability [1, 11]. It has been shown that the lack of convergence is not necessarily related to genuine physical phenomena causing harmonic instabilities but is often a consequence of the main characteristic of IHA: the use at each iteration of two different stages for the simulation of converters and the supply system [9, 11, 12].

The convergence properties of the IHA method have been the subject of considerable debate. A significant improvement of the IHA has been obtained in [9] by adjusting simulation parameters during the iterative process. In practice, the use of shifting the converter terminal busbar, inserting a pair of equal and opposite reactances, without affecting the physical system, overcomes the convergence difficulties resulting from the low SCR of the system. In the subsequent text, this technique will be called the "Series Reactance Pair Technique" (S-RPT).

When the lack of convergence depends on the supply system impedance resonances, divergence problems may still remain, even when utilizing the S-RPT. To overcome such problems, an improvement of the S-RPT based upon the use of a pair of equal and opposite frequency dependent impedances, instead of the reactance pair, has been proposed in [10]. The numerical experiments performed have shown good results for the possibility of improving the IHA convergence also in the presence of an AC system with high shunt resonances. In the following text, this technique will be called the "Series Impedance Pair Technique" (S-IPT).

Two different possibilities of modifying the IHA to allow for the introduction of the aforementioned series impedance pair are indicated in [10]. The first possibility is that of substituting the traditional converter mathematical model of IHA with a time domain simulation. The second possibility is that of using the advanced mathematical models of line-commutated converters developed in [13], which assure high result accuracy along with the possibility of inserting impedances in series with the converter-transformer group. Whichever converter model is utilized, the implications of the S-IPT are manifested as: (i) an increase in computational effort, (ii) the need of evaluating appropriate frequency dependent impedance equivalent circuits for each specific application and (iii) problems in modelling different converters supplied by the same busbar.

Both the S-RPT and S-IPT may be classified as Series Compensation Techniques (SCT). Their main drawback is a consequence of the way in which they act on the converter side, i.e., by modifying the whole converter model, in particular its commutation process.

In the paper, to overcome the SCT disadvantages and applicability limits, the authors propose a "Parallel Compensation Technique " (PCT), on the analogy of the PCT used in [14] for convergence improvement of relaxation methods applied to stiff non-linear communication circuits. The PCT presented here is based upon the addition of a pair of equal and opposite frequency dependent parallel impedances at the converter terminal busbars. The frequency dependent impedance has to be chosen to decrease the converter side sensitivity to the supply voltage harmonic variations at resonance frequencies, without introducing undesired effects at other frequencies. It is important to explicitly state that, unlike the SCT, the PCT does not require any modification of the converter model (in particular that of the traditional IHA) nor of the supply system model. This property makes the proposed compensation technique suitable for all iterative methods based on sequential substitutions Gauss-Seidel type, regardless of the particular method formulation. Numerical experiments of a particularly critical case-study demonstrate the effectiveness of the PCT in solving the convergence problems of IHA. It is important to underline the absence of a particular sensitivity to deviations of the selected parallel impedances from their optimal values.

In the following, firstly some theoretical remarks on the IHA algorithm and its convergence properties are reported. Then, the SCTs and their applicability limits are critically analyzed. Following this, the new PCT is presented and discussed. Finally, the numerical experiments are reported.

II. Remarks on Iterative Harmonic Analysis

A. *Algorithm*

The IHA algorithm can be summarized by the following fundamental steps:

(1) input information is obtained on system component characteristics and AC system impedances, then, initial conditions are evaluated by means of an AC/DC three-phase power-flow for each node supplying a converter;

(2) starting from the voltage waveforms at the converter terminals, the steady-state current waveform i(t) is estimated for each converter;

(3) the steady-state time domain current waveforms previously obtained for each converter are subject to a FFT in order to obtain the vector $\mathbf{I}^h$ of their harmonic components;

(4) the current harmonics $\mathbf{I}^h$ obtained in (3) are utilized to perform steady-state harmonic analyses of the AC system by means of the harmonic impedances of the AC system to obtain the corresponding updated vector $\mathbf{V}^h$ of voltage harmonics at the converter busbars.

Steps (2) to (4) are repeated until convergence is achieved, i.e. until the voltage harmonics at each converter supplying busbar do not change significantly compared to those obtained in the previous iteration with respect to a defined convergence threshold.

In a version proposed in [12] the IHA algorithm has been modified to take into account the fundamental power variations produced by the converters by including the three-phase load flow stage during the iteration process.

In detail, this means that, for each converter, step (2) is extended by:

a) assuming a set of busbar voltages, either balanced or unbalanced, as initial conditions;

b) determining the valve firing instants using any type of firing angle model;

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c) calculating the DC side harmonic voltages using the firing instants and the AC busbar voltages;
d) obtaining the DC currents knowing the DC side harmonic impedance;
e) calculating the AC side harmonic currents from the DC side currents, AC busbar voltages, AC impedances, firing instants, etc..

Steps a) to e) are described in [7, 8, 12].

B. Convergence properties

The sequential nature of the IHA numerical solution is not particularly reliable. In critical conditions, the solution given by an IHA method may not be acceptable. Specifically, it has been demonstrated that the solution might be:
a) divergent;
b) physically meaningless for the case under study;
c) not computationally efficient.

Mathematically speaking, the following consideration can be made: at the k-th iteration, steps (2) and (3) can be summarized by:

$$\mathbf{I}^h(k+1)= f(\Delta\mathbf{V}^h(k)) \tag{1}$$

and step (4) by

$$\Delta\mathbf{V}^h\,(k+1) = [\mathbf{Y}^h]^{-1}\ \mathbf{I}^h(k+1) \tag{2}$$

where ($\Delta\mathbf{V}^h$) is the vector of harmonic voltage drops, $f(\Delta\mathbf{V}^h)$ represents the device behavior in the frequency domain, and $[\mathbf{Y}^h]$ is the supply system harmonic admittance matrix. Eliminating the currents from equations (1) and (2) gives:

$$\Delta\mathbf{V}^h(k+1) = [\mathbf{Y}^h]^{-1}\ f(\Delta\mathbf{V}^h(k))\ . \tag{3}$$

Starting from a Lipschitz sufficient condition, Callaghan and Arrillaga [9] have demonstrated a sufficient condition for convergence onto the solution of the sequence of equation (3). Convergence is assured when, for some $\varepsilon<1$ in a neighborhood centered on the solution:

$$\mathrm{big}([\mathbf{J}^h](\Delta\mathbf{V}^h(k+1)-\Delta\mathbf{V}^h(K))) < \varepsilon\cdot \mathrm{big}(\Delta\mathbf{V}^h(k+1)-\Delta\mathbf{V}^h(k)) \tag{4}$$

where

$$[\mathbf{J}^h] = [\mathbf{Y}^h]^{-1}\ [\delta f/\ \delta(\Delta\mathbf{V}^h)] \tag{5}$$

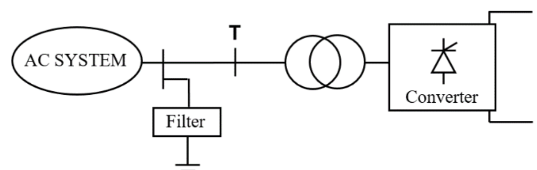
and the function big(x) represents the magnitude of the numerically largest element of its vector argument x, with $\Delta\mathbf{V}^h(k+1)$ and $\Delta\mathbf{V}^h(k)$ both lying within the neighborhood centered on the solution.

The convergence constraint (4) provides only qualitative information in practical cases, due to the complexity of equation (1) (because of the presence of function f expressing the converter behavior) which implicitly appears in (4) through (2), (3) and (5). In particular, when the system is unbalanced, to analytically define equation (1) is extremely complicated, if not impossible. However, the convergence behavior essentially refers to the magnitude of the elements of $[\mathbf{J}^h]$. Thus, taking into account (5), one can conclude that the divergence of IHA is likely when either (a) the system impedance is "large" or (b) the device behavior is particularly sensitive to changes in harmonic voltage levels (i.e. elements of $\delta f/\delta\Delta\mathbf{V}^h$ are "large").

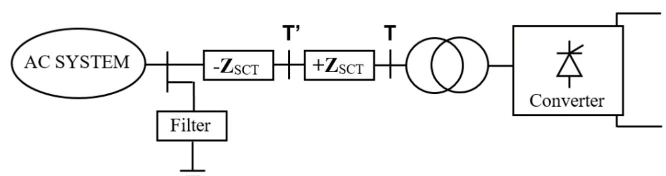
III. Series Compensation Techniques

The fundamental consideration that SCTs are based on is that the convergence characteristic of IHA may be improved by either (i) increasing the effective converter commutating impedance or (ii) decreasing the effective supply network impedance, as can easily be observed, starting from (4).

The SCTs can be illustrated with reference to the test system depicted in Fig. 1a. The physical system can be modified by inserting a pair of equal and opposite impedances, Z_{SCT} and $-Z_{SCT}$, between the filters and the transformer as shown in Fig. 1b. The series combination of these inserted elements amounts to a zero value impedance and ensures that the representation of the physical system under study is unaffected by the insertion. The pair of impedances inserted allows a new shifted observation point (T' in Fig. 1b) to be considered in IHA.



a)



b)

Fig. 1. Series compensation technique: a) physical system; b) SCT modified system.

The IHA can be easily applied to the modified system by first obtaining a solution for the vector of voltage harmonics at the fictitious busbar T', $\mathbf{V}^h(T')$, and then evaluating the vector of voltage harmonics at the actual busbar T, $\mathbf{V}^h(T)$.

A. The Reactance Pair Technique

In [9] a reactance pair, X_{SCT} and - X_{SCT}, is used. The positive reactance is absorbed into the commutating impedance of the converter and the negative reactance into the system impedance. This has the effect of increasing the commutating reactance, while the exact effect on the effective system impedance depends largely on the original system impedance locus. That means that, with respect to the possible improvements of the SCT previously discussed, effect (i) is always attained, while effect (ii) may or may not be attained.

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This series reactance pair technique, S-RPT, can improve the applicability of IHA by obtaining a solution for some otherwise divergent cases (i.e. for cases with unfavourable SCR). This is due to the ability of S-RPT to modify an unfavorable SCR.

It has been demonstrated in [9] that the S-RPT does not affect solution accuracy, provided that the convergence occurs. This characteristic allows the S-RPT to cope with convergence problems also in unbalanced systems.

Unfortunately, in most cases (i.e. in the presence of passive filters), the origins of divergence are often connected to the presence of AC system parallel resonances, that is to say, as it is well known, very high impedance values in correspondence to specified frequencies, generally some order higher than that of the fundamental. In such cases, the S-RPT may be inadequate to obtain convergence without affecting the model of the physical system [10].

The problem is that the reactance $-X_{SCT}$ to be added to the AC system has effects at all frequencies and these effects are proportional to the frequency. This causes problems at frequencies lower or higher than resonance frequencies, particularly in the frequency ranges in which the system impedance is capacitive. In such conditions, the addition of $-X_{SCT}$ will clearly increase the modulus of the system impedance, and, therefore, contribute to solution divergence and not to convergence.

B. The Impedance Pair Technique

In [10] the divergence problem caused by resonances has been overcome by adjusting the simulation parameters in a more complex way, therefore allowing IHA to converge in a wider range of practical situations. This is realized by utilizing an impedance pair, $\mathbf{Z}_{SCT}$ and - $\mathbf{Z}_{SCT}$, instead of a reactance pair. The impedance - $\mathbf{Z}_{SCT}$ has to be chosen to decrease the effective supply network impedance, in particular at the resonance frequencies, without introducing undesired effects at the other frequencies. This means the $\mathbf{Z}_{SCT}$ has to present impedance-frequency characteristics similar to that of the AC system, in particular, the same main shunt-resonances.

The $\mathbf{Z}_{SCT}$ to be adopted has to assure only the convergence and not the accuracy of the final solution, which remains related to the nature of the IHA approach. Therefore, the series impedance pair technique, S-IPT, is able to cope with convergence problems also in unbalanced systems.

The frequency dependent equivalent circuit reproducing $\mathbf{Z}_{SCT}$ needs two or more parallel branches according to resonances and at least one of the branches must include capacitance. Such an equivalent circuit has to be connected in series with the simple RL commutation impedance utilized for the IHA converter model, as reported in Fig. 2a, where $v_a(t)$ and $v_b(t)$ stand for the supplying voltages of phase *a* and *b* involved in the commutation. The undesired effect of this modification is that of destroying the simplicity of the commutation models that can be used, as, for instance, that proposed in [8], and, consequently, their whole converter model cannot be used (see Fig. 2b). Therefore, a modification of the IHA is needed.

Two different possibilities of modifying the IHA to allow for the introduction of the aforementioned impedance pair are indicated in [10].

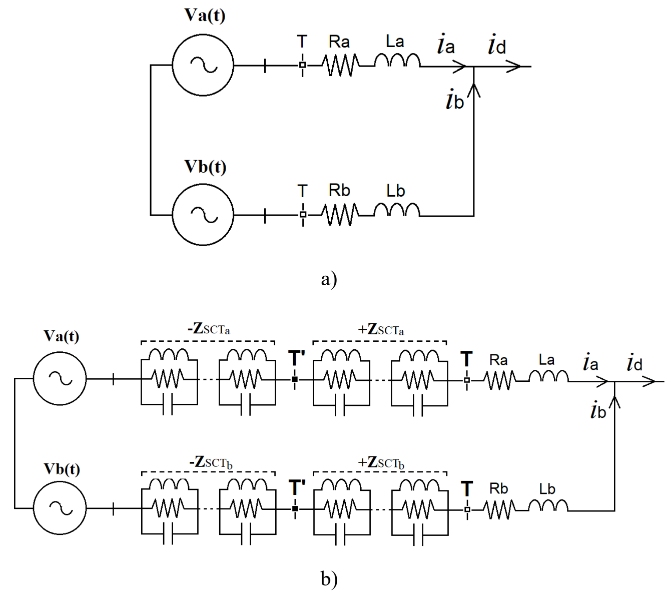

Fig. 2. Converter equivalent circuit during commutation: a) typical IHA model; b) S-IPT model.

The first possibility is substituting the model used in step (2) of the IHA (Section II.A) with a time domain simulation of the converter. This possibility relies on electromagnetic transient simulation programs for its implementation and this may require long computation times.

The second possibility can be accomplished by the following steps:

- a maximum number of shunt resonances to be reproduced is chosen;
- an equivalent circuit general structure able to represent the fixed number of parallel resonances is obtained;
- general analytical expressions are derived in order to describe the converter current waveforms in presence of a commutation circuit which includes the general equivalent previously obtained (see Fig. 2b).

The first possibility was adopted in [10] to perform numerical experiments to show the applicability and usefulness of the IPT. The second is possible as new mathematical models of line-commutated converters have been developed [13]; such models assure high result accuracy along with the possibility of inserting impedances in series with the converter-transformer group.

C. Applicability limits

The S-RPT is not generally applicable as it can fail in cases when the divergence problems are due to the AC system shunt resonances. Therefore, the SCTs are reduced to the S-IPT when the divergence problems are due to the AC system high shunt resonances. In practical applications, the S-IPT is characterized by the following applicability limits (and disadvantages):

a) the frequency dependent equivalent circuits to be adopted need to be developed for each specific application, and this may not be a trivial task;

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b) the converter model solution, both by transient simulation programs or by the mathematical model [13], may require high computational effort;

c) difficulties may arise in detecting the zero-crossing of the line-to-line voltages at the transformer busbar T (see Fig. 2b), especially when filtered voltages in the physical system are utilized for this purpose;

d) in the case of n different converters supplied by the same busbar, it is necessary to introduce n fictitious busbars, T'_1, T'_2, … T'_n, as shows in Fig. 3, therefore changing the structure and increasing the dimensions of the AC system admittance matrix [**Y**];

e) the virtual impossibility of automating the technique.

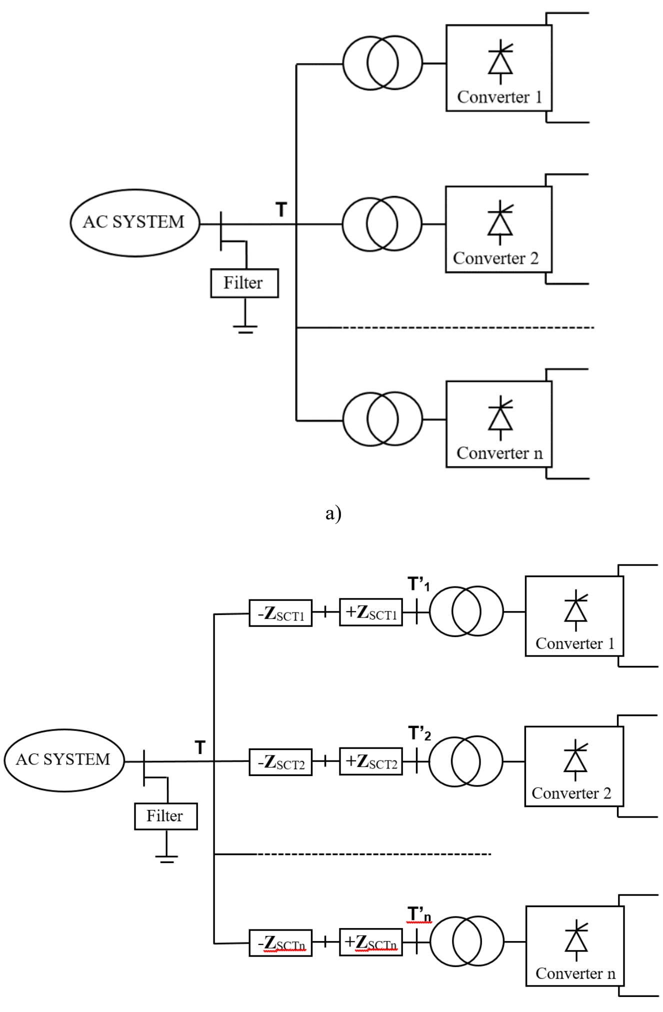

Fig. 3. S-IPT application to a busbar supplying a multiconverter system circuit during commutation: (a) original system (b) S-IPT modified system.

IV. The Parallel Compensation Technique

To overcome all limitations of the SCTs and to provide a generally applicable solution for IHA, a new proposal named Parallel Compensation Technique (PCT) is presented here. The fundamental consideration that the PCT is based on is that the convergence characteristic of IHA may be improved by decreasing the magnitude of the elements of $[\mathbf{J}^h]$ defined by (5), that is to say the product between the non-linear sensitivity to change in harmonic voltage levels (i.e. elements of $[\partial f/\partial \Delta \mathbf{V}^h]$) and the AC system impedance $[\mathbf{Y}^h]^{-1}$.

The PCT can be illustrated with reference to the test system depicted in Fig. 4a. The physical system can be modified by adding a pair of equal and opposite parallel impedances, Z_{PCT} and $-Z_{PCT}$, as shown in Fig. 4b. The parallel combination of these impedances amounts to an infinite value impedance and ensures that the physical system under study is unaffected.

The most important aspect is that the pair of impedances added to the system does not modify the observation point T to be considered in IHA, contrary to the SCTs. Furthermore, during the iterative process, the following additional and important advantages are obtained:

- the positive impedance is absorbed into the AC system and is utilized to mitigate the present impedance resonances;
- the negative impedance introduces a new modified non-linear load along with the converter, but the converter solution remains unaffected by $-Z_{PCT}$ because this is in parallel.

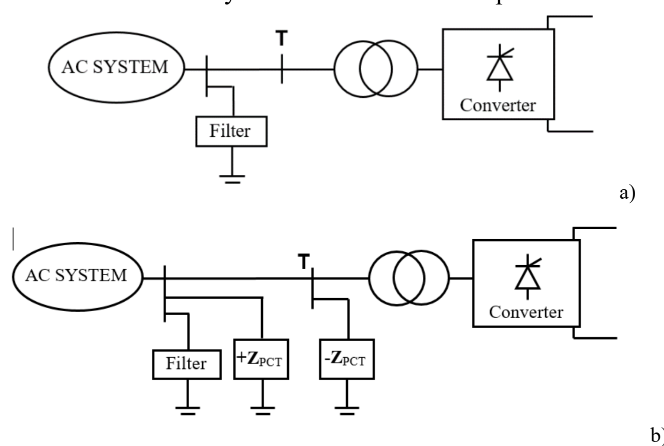

Fig. 4. Parallel compensation technique: a) physical system; b) PCT modified system

A. Convergence improvement

As a consequence of the PCT application, (1) is modified as:

$$\mathbf{I}^{h*}(k+1) = f^*(\Delta \mathbf{V}^h(k)) , \tag{6a}$$

where:

$$f^*(\Delta \mathbf{V}^h(k)) = f(\Delta \mathbf{V}^h(k)) + \Delta \mathbf{V}^h(k) \cdot 1/\mathbf{Z}^h_{PCT} , \tag{6b}$$

and (2) as:

$$\Delta \mathbf{V}^h(k+1) = [\mathbf{Y}^{h*}]^{-1} \cdot \mathbf{I}^{h*}(k+1) , \tag{7a}$$

where :

$$[\mathbf{Y}^{h*}] = ([\mathbf{Y}^h] + [\mathbf{Y}^h_{PCT}]) , \tag{7b}$$

with $[\mathbf{Y}^h_{PCT}]$ the diagonal matrix whose elements are obtained as $1/\mathbf{Z}^h_{PCT}$ for each node of the analyzed system, assuming $1/\mathbf{Z}^h_{PCT} = 0$ for the nodes which are not subject to compensation at the h-th harmonic order.

The convergence constraint (4) can be rewritten as:

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$$\text{big}([\mathbf{J}^{h*}](\Delta\mathbf{V}^h(k+1)-\Delta\mathbf{V}^h(k))) < \varepsilon\cdot\text{big}(\Delta\mathbf{V}^h(k+1)-\Delta\mathbf{V}^h(k)) \,, \quad (8)$$

where:

$$[\mathbf{J}^{h*}] = [\mathbf{Y}^{h*}]^{-1}\,[\delta f^*/\delta(\Delta\mathbf{V}^h)] \,. \quad (9)$$

An appropriate choice of $\mathbf{Z}_{PCT}$ has to assure that:

$$(|\mathbf{J}^{h*}| - |\mathbf{J}^h|) < 0 \qquad \forall h \quad (10)$$

in order to improve the convergence.
The optimal choice of $\mathbf{Z}_{PCT}$ value can be obtained solving the following optimization problem:

$$\min_{\mathbf{Z}_{PCT}} (|\mathbf{J}^{h*}| - |\mathbf{J}^h|) \qquad \forall h \quad (11)$$

therefore, minimizing the number of iterations needed for convergence.

The following conditions can arise:

a) $[\mathbf{Y}^{h*}]^{-1} < [\mathbf{Y}^h]^{-1}$ and $\delta f^*/\delta(\Delta\mathbf{V}^h) < \delta f/\delta(\Delta\mathbf{V}^h)$,

b) $[\mathbf{Y}^{h*}]^{-1} \geq [\mathbf{Y}^h]^{-1}$ and $\delta f^*/\delta(\Delta\mathbf{V}^h) < \delta f/\delta(\Delta\mathbf{V}^h)$,

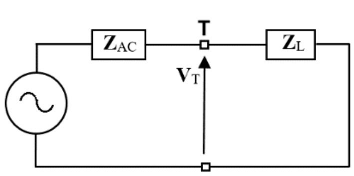
c) $[\mathbf{Y}^{h*}]^{-1} < [\mathbf{Y}^h]^{-1}$ and $\delta f^*/\delta(\Delta\mathbf{V}^h) \geq \delta f/\delta(\Delta\mathbf{V}^h)$.

Condition a) is sufficient to assure the convergence, but (10) may be verified also for conditions b) or c) so obtaining convergence. This makes the solution flat around the optimum, and this will be shown in the next section by means of a simple numerical example.

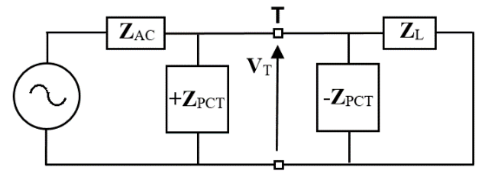
Aspects of particular importance and effectiveness consequent to the fact that $\mathbf{Z}_{PCT}$ plays only a numerical role are:
- $\mathbf{Z}_{PCT}$ is evaluated independently for each harmonic order h;
- the physical feasibility of $\mathbf{Z}_{PCT}$ and of - $\mathbf{Z}_{PCT}$ is not necessary.

B. Evaluation of the compensation impedance

In Fig. 5a, a simple mono-frequency linear system is depicted in order to discuss the convergence properties. It is possible to solve the system by the application of a very simple iterative algorithm similar to the IHA process described in section II.A. The iteration process of the physical system in Fig. 5a will diverge when the system impedance $\mathbf{Z}_{AC}$ is greater than or equal to the load Impedance $\mathbf{Z}_L$, i.e. $\mathbf{Z}_{AC} \geq \mathbf{Z}_L$.



a)



b)

Fig. 5. Linear system for convergence property tests: a) physical system; b) PCT modified system

In Fig. 6 the convergence behavior obtained for the modified system of Fig. 5b is illustrated for different choices of the parallel compensating impedance $\mathbf{Z}_{PCT}$, introducing as a parameter the ratio $\mathbf{Z}_{AC}/\mathbf{Z}_L$ to characterize the "divergence entity" of the physical system. In Fig. 6, the number of iterations needed to achieve convergence - assuming a convergence criterion of a difference less than 0.1% between two consecutive values of $\mathbf{V}^h{}_T$ - are reported versus the values of the ratio $\mathbf{Z}_{PCT}/\mathbf{Z}_L$.

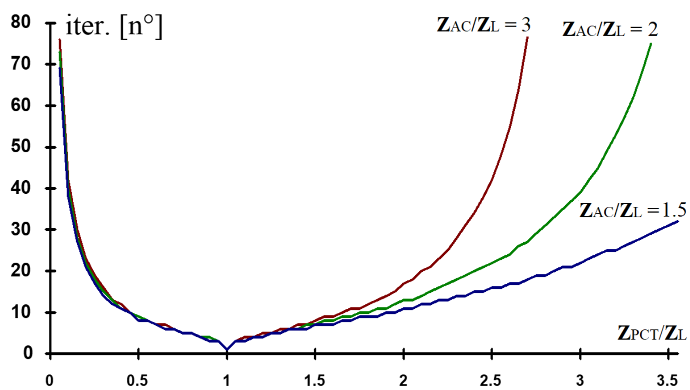


Fig. 6: Number of iterations needed for convergence, for the linear system in Fig. 5b versus the values of the ratio $\mathbf{Z}_{PCT}/\mathbf{Z}_L$ for different $\mathbf{Z}_{AC}/\mathbf{Z}_L$ values.

The following observations can be made:

a) the optimal behavior is obtained for $\mathbf{Z}_{PCT}/\mathbf{Z}_L = 1$, regardless of the value of the "divergence ratio" $\mathbf{Z}_{AC}/\mathbf{Z}_L$;

b) for values of $\mathbf{Z}_{PCT}/\mathbf{Z}_L$ from about 0.5 to about 2.0, the number of iterations is never significantly larger than 10, with little influence of the different "divergence ratios" considered;

c) as $\mathbf{Z}_{PCT}/\mathbf{Z}_L$ tends towards zero, the number of iterations increases hyperbolically for every "divergence ratio" of the physical system;

d) for $\mathbf{Z}_{PCT}/\mathbf{Z}_L$ values greater than two – that is to say when the modified system is approaching the original physical system – the number of iterations quickly increases, and the rate of this increase is proportional to the "divergence ratio" of the physical system.

The problem is to extend the results obtained for the simple linear circuit to the case of loads constituted by a transformer-converter group. To this end, it is necessary to obtain an estimation of the elements of $[\partial f/\partial\Delta\mathbf{V}^h]$ of (5). It is possible to utilize the results obtained in [9] by Callaghan and Arrillaga. They have provided a simplified solution on the grounds that its purpose is to estimate an effect, rather than produce an accurate

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result. In particular, with reference to the simple equivalent circuit for analyzing the commutation reported in Fig. 2a, they have shown that, during commutation from phase *a* to phase *b*:

$$\frac{di_b}{dv_{ba}} = \frac{1}{l_{ab}} v_{ba} \frac{1}{\frac{dv_{ba}}{dt}} \quad (12)$$

where $l_{ab}=(l_a+l_b)$; similar results also apply for the other converter commutations. This means that the magnitudes of the derivative of the converter currents are inversely proportional to the magnitude of the transformer impedance.

Starting from this result, and from the analysis of numerous experiments, the authors propose to estimate the elements of $[\partial f/\partial \Delta \mathbf{V}^h]$, around the solution point, as:

$$\partial f/\partial \Delta \mathbf{V}^h = \frac{1}{\gamma\sqrt{R_T^2 + (2\pi h f_1 L_T)^2}} \quad (13)$$

where R_T and L_T are the transformer resistance and inductance respectively, f_1 is the system fundamental frequency and γ is a real factor whose values the authors propose to find in the interval of real numbers $\gamma = [1, 3]$.

This means that the sensitivity to voltage variation of the converter, at a given harmonic of order h, are related to the equivalent converter impedance, $\mathbf{Z}^h_{conv}$, which can be expressed by:

$$\mathbf{Z}^h_{conv} = \gamma(R_T + j2\pi h f_1 L_T) . \quad (14)$$

Other types of nonlinear loads (as arc furnaces, FACTS devices, etc.) can be analytically modeled in a different way or subjected to a numerical sensitivity analysis for estimating the numerical value of their impedance at the h-th harmonic.

Once an equivalent impedance for the converter sensitivity to the supply harmonic voltage variations has been estimated, the generalization of the results obtained for the linear system of Fig. 5, gives the following solution for the optimal choice of $\mathbf{Z}_{PCT}$:

$$\mathbf{Z}_{PCT} = \mathbf{Z}^h_{conv} . \quad (15)$$

Since the estimation of $\mathbf{Z}^h_{conv}$ may be uncertain, due to the linearization adopted in modelling the converter sensitivity, it is useful to formulate a sufficient condition for convergence and, then, to take into account the possible error in estimating $\mathbf{Z}^h_{conv}$. With reference to Fig. 5, once $\mathbf{Z}_L$ is substituted by $\mathbf{Z}^h_{conv}$, the following convergence constraint applies:

$$\left| \mathbf{Z}^h_{AC} // \mathbf{Z}^h_{PCT} \right| < \left| \mathbf{Z}^h_{conv} // (-\mathbf{Z}^h_{PCT}) \right| \quad (16)$$

which can easily be rewritten as:

$$\left| (\mathbf{Z}^h_{AC} + \mathbf{Z}^h_{PCT})/(\mathbf{Z}^h_{conv} - \mathbf{Z}^h_{PCT}) \right| > \left| \mathbf{Z}^h_{AC} / \mathbf{Z}^h_{conv} \right| \quad (17)$$

The constraint (17) clearly indicates the effect that $\mathbf{Z}^h_{PCT}$ has to produce. Starting from (17) it is possible to take into account the aforementioned linearization errors in estimating the converter equivalent impedance $\mathbf{Z}^h_{conv}$.

C. *IHA modification*

The IHA algorithm described in section II.A has to be modified by adding steps 1-bis and 3-bis as follows:

(1) unmodified;

(1-bis) the shunt resonances overcoming pre-established values are detected using the $[\mathbf{Y}^h]$ matrix; then appropriate $\mathbf{Z}^h_{PCT}$ values and the consequent $[\mathbf{Y}^{h*}]$ matrices defined in (7b) are evaluated;

(2) unmodified;

(3) unmodified;

(3-bis) starting from the vector of the harmonic components of the voltages at the converter terminal, the vector $\mathbf{I}^h_{PCT}$ of the harmonic components of the currents absorbed by the $-\mathbf{Z}^h_{PCT}$ impedance is evaluated in the frequency domain;

(4) the current harmonics $\mathbf{I}^h$ obtained in step (3) are added to the current harmonic $\mathbf{I}^h_{PCT}$ obtained in step (3-bis), then, the resulting vector $\mathbf{I}^{h*}=(\mathbf{I}^h + \mathbf{I}^h_{PCT})$ is used to perform steady-state harmonic analyses of the AC system by means of the corresponding harmonic impedances $[\mathbf{Y}^{h*}]^{-1}$ of the modified AC system, to obtain the corresponding updated vector $\mathbf{V}^h$ of voltage harmonic components at the converter busbars.

Steps (1) to (4) are repeated until convergence is achieved.

It is important to note that the effects of the added impedances, $\mathbf{Z}^h_{PCT}$ and $-\mathbf{Z}^h_{PCT}$, are evaluated entirely in the frequency domain.

D. *Comments*

The main advantage of the PCT is that it overcomes all of the disadvantages evidenced in the section III.C for the SCTs. Therefore, PCT assures convergence, is easier to apply and more effective than SCT approaches.

The choice of the parallel compensating impedance, which plays a central role in obtaining an optimal convergence behavior, is very easy to automatize. Moreover, as evidenced by the simple case study of Fig. 5, the convergence behavior is not sensitive to the choice of the compensating impedance $\mathbf{Z}^h_{PCT}$ in a large neighborhood around its optimal value. Furthermore, no disadvantages arise for compensating situations that are already intrinsically convergent.

V. NUMERICAL EXPERIMENTS

Numerical experiments were carried out using all of the previously discussed techniques. The aim of the study is the analysis of the behavior of the previously considered iterative methods in the presence of supply system parallel resonances, as these are the main cause of divergence. The same simple case-study of [10], which consists of a 6-pulse bridge, a symmetrical system and a single parallel resonance, was considered.

The system considered is shown in Fig. 7. The AC system is symmetrical and its behavior at the point of common coupling (T) of the converter group is characterized by the harmonic

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impedances, whose magnitude in pu is shown in Fig. 8 as a function of frequency, and by the AC system open-circuit node voltage of value 1.02 pu.

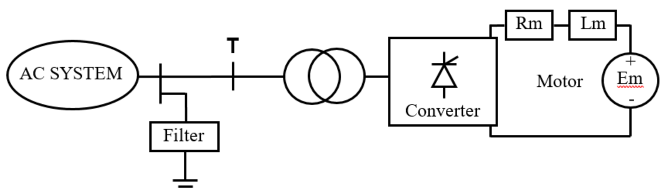

Fig. 7: The system considered for numerical experiments

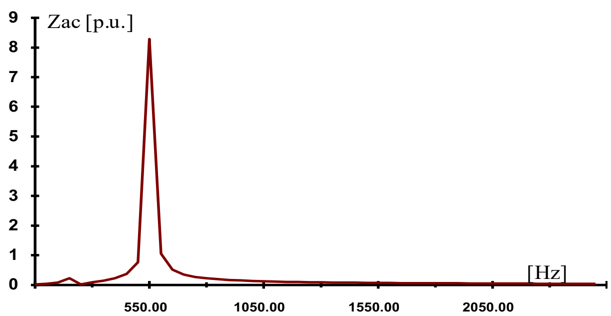

Fig. 8: impedance-frequency characteristics at the PCC, **T**.

It should be noted that the parallel resonance at the 11-th harmonic, that is a characteristic harmonic, is related to the aforementioned exigency of using a case study in which divergence causes are present and, at the same time, clearly identified. However, this case is not removed from reality and corresponds to the practical case of a large power converter to be analyzed during the outage of the 11-th harmonic filter and supplied by a system whose impedance has an 11-th harmonic parallel resonance with the remaining filters.

The transformer converter load-group is based on a six-pulse converter with controlled valves. The transformer converter load group main characteristics are reported in Table I.

TABLE I
TRANSFORMER CONVERTER LOAD GROUP MAIN CHARACTERISTICS

R_T (pu)	X_T (pu)	α (°)	R_M (pu)	L_M (pu)	E_M (pu)
$4.80 \cdot 10^{-3}$	0.150	12.7	0.0248	$1.30 \cdot 10^{-3}$	0.865

A normal IHA application and two different S-RPT applications were reported in [10], and convergence to a solution was not obtained. The S-IPT applications referred to in [10] gave convergence. Further numerical experiments, in which the impedance of the system was progressively decreased in the neighborhood of the resonant frequency, were also referred to in [10]. It was shown that once values below a proper threshold were reached, convergence was also obtained for IHA, therefore, clearly demonstrating that the strong resonance at 550 Hz was the cause of divergence in the previous case.

The case study has been solved successively by means of a certain number of PCT applications, each utilizing a different compensation impedance value, and by a whole system time domain simulation (TS).

The equivalent circuit utilized for TS is not reported here for the sake of brevity; however, the equivalent circuit which represents AC system and filters perfectly represents the impedance-frequency characteristic depicted in Fig. 8. Therefore, the TS results can be assumed as a reference.

Since the cause of the IHA divergence is the 11-th harmonic impedance resonance, the $\mathbf{Z}_{PCT}$ values have been chosen according to the following rule:

$$\mathbf{Z}^{h}_{PCT} = \infty \quad \forall h \# 11,$$

$$\mathbf{Z}^{h}_{PCT} \# \infty \quad h=11;$$

specifically, different finite values of $\mathbf{Z}^{11}_{PCT}$ were tested.

The results of the application of the PCT are reported in Table II, in terms of the number of iterations required for solution convergence for some of the different values of $\mathbf{Z}^{11}_{PCT}$ adopted. Moreover, in Fig. 9 the number of iterations needed for convergence are plotted as a function of the ratio $| \mathbf{Z}^{11}_{PCT} / \mathbf{Z}^{11} |$, where $\mathbf{Z}^{11}_{T}$ is the transformer 11-th harmonic impedance, which gives an idea of the degree of compensation.

It is possible to observe that convergence is obtained for a large interval of $\mathbf{Z}^{11}_{PCT}$ values. For very high values of $\mathbf{Z}^{11}_{PCT}$, divergence is observed at both extremes, i.e. the compensating effect is negligible. In such cases, the PCT application coincides with the normal IHA application, and the technique gives divergence; this occurs as a consequence of selecting a 'bad' value of $\mathbf{Z}^{11}_{PCT}$.

TABLE II
ITERATIONS NEEDED FOR CONVERGENCE FOR DIFFERENT PARALLEL COMPENSATING IMPEDANCE $\mathbf{Z}^{11}_{PCT}$, VALUES

$\mathbf{Z}^{11}_{PCT}$ [pu]	iterations to convergence
.....	.....
0.8	diverged
1.3	15
1.7	10
2.6	8
3.9	5
7.8	5
8.5	8
.....	.....
∞	diverged

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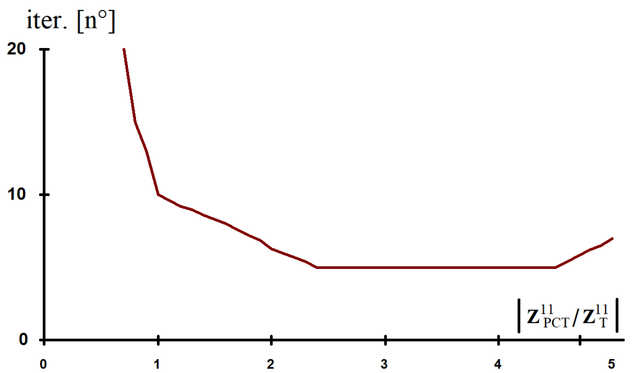

Fig. 9. PCT iterations needed for convergence versus the ratio $|\mathbf{Z}^{11}_{PCT}/\ \mathbf{Z}^{11}_{T}|$

The agreement between the PCT solution and the TS solution is very good. Table III shows in detail, harmonic by harmonic, the agreement between the TS, S-IPT and PCT solutions; the S-IPT results are taken from [10].

TABLE III
COMPARISON OF TS, S-IPT AND PCT RESULTS

harmonic order	Voltage harmonics at busbar **T** [%]		
	TS	S-IPT	PCT
5	0.313	0.280	0.276
7	1.23	1.22	1.18
11	13.7	13.6	13.9
13	1.33	1.26	1.31
17	0.195	0.192	0.166

Therefore, the PCT is able to assure convergence, whilst, at the same time, eliminating, the disadvantages of the SCTs, i.e. the increased computational effort and the difficulties in use and in modelling the equivalent converter impedance.

Recently, some further tests using different test systems (also multi-resonant and unbalanced) have been developed in [2] and they have given very similar results. They are not reported here for the sake of brevity; however, they confirm the value of the PCT.

VI. CONCLUSION

The paper has focused on the convergence difficulties of the iterative approaches based on sequential substitutions Gauss-Seidel type for updating converter injected currents and supply harmonic voltages for harmonic analyses in power systems. Explicit reference has been made to the Iterative Harmonic Analysis (IHA) in its classical formulation, even if the results obtained apply to a wider set of cases. Two proposed series compensation techniques to improve IHA convergence have been described and their applicability limits have been analyzed. To overcome the limits, a parallel compensation technique has been presented. A critical case-study, characterized by the presence of AC system shunt resonances, which cause divergence of IHA, has been solved and the results obtained by different compensation impedance values have been analyzed and discussed.

The main outcome of the paper is that the parallel compensation technique succeeds in solving the convergence problems of IHA also in the presence of AC systems with high shunt resonances. The PCT also eliminates the disadvantages of the series compensation techniques, i.e. the increase of computational effort and the difficulties in use and in modelling.

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REFERENCES

[1] A. Medina, J. Segundo-Ramirez, P. Ribeiro, W. Xu, K. L. Lian, G. W. Chang, V. Dinavahi, and N. R. Watson, "Harmonic Analysis in Frequency and Time Domain". *IEEE Trans. on Power Delivery,* Vol. 28, N.3, July 2013. DOI 10.1109/TPWRD.2013.2258688.

Energia Elettrica Supplement Journal – Vol. 97 – DOI: 10.36156/ENERGIA06_04

[2] S. Z. Djokic, R. Langella, I. Papič, A. Špelko, A. Testa, “Application of Voltage Harmonic Vector Method at the Interface between HV and MV Systems”. *2020 19th IEEE International Conference on Harmonics and Quality of Power (ICHQP),* 6-7 July 2020, Dubai.

[3] Dean Culver, Earl Dowell: “Iterative techniques for analyzing nonlinear vibrating dynamical systems in the frequency domain”. *Nonlinear Dynamics, Springer Science+Business Media B.V.,* March 2018, Volume 91, Issue 4, pp 2113–2140. https://doi.org/10.1007/s11071-017-4005-0.

[4] M. Carpita, P. Caramia, E. Ascione, D. Proto, P. Varilone, ”The Iterative Harmonic Analysis for Waveform Distortion Assessment of Low Voltage Distribution Systems with Plug-in Electric Vehicles”. *International Symposium on Power Electronics, Electrical Drives, Automation and Motion (SPEEDAM),* 27 August 2018, Amalfi, Italy. DOI: 10.1109/SPEEDAM.2018.8445356.

[5] Bula, D. and Lewandowski, M., “Steady state simulation of a distributed power supplying system using a simple hybrid time-frequency model”. *Elsevier Applied Mathematics and Computation,* Volume 319, pp. 195–202, 15 February 2018. https://doi.org/10.1016/j.amc.2017.02.028.

[6] Thainan S. Theodoro, M. T. Correia de Barros, P. G. Barbosa, A. M. Variz, “Iterative Time-Frequency Harmonic Analysis: Problems and Solutions”. *Proceddings of IEEE PES Conference: Innovative Smart Grid Technologies Europe (ISGT-Europe),* September 2019. DOI: 10.1109/ISGTEurope.2019.8905727. Electronic ISBN: 978-1-5386-8218-0.

[7] R. Yacamini and J. C. de Oliveira, "Harmonics in multiple convertor systems: a generalised approach". In *IEE Proceedings B - Electric Power Applications,* vol. 127, no. 2, pp. 96-106, March 1980, doi: 10.1049/ip-b.1980.0012.

[8] R. Yacamini and J. C. de Oliveira, *"Comprehensive calculation of convertor harmonics with system impedances and control representation".* In *IEE Proceedings B - Electric Power Applications,* vol. 133, no. 2, pp. 95-102, March 1986, doi: 10.1049/ip-b.1986.0014.

[9] C.D. Callaghan, J. Arrillaga, “Convergence criteria for iterative harmonic analysis and its application to static convertors”*. Proceedings of IV IEEE International Conference on Harmonics in Power Systems (ICHPS),* Budapest - Hungary, 4-6 October 1990.

[10] R. Carbone, M. Fantauzzi, F. Gagliardi and A. Testa, "Some considerations on the iterative harmonic analysis convergence". In *IEEE Trans. on Power Delivery,* vol. 8, no. 2, pp. 487-493, April 1993, doi: 10.1109/61.216850.

[11] J. Arrillaga, N. R. Watson, J. F. Eggleston and C. D. Callaghan, "Comparison of steady-state and dynamic models for the calculation of AC/DC system harmonics". In *IEE Proceedings C - Generation, Transmission and Distribution,* vol. 134, no. 1, pp. 31-37, January 1987, doi: 10.1049/ip-c.1987.0005.

[12] J. Arrillaga and C. D. Callaghan, "Three phase AC-DC load and harmonic flows". In *IEEE Trans. on Power Delivery,* vol. 6, no. 1, pp. 238-244, Jan. 1991, doi: 10.1109/61.103743.

[13] G. Carpinelli, F. Gagliardi, M. Russo and D. Villacci, "Generalised convertor models for iterative harmonic analysis in power systems". In *IEE Proceedings - Generation, Transmission and Distribution,* vol. 141, no. 5, pp. 445-451, Sept. 1994, doi: 10.1049/ip-gtd:19941212.

[14] Ushida and L. O. Chua, "Steady-state response of non-linear circuits: a frequency-domain relaxation method", *International Journal of Circuit Theory and Applications, Jonh Wiley & Sons*, vol.17, 249-269 (1989).