Operations Research Perspectives 4 (2017) 136–141

Contents lists available at ScienceDirect

Operations Research Perspectives

journal homepage: www.elsevier.com/locate/orp

A robust optimization model for a decision-making problem: An application for stock market

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ARTICLE INFO

Article history:
Received 27 March 2017
Revised 19 August 2017
Accepted 11 October 2017
Available online 12 October 2017

Keywords:
Decision making
Robust LINMAP
Stock

ABSTRACT

In this paper we apply robust linear programming technique for multidimensional analysis of preference (LINMAP) method for a decision making problem. During the last two decades, many methods have been extensively used for decision making problems. However, there is no investigation among many existing studies where the uncertainty in data is possible. The robust LINMAP method with the assumption of uncertainty on parameters is implemented in the stock market in order to rank priorities of the stocks.

1. Introduction

Multi attribute decision making (MADM) methods are practical and useful techniques for real-world decision making situations. Many financial decision problems which include several criteria applied MADM methods as an effective tool. Optimum choice of stock is an issue which investors are tackling permanently. A large number of studies have been expanded in this field. Diakoulaki et al. [7] presented a MADM method for assessment of the companies' operation and applied the results of a multi criteria analysis to a large sample of Greek pharmaceutical industries. A multi criteria industrial evaluation system was provided by Mareschal and Bransj [15]. which is useful for decision-makers when they want to make decisions about their industrial clients. Samaras et al. [18] introduced a system to utilize multi criteria analysis methodologies in order to evaluate and rank the Athens Stock Exchange (ASE) stocks. MADM problems can be divided into different categories. Technique for order preference using similarity to the ideal solution (TOPSIS) is a method based on distance measures which has been introduced by Hwang and Yoon [13]. The other method which is similar to the TOPSIS is a linear programming technique for multidimensional analysis of preference (LINMAP) developed by Srinivasan et al. [20]. Nevertheless, the TOPSIS and LINMAP methods need various kinds of data and decision conditions. In the LINMAP method decision makers compare alternatives in form of pairs and the best solution is the alternative that has shortest distance to the positive ideal solution (PIS), while in the TOPSIS method shortest distance to the PIS and the farthest from the negative ideal solution (NIS) is considered.

Ordinal regression is one of the methods used in decision making problems. It can represent a set of holistic preference information provided by the Decision Maker (DM). Greco et al [11]. presented an ordinal regression method for multiple criteria ranking of alternatives. A Regression with Intensities of Preference (GRIP) method was presented for ranking a finite set of actions which was based on indirect preference information and the ordinal regression paradigm [8]. Greco et al. [12] introduced an ordinal regression model for multiple criteria problems by using a set of additive value functions computed through the resolution of linear programs. Robust Ordinal Regression (ROR) is one of the recent approaches concerning the development of preference models. ROR designed for multiple criteria ranking is a non-statistical methodology of preference learning. The basic concepts and the main developments of ROR were introduced by Corrente et al. [6]. Corrente et al. [5] clarified the specific interpretation of the concept of preference learning adopted in ROR. They focused on ROR, which is closer to preference learning practiced in Machine Learning.

Nevertheless, in many real-world decision problems results produced by deterministic approaches could lead to neglect of inaccurate information. Consequently, a large number of methods were

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https://doi.org/10.1016/j.orp.2017.10.001

developed to manage uncertainty on the decision problems, like robust solution, stochastic and fuzzy programming . The concept of fuzzy logic, initially introduced by Zadeh et al. [22] is more applicable when there is no access to historical information and the information are based on decision maker's prejudgment. Recently, fuzzy logic has been widely applied in decision making problems. Chen and Tan [4] developed a LINMAP method to deal with multiple criteria decision analysis problems based on interval type-2 trapezoidal fuzzy numbers. Wang and Lie [21] presented a fuzzy logic approach to solve multi-attribute group decision making problems in which all the preference information provided by the decision makers and the preference data about the alternatives are generally unknown. Zarghami et al. [23] introduced a fuzzy-stochastic modelling of MCDM problems by using the stochastic and fuzzy approaches in order to obtain a robust decision under uncertainly. Different fuzzy methods that are multi-criteria decision making were introduced by Li et al. [14].

However, the approaches have been applied to decision-making problems without assuming any uncertainty in information, have been under some severe examination where a minor perturbation can make a significant modification on the ranking. Ben-Tal and Nemirovski [1] showed a small perturbation on information might lead to infeasible solutions and the results of the ranking might be unreliable. Recent researches on robust optimization have developed some models which are capable of considering uncertainty in data and generating the ranking that is more reliable and a minor modification in input and output parameters might not change the outcomes.

When input and output data in a mathematical linear programming are uncertain, could not solve by traditional methods, robust optimization can cope with this uncertainty on decision making problem. Soyster [19] introduced robust optimization method but it was too conservative. A new robust optimization for handling uncertainty on linear programming was presented by Ben and Nemirovski [1]. A robust optimization which has been widely used for MCDM problems was introduced by Bertsimas and Sim [3]. In some studies the robust optimization method was applied in the industrial cases. It was utilized for measuring the efficiency of telecommunication companies [17]. In addition, Sadjadi and Omrani [16] were presented Data Envelopment Analysis model (DEA) with uncertain data for performance assessment of electricity distribution companies. In this paper, the proposed robust optimization technique is implemented to stock market in order to select best stock. The task of the choosing stocks including several fundamental indicators to invest is a decision making process. There are many unknown and uncertain criteria and an investor should take into account all available data. The main objective of this paper is to apply a method considering all parameters for selecting stocks. As, there are many criteria for choosing stock that some of them are uncertain and some criteria are changing during the time, applying robust LINMAP method can cope with uncertainly in data. We organize the paper as follows. Section 2 contains description of the LINMAP method. Robust form of LINMAP is presented in Section 3. Finally, the method is implemented in a real case in Section 4.

2. LINMAP formulation for multiple attribute

A LINMAP issue is to catch the best compromise solution from all appropriate alternatives assessed on multiple attributes. Suppose that there is a collection existing of V decision makers who choose one(s) of (or rank) m alternatives based on n attributes. Alternatives composed of attributes are represented as m points in the n-dimensional space. Assume that ratings of alternatives on attributes are given using LINMAP through judgments of the decision makers. A decision maker considers an ideal point in his mind based on his preference. Then the alternatives which have the shortest distance from the ideal point are selected. Therefore, for each alternative A_i the distance from ideal point is shown by d_i as follow [13]:

$$d_i = \left[\sum_{j=1}^{n} w_j (x_{ij} - x_j^*)^2 \right]^{1/2}, \qquad i = 1, 2, \ldots, m,$$

where weights of attributes are w_j (j=1,2,... ,n). Weights are unknown and must be determined, x_{ij} is the value of i_{th} alternative based on j_{th} attribute in decision matrix and x_j^* is the ideal point value, so that the square of the distance from the ideal point is;

$$s_i = d_i^2 = \sum_{j=1}^{n} w_j (x_{ij} - x_j^*)^2, \qquad i = 1, 2, \ldots, m.$$

Decision makers give the preference between alternatives by $\Omega = \{(k, l)\}$ that denotes a set of ordered pairs (k, l), where k represents the preferred alternative basis on results from a pair wise comparison involving alternatives k and l. Generally not perforce Ω has all alternatives. For each pair in Ω, the solution (w, x^*) might be consistent with the weighted Euclidean distance while the following condition holds,

$$s_l \geq s_k.$$

Otherwise if $s_l < s_k$ means an error happened and we generally define

$$(s_l - s_k)^- = \begin{cases} 0 & \text{if} \quad s_l \geq s_k \\ s_k - s_l & \text{if} \quad s_l < s_k, \end{cases} \tag{2.1}$$

that could measure inconsistency between the ranking order of alternatives a_k and a_l determined by s_k and s_l and the preference relation $(k, l) \in \Omega$ given by the decision maker. Obviously, the index in (2.1) can be rewritten as follows:

$$(s_l - s_k)^- = \max\{0, \ (s_k - s_l)\},$$

and $(s_l - s_k)^-$ represents error for the pair of $(k, l) \in \Omega$. We define a total inconsistency index of the decision maker by:

$$B = \sum_{(k,l)\in\Omega} (s_l - s_k)^- = \sum_{(k,l)\in\Omega} \max\{0, \ (s_k - s_l)\}.$$

By definition, $(s_l - s_k)^-$ and B are nonnegative. Finding (w, x^*) based on which B is minimal is our problem. Similar to B, we define a total consistency index of the decision maker by:

$$G = \sum_{(k,l)} (s_l - s_k)^+,$$

where:

$$(s_l - s_k)^+ = \begin{cases} s_l - s_k & \text{if} \quad s_l \geq s_k \\ 0 & \text{if} \quad s_l < s_k. \end{cases}$$

If $G > B$ we define $G - B = h$, where h is a positive number. It suddenly shows that $(s_l - s_k)^+ - (s_l - s_k)^- = (s_l - s_k)$. Furthermore, h can be extended as $\sum_{(k,l)\in\Omega} (s_l - s_k)$.

We construct the auxiliary mathematical programming model to determine w; thus (w, x^*) could be acquired by solving the following model,

$$\min B = \sum_{(k,l)\in\Omega} \max\{0, \ (s_k - s_l)\},$$

$$\text{subject to} \quad \sum_{(k,l)\in\Omega} (s_l - s_k) = h,$$

or equivalently:

$$\min \sum_{(k,l)\in\Omega} Z_{kl},$$

$$\text{subject to} \quad (s_l - s_k) + Z_{kl} \geq 0, \qquad \text{for} \quad (k,l) \in \Omega,$$

$$\sum_{(k,l)\in\Omega} (s_l - s_k) = h, Z_{kl} \geq 0, \qquad \text{for} \quad (k,l) \in \Omega.$$

Finally we can make the linear programming (LP) formulation of ready to solve by substituting s_l and s_k to obtain:

$$(s_l - s_k) = \sum_{j=1}^{n} w_j (x_{lj} - x_j^*)^2 - \sum_{j=1}^{n} w_j (x_{kj} - x_j^*)^2$$

$$= \sum_{j=1}^{n} w_j (x_{lj}^2 - x_{kj}^2) - 2\sum_{j=1}^{n} w_j x_j^* (x_{lj} - x_{kj}).$$

Since x_j^* is an indeterminate constant, we define $v_j := w_j x_j^*$. Hence, the LP model is as follows:

$$\min \sum_{(k,l)\in\Omega} Z_{kl},$$

$$\text{subject to} \quad \sum_{j=1}^{n} w_j (x_{lj}^2 - x_{kj}^2) - 2\sum_{j=1}^{n} v_j (x_{lj} - x_{kj}) + Z_{kl} \geq 0,$$

$$\text{for} \quad (k,l) \in \Omega.$$

$$\sum_{j=1}^{n} w_j \sum_{(k,l)\in\Omega} (x_{lj}^2 - x_{kj}^2) - 2\sum_{j=1}^{n} v_j \sum_{(k,l)\in\Omega} (x_{lj} - x_{kj}) = h,$$

$$w_j \geq 0, \quad Z_{kl} \geq 0, \quad \text{for} \quad (k,l) \in \Omega \quad v_j = w_j x_j^*;$$

$$j = 1, 2, \ldots, n. \tag{2.2}$$

I) If $w_j^* > 0$, then $x_j^* = v_j^* / w_j^*$,
II) If $w_j^* = 0$ and $v_j^* = 0$ define $x_j^* = 0$,
III) If $w_j^* = 0$ and $v_j^* > 0$, then $x_j^* = +\infty$,
IV) If $w_j^* = 0$ and $v_j^* < 0$, then $x_j^* = -\infty$,

where w_j^* and v_j^* are the ideal points value for j_{th} attribute. The following formula is the square distance from the x^*.

$$S_i = \sum_{\alpha} w_\alpha^* (x_{i\alpha} - x_\alpha^*)^2 - 2\sum_{\beta} v_\beta^* x_{i\beta}, \qquad i = 1, 2, \ldots, m, \tag{2.3}$$

where $\alpha = \{j | w_j^* \geq 0\}$, $\quad \beta = \{j | w_j^* = 0 \quad \text{and} \quad v_j^* \neq 0\}$.

3. Robust LINMAP optimization model

There are various attitudes of modelling uncertainty in information. Typical modelling approaches in operations research under uncertainty suppose a full probabilistic characterization. Actually, in most of the models the uncertainty is disregarded altogether and a representative nominal use of the data is considered. The typical approach to deal with uncertainty is the stochastic programming (SP) [17]. Robust optimization can be examined as a corresponding replacement to sensitivity analysis and stochastic programming. To show the robust structure introduced by Bertsimas and Sim [1], let c is a n-vectors, A is a m^*n matrix and b is a m-vector. Assume a given linear programming problem of the following form:

$$\min c^T x,$$

$$\text{subject to} \quad Ax \leq b, \forall a_1 \in U_1, \ldots, a_m \in U_m$$

$$x \in X,$$

where a_i represents the i_{th} row of the uncertain matrix A, and takes values in the uncertainty set $U_i \subseteq R^n$. Then, $a_i^T\ x \leq b_i, \quad \forall$ $a_i \in U_i$ if $\max_{a_i \in u_i} a_i^T\ x \leq b_i\ \forall i$ [2]. Given an uncertain matrix $A = [a_{ij}]$, suppose that in row i, the entries a_{ij} for $j \in J_i \subseteq \{1, \ldots, n\}$ vary in some intervals based on their nominal value, namely, simply gets its value in the interval $[a_{ij} - \delta_{ij}, a_{ij} + \delta_{ij}]$; in which δ_{ij} is the maximum variation of a_{ij}. X is a polyhedron as well. Only the elements of the matrix A are effected by uncertainly, and assume j_i representing the set of coefficients subordinate to uncertainly in a specific row i [3].

Every parameter can deviate and Γ_i coefficients to deviate. Parameter Γ_i for $i = 1, 2, \ldots, m$ not necessarily integer, determines the uncertainty related to each input parameter. Γ_i is the budget of uncertainty for constraint i. When $\Gamma_i = 0$ there is no uncertainty. As Γ_i increases, the uncertainty also increases. The robust formulation becomes:

$$\min c^T x,$$

$$\sum_j a_{ij} x_j + \max_{\{S_i \subseteq J_i : |S_i| = \Gamma_i\}} \sum_{j \in S_i} \delta_{ij} y_j \leq b_i \qquad 1 \leq i \leq m,$$

$$-y_j \leq x_j \leq y_j \qquad \forall j = 1, \ldots, n,$$

$$l_j \leq x_j \leq u_j \qquad \forall j = 1, \ldots, n,$$

$$y_j \geq 0, \qquad \forall i, j \in J_i.$$

Taking the dual of the inner maximization problem and regarding to the assumption of Bertsimas and Sim [3] the robust model can be rewritten as follows:

$$\max c^T x,$$

$$\sum_j a_{ij} x_j + z_i \Gamma_i + \sum_{j \in J_i} p_{ij} \leq b_i \qquad \forall i,$$

$$z_i + p_{ij} \geq \delta_{ij} y_j \qquad \forall i, j \in J_i,$$

$$-y_j \leq x_j \leq y_j \qquad \forall j = 1, \ldots, n,$$

$$l_j \leq x_j \leq u_j \qquad \forall j = 1, \ldots, n,$$

$$p_{ij} \geq 0,\ y_j \geq 0,\ z_i \geq 0 \qquad \forall i, j \in J_i,$$

Since model (2.2) is a linear programming problem we apply the idea of robust optimization as follow [9].

$$\max - \sum_{(k,l)\in\Omega} Z$$

subject to

$$-\sum_j w_j (x_{lj}^2 - x_{kj}^2) + 2\sum_j v_j (x_{lj} - x_{kj}) - Z_{kl} + \Psi_{kl}\Gamma$$

$$+ \sum_j (P_{klj} + q_{klj}) \leq 0 \quad \text{for} \quad (k,l) \in \Omega,$$

$$\Psi_{kl} + p_{kjl} \geq \triangle_1 (x_{lj}^2 - x_{kj}^2) y_j \qquad (k,l,j) \in \acute{\Omega},$$

$$\Psi_{kl} + q_{kjl} \geq \triangle_2 (x_{lj} - x_{kj}) \Phi_j \qquad (k,l,j) \in \acute{\Omega},$$

$$-y_j \leq w_j \leq y_j, \qquad \forall j = 1, 2, \ldots, n,$$

$$-\Phi_j \leq v_j \leq \Phi_j, \qquad \forall j = 1, 2, \ldots, n,$$

$$\sum_j w_j \sum_{kl\in\Omega} (x_{lj}^2 - x_{kj}^2) - 2\sum_j v_j \sum_{kl\in\Omega} (x_{lj} - x_{kj}) = h,$$

$$w_j,\ \Psi_{kl},\ Z_{kl} \geq 0, \quad \text{for} \quad (k,l) \in \Omega, \quad v_j = w_j x_j^*; \quad j = 1, 2, \ldots, n, \tag{3.1}$$

where p, q and Ψ are the dual variables and are new dummy non-negative variables related to the uncertain parameters in (2.2). $\acute{\Omega} = \{(k,l,j) \mid (k,l) \in \Omega, j \in J_i\}$, $\triangle_1$ and $\triangle_2$ are perturbation in parameters. A simple didactic example is considered for introducing the model. Assume a decision making problem in which the decision maker takes into consideration the three attributes in evaluating three candidates. First, the decision maker provides his preferences between the candidates regarding to his experience and his knowledge. Assume that the decision maker provides his preferences between the candidates as follows: $\Omega = \{(1,2),(2,3)\}$. Since there are uncertainties associated with possible decision maker preferences, robust LINMAP could be adopted to solve the resulted problem.

Then, the decision matrix is marked based on alternatives and criteria. Since a small perturbation could make a big change on the ranking we consider the perturbation in data, we assume perturbation in parameters by using Δ in the model. We suppose 10%% disturbance in elements, i.e. $\Delta = 0.1$. We also assume $\Gamma = 1.5$ which represents 0.95%% guarantee in holding the constraints. The parameter $h = 1$ is considered. Finally, by using the robust LINMAP model we have formulated (3.1), the values of the decision matrix, and Ω we can construct the following L.P.P:

$$
\begin{aligned}
&\max -(Z_{12}+Z_{23})\\
&\text{subject to}\\
&-\sum_{j=1}^{3} w_j(x_{2j}^2-x_{1j}^2)+2\sum_{j=1}^{3} v_j(x_{2j}-x_{1j})-Z_{12}+1.5\Psi_{12}\\
&\quad+\sum_j (P_{12j}+q_{12j})\le 0,\\
&-\sum_{j=1}^{3} w_j(x_{3j}^2-x_{2j}^2)+2\sum_{j=1}^{3} v_j(x_{3j}-x_{2j})-Z_{23}+1.5\Psi_{23}\\
&\quad+\sum_j (P_{23j}+q_{23j})\le 0,\\
&\Psi_{12}+p_{12j}\ge 0.1(x_{2j}^2-x_{1j}^2)y_j, \qquad \forall j=1,2,3,\\
&\Psi_{23}+p_{23j}\ge 0.1(x_{3j}^2-x_{2j}^2)y_j, \qquad \forall j=1,2,3,\\
&\Psi_{12}+q_{12j}\ge 0.1(x_{2j}-x_{1j})\Phi_j \qquad \forall j=1,2,3,\\
&\Psi_{23}+q_{23j}\ge 0.1(x_{3j}-x_{2j})\Phi_j \qquad \forall j=1,2,3,\\
&-y_j\le w_j\le y_j, \qquad \forall j=1,2,3,\\
&-\Phi_j\le v_j\le \Phi_j, \qquad \forall j=1,2,3,\\
&\sum_{j=1}^{3} w_j(x_{2j}^2-x_{1j}^2)+(x_{3j}^2-x_{2j}^2)-2\sum_{j=1}^{3} v_j(x_{2j}-x_{1j})+(x_{3j}-x_{2j})\\
&=1,\\
&w_j\ge 0,\ \Psi_{12}\ge 0,\ Z_{12}\ge 0,\ \Psi_{23}\ge 0,\ Z_{23}\ge 0.
\end{aligned}
$$

Substituting the values from the decision matrix, we can obtain the optimal solution by using MATLAB. The unknown weight vectors w and v are determined by solving model. x^* can be calculated. Consequently, ranking order of three candidates is generated by the square of the distance of each alternative from the PIS which can be obtained by using equation (2.3).

Robust and stochastic optimization are two methods to deal with data uncertainty in optimization. Stochastic optimization has an important assumption, i.e., the true probability distribution of uncertain data has to be known or estimated. If this condition is met and the reformulation of the uncertain optimization problem is computationally tractable, then SO is the methodology to solve the uncertain optimization problem at hand. Robust optimization, on the other hand, does not assume that probability distributions are known, but instead it assumes that the uncertain data resides in a so-called uncertainty set. RO is popular because of its computational tractability for many classes of uncertainty sets and problem types [10]. Robust Optimization is an approach to optimization under uncertainty, in which the uncertainty model is not stochastic, but rather deterministic and set-based. Instead of seeking to immunize the solution in some probabilistic sense to stochastic uncertainty, here the decision-maker constructs a solution that is optimal for any realization of the uncertainty in a given set. Robust Optimization constructs solutions that are deterministically immune to realizations of the uncertain parameters in certain sets. This approach may be the only reasonable alternative when the parameter uncertainty is not stochastic, or if no distributional information is available. But even if there is an underlying distribution, the tractability benefits of the Robust Optimization paradigm may make it more attractive than alternative approaches from Stochastic Optimization [2].

4. Case study

In this section, we show the implementation of LINMAP and proposed robust LINMAP. Since selecting the best stock is a MCDM problem, there are many criteria which influence choosing the best stock. Moreover, some criteria are uncertain and time can effect some of them. With applying robust LINMAP and considering a tolerance for criteria, we can reduce the effects of any uncertainty. Therefore, results in this method are more reliable. We have alternatives with some criteria and our goal is selecting the best alternative based on uncertainty value of criteria and group decision making. In this process, first step is determination of ingredients and influential variables for investing in the stock market. These criteria are obtained through evaluation, initial observation and interviews with financial experts in stock exchange. Some criteria that affect investor's decisions for selecting stocks are, volume, market capitalization, earnings price ratio (P/E), earnings per share (EPS), liability ratio to equity (L/E), return on equity (ROE) which are defined as follows: volume is a measure of how much of a given financial asset has been traded in a given period of time. Market capitalization shows the size of a company and companies can be ranked according to their market capitalizations. Earnings price ratio (P/E) is the ratio for valuing a company that measures its current share price relative to its per-share earnings. In general, a high P/E suggests that investors are expecting higher earnings growth in the future compared to companies with a lower P/E. Earnings per share (EPS) is the portion of a company's profit allocated to each outstanding share of common stock. Earnings per share serves as an indicator of a company's profitability. Liability ratio to equity (L/E) is calculated by dividing the company's total liabilities by its shareholder's equity the real use of debt-equity is in comparing the ratio for firms in the same industry. Return on equity (ROE) measures a corporation's profitability by revealing how much profit of a company generates with the money invested by shareholders. The ROE is useful for comparing the profitability of a company to that of other firms in the same industry. These criteria are denoted by X_1, ... ,X_6. In the next step, the elements of decision matrix D which are values associated with each alternative, are defined. Also decision- makers are questioned and asked to compare each alternative with others. Moreover, with defining criteria and alternatives, the model is designed. Finally, LINMAP and robust LINMAP are applied for finding the high priority and choosing the best stock.

In this paper, the companies acting on global financial markets, have been chosen from a financial portal (Investing.com) which contains the valuable information of international companies. Selected companies which are the elements of the decision matrix are engaged in a variety of health services. All scripts Health care Solutions Inc. (MDRX) are operating through three segments: Clinical and Financial Solutions, Population Health, and software and technology. CVS Health Corp that is an integrated pharmacy health care company. Health care Realty Trust Incorporated (HR), the Company owns, leases, manages, acquires, finances, develops and redevelops real estate properties associated primarily with the delivery of outpatient health care services across the United States. Cardinal Health, Inc. (CAR) is a health care services and products company. The Company operates through two segments: Pharma-

Table 1
Different weights for three decision makers using LINMAP.

	DM1	DM2	DM3
w_1	0	0	0
w_2	0	0	0
w_3	0	0	0
w_4	0	0	0
w_5	0	0	0
w_6	0	0	0
v_1	0	0	−0.000001
v_2	0.001471	0	−0.021850
v_3	0	0	0.000260
v_4	0	0	−0.300286
v_5	0	−0.002444	−0.129535
v_6	0	−0.085299	6.299720

Table 2
The relative and geometric average absolute distances using LINMAP.

	DM1	DM2	DM3	Average
S_{A_1}	−0.0503	10.1122	−648.269	6.908594
S_{A_2}	−0.0271	7.4752	−490.474	4.631653
S_{A_3}	−0.0488	9.0766	−599.528	6.42763
S_{A_4}	−0.0893	6.8277	−312.727	5.755683
S_{A_5}	−0.0354	8.6677	−573.281	5.603056
S_{A_6}	−0.095	8.1103	−419.28	6.861537
S_{A_7}	−0.0793	9.2377	−577.307	7.506103
S_{A_8}	−0.0498	8.1353	−520.935	5.953818

Table 3
Different weights of three decision makers using robust LINMAP.

	DM1	DM2	DM3
w_1	0	0	0
w_2	0	0.000005	0
w_3	0	0.000024	0
w_4	0	0	0
w_5	0	0	0.000359
w_6	0	0	0.000322
v_1	0	0	0.00007
v_2	0.001468	0	−0.000002
v_3	0	0.003651	0
v_4	0	0	−0.000062
v_5	0	0	0.000004
v_6	0	0	0

Table 4
The relative and geometric average absolute distances using robust LINMAP.

	DM1	DM2	DM3	Average
S_{A_1}	−0.052	0.0963	2.3372	−0.224393
S_{A_2}	−0.0271	0.125	1.2397	−0.161336
S_{A_3}	−0.0487	0.0561	1.6884	−0.166465
S_{A_4}	−0.0891	0.1108	1.5339	−0.225007
S_{A_5}	−0.0353	0.0769	1.9164	−0.173272
S_{A_6}	−0.0948	0.0329	1.4399	−0.164985
S_{A_7}	−0.0792	0.0527	1.9306	−0.200482
S_{A_8}	−0.0497	0.059	1.3963	−0.159979

ceutical and Medical. Omega Health care Investors Inc. (OHI), the company maintains a portfolio of long-term health care facilities and mortgages. Community Health Systems Inc. (CYH) is an operator of general acute care hospitals and outpatient facilities in communities across the country. The Company operates through hospital operations segment, which includes its general acute care hospitals and related health care entities that provide inpatient and outpatient health care services. Magellan Health Services Inc. (MGLN) is engaged in the health care management business. The company's segments include health care, pharmacy management and corporate. It is focused on managing special populations, complete pharmacy benefits and other specialty areas of health care. Well Care Health Plans Inc. (WCG) is a company that focuses on Medicare Advantage (MA) and Medicare Prescription Drug Plans (PDPs), to families, children, seniors and individuals with medical needs. The company operates through three segments: Medicaid Health Plans, Medicare Health Plans and Medicare PDPs. The companies will be shown with $A_1, \ldots, A_8$ as alternatives respectively. The criteria values associated with the different alternatives formed the decision matrix D. For the implementation of the robust LINMAP method first, we suppose 10%% disturbance in elements i.e. $\Delta = 0.1$. Moreover Δ is considered 0.05, 0.2 and 0.5, respectively. We also assume $\Gamma = 1.5$ which represent 0.95%% guarantee in holding the constraints. Finally, the parameter $h = 1$ is an arbitrary positive number and is advised by the model.

$$D = \begin{bmatrix} 1547724 & 2.47 & 256.06 & 0.05 & 0.931 & 0.0075 \\ 3816660 & 112.61 & 22.75 & 4.61 & 1.485 & 0.141 \\ 764801 & 3.25 & 49.46 & 0.63 & 1.231 & 4.99 \\ 2641088 & 2.47 & 10.33 & 2.53 & 30.503 & 60.76 \\ 1667218 & 6.18 & 27.2 & 1.21 & 1.323 & 0.0868 \\ 6080354 & 1.43 & 12.86 & 0.98 & 5.41 & 2.85 \\ 252484 & 1.65 & 43.2 & 1.54 & 0.994 & 0.0332 \\ 462682 & 4.11 & 29.87 & 3.11 & 1.891 & 0.0817 \end{bmatrix},$$

Three decision makers compare alternatives and give the pairwise comparisons of alternatives as follows:

$$\Omega_1 = \{(A_2, A_1), (A_2, A_3), (A_2, A_8), (A_6, A_1), (A_6, A_7), (A_6, A_8), (A_8, A_7), (A_1, A_7)\},$$

$$\Omega_2 = \{(A_7, A_4), (A_7, A_6), (A_1, A_7), (A_2, A_6), (A_1, A_4), (A_2, A_4), (A_3, A_4)\},$$

$$\Omega_3 = \{(A_3, A_2), (A_3, A_7), (A_3, A_8), (A_3, A_4), (A_2, A_1), (A_1, A_4), (A_5, A_4), (A_5, A_7)\}.$$

We apply LINMAP method for the case study and Table 1 shows the optimal weights for all alternatives which have been acquired with Lingo 8 for decision maker 1 (DM_1), decision maker 2 (DM_2) and decision maker 3 (DM_3).

From the results of Table 2 the highest priorities belong to CVS company and MGLN company has the last priority for selecting in LINMAP method. The results have been obtained by applying MATLAB software. The following order represents the priorities of all alternatives supplied by LINMAP method.

$S_{A_2} < S_{A_5} < S_{A_4} < S_{A_8} < S_{A_3} < S_{A_6} < S_{A_1} < S_{A_7}$. Then we apply the proposed robust LINMAP method to the case study first for $\Delta = 0.1$ then we obtain results for $\Delta = 0.05, 0.2$ and 0.5 and compare them. Table 3 shows the weights which are acquired with robust LINMAP method by running Lingo 8.

The ultimate results applied by MATLAB software for electing best alternative with minimum distance are presented in the following table. The distances related to decision maker 1, 2 and 3 are summarized in columns two, three and four of Table 4 respectively. Column five of Table 4 shows the absolute geometric average of the evaluations of three decision makers.

The orders of priorities for all alternatives in robust LINMAP method are as follows: $S_{A_4} < S_{A_1} < S_{A_7} < S_{A_5} < S_{A_3} < S_{A_6} < S_{A_2} < S_{A_8}$.

Based on Table 4, A_4 which is the implementation of CAR company has the highest priority that can be selected as the best stock and next priorities went to MDRX, MGLN, OHI, HR, CYH, CVS and WCG respectively.

Table 5
Average absolute distance using robust LINMAP for different perturbation.

	$\Delta = 0.05$	$\Delta = 0.2$	$\Delta = 0.5$
S_{A_1}	−0.292283	−0.183424	−0.183083
S_{A_2}	−0.208204	−0.130512	−0.130371
S_{A_3}	−0.217629	−0.131111	−0.130857
S_{A_4}	−0.397356	−0.18500	−0.184671
S_{A_5}	−0.216546	−0.145957	−0.145697
S_{A_6}	−0.270619	−0.131312	−0.131093
S_{A_7}	−0.266817	−0.162927	−0.162599
S_{A_8}	−0.212431	−0.127202	−0.126964

Table 5 shows the absolute geometric average of three decision makers for $\Delta = 0.05, 0.2$ and 0.5.

The orders of priorities for all alternatives in robust LINMAP method for $\Delta = 0.05, 0.2$ and 0.5 are as follows respectively:

$$S_{A_4} < S_{A_1} < S_{A_6} < S_{A_7} < S_{A_3} < S_{A_5} < S_{A_8} < S_{A_2},$$
$$S_{A_4} < S_{A_1} < S_{A_7} < S_{A_5} < S_{A_6} < S_{A_3} < S_{A_2} < S_{A_8},$$
$$S_{A_4} < S_{A_1} < S_{A_7} < S_{A_5} < S_{A_6} < S_{A_3} < S_{A_2} < S_{A_8}.$$

The results of the robust LINMAP show that different perturbation cannot change the results significantly, for $\Delta = 0.1, 0.2$ and 0.5 results are identical which means this method is reliable.

5. Conclusion

We have introduced a robust LINMAP method where there is an uncertainly in the parameters. LINMAP and robust LINMAP methods have been implemented for a case study of decision making problem. The proposed models have been applied to stock market in order to select the best stock under uncertain conditions. There were eight alternatives with some criteria which influenced investors' decisions for choosing stocks and three decision makers have been asked for the relative significance of these alternatives. The alternatives have been prioritized by using the two approaches. The achievements indicated that the robust LINMAP approach can be a more flexible and reliable method.

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