Feasible Trajectory Planning Algorithm for a Skid-Steered Tracked Mobile Robot Subject to Skid and Slip Phenomena

Feasible trajectory planning algorithm for a skid-steered tracked mobile robot subject to skid and slip phenomena

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***Abstract*—This paper proposes an approach for the motion planning of a constrained skid-steered tracked mobile robot under the hypothesis of non-negligible skid and slip phenomena. Operating environment is firstly discretized with a finite dimensional grid. Then, a weighted graph is defined whose nodes are the above mentioned grid points, and whose arcs denote the trajectory segments. A modified shortest path search algorithm is then proposed to find a trajectory, in terms of succession of arcs, connecting starting and ending nodes. Trajectory feasibility is guaranteed by recurring to set-based arguments. In order to show the effectiveness of the proposed approach, some numerical examples are finally discussed.**

I. Introduction

Nowadays, skid-steered mobile robots play a key role in several applications fields such as agricultural [1], rescue missions [2], surveillance [3], land-mine detection [4].

In a skid-steered mobile robot, the steering action is performed by controlling the differential velocity of left and right side of the vehicle [5]. Skid-steering drive mechanism offers significant advantages compared to alternative steering mechanisms: it is mechanically robust and simple, providing at the same time a good manoeuvrability (i.e. zero radius turning capability).

Such kind of locomotion scheme poses some problems from the point of view of motion control due to the presence of non negligible skidding and slipping effects. Classical mathematical representations of a mobile robot involve nominal nonholonomic kinematic equations for the mobility description [6]. Nonholonomic constraints are violated by the presence of skid and slip phenomena, thus requiring different mathematical models to be considered. In [8] it is introduced a classification of mobile robots based upon the influence of skidding and slipping effects on the kinematic model.

In literature two main alternative approaches are proposed to properly take into account skidding and slipping effects. As shown in [9], [10], [11], [12] reliable representations involve dynamic model of skid-slip phenomena providing the possibility to reach best accuracy at the price of significant growth of complexity of mathematical models [13]. An alternative approach lies in kinematic description of effects of skid-slip phenomena. Such an approach is attractive from a practical point of view, seeming reasonable since skidding and slipping effects can be viewed at the kinematic level in terms of disturbances of control velocities [14], [15].

The presence of skid and slip phenomena poses some difficulties when the prediction of motion of vehicle is required [16]. Thus, the computation of a feasible trajectory may become a non-trivial task. Feasibility denotes that starting and ending points, the path and control constraints and the equations of motion are all satisfied taking into account the presence of skid and slip phenomena. Generally, a trajectory planning algorithm computes a path on which a timing law is specified so that the mobile robot is able to reach the target point in a prescribed time avoiding sensed obstacles. Optimality and smoothness can be required at the price of increasing the complexity of trajectory generation problem [17]. Moreover, the complexity of trajectory planning problem depends on the assumptions related to the operating environment (i.e. a-priori known obstacles, static or dynamic environment). In [18] the solutions of motion planning problem is categorized as follows: planning by construction (PBC) and planning by modification (PBM). PBC method extends a trajectory by attaching new points until the target is reached. Due to discretization of operating environment, obtained trajectories are typically non-smooth. Different examples of PCB approach can be seen in [17], [19], [20] and references therein. PBM method perturbs a given trajectory such that a set of prescribed properties are fulfilled. Such kind of approach can provide smoother trajectories but typically it requires the solution of highly non-convex optimization problem [18], [21].

This paper tackles the problem of computing an optimal feasible trajectory in a known cluttered environment for a constrained skid-steering tracked mobile robot in presence of bounded skid and slip phenomena. Operating environment is firstly discretized with a finite dimensional grid and a weighted graph is defined whose nodes are the above mentioned grid points whereas arcs denote the trajectory segment to cross with a given velocity. A shortest path search algorithm is considered in order to find optimal trajectory, in terms of a succession of arcs, connecting two given nodes. The feasibility of trajectory is guaranteed by recurring to conservative set-

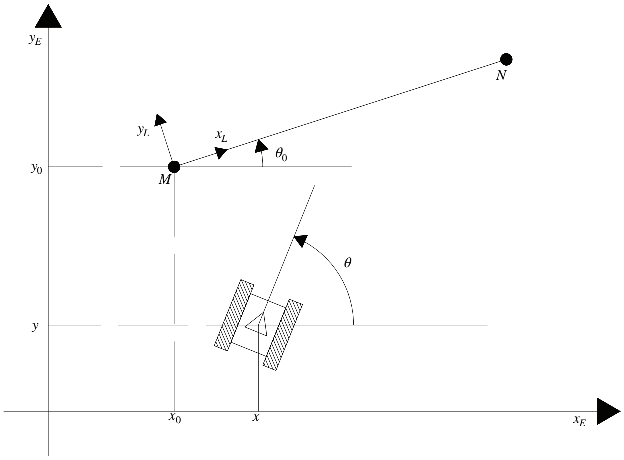

Fig. 1. Coordinate Frames for the skid-steering tracked robot.

based arguments taking into account prescribed constraints involving control inputs and trajectory tracking error. In Section II mathematical modelling of skid-steered mobile robot is considered. In Section III the tracking trajectory control design for the mobile robot is proposed. In Section IV the feasible motion planning algorithm is introduced. Finally, in Section V numerical examples are discussed in order to show the effectiveness of the proposed approach.

II. Mathematical modelling

Be $q = \begin{bmatrix} x \, y \, \theta \end{bmatrix}^T$ the vector of coordinates for a skid-steered tracked mobile robot expressed in a given inertial reference frame **E**, see Fig.1. The classical first-order kinematic model is

$$\dot{q} = G(q) \cdot \tilde{u} \tag{1}$$

being

$$G(q) = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \tag{2}$$

and $\tilde{u} = \begin{bmatrix} \tilde{V} \, \tilde{\omega} \end{bmatrix}^T$ where $\tilde{V}$ ($\tilde{\omega}$) denotes the *effective* forward (rotational) velocity. According to the reference coordinates shown in Fig.1 the kinematic relation between $\tilde{u}$ and the vector of effective angular velocities of the tracks sprockets $\tilde{w} = \begin{bmatrix} \tilde{w}_R \, \tilde{w}_L \end{bmatrix}^T$ is:

$$\tilde{u} = J \cdot \tilde{w} \tag{3}$$

being

$$J = \begin{bmatrix} R/2 & R/2 \\ R/d & -R/d \end{bmatrix} \tag{4}$$

where R represents the radius of track sprocket and d the distance between the two tracks. During rotation a skid-steered tracked vehicle experiences both skidding (inner wheel) and slipping (outer wheel) effects. According to [22], the skidding and slipping effects can be modelled as terrain-dependent possibly time-varying friction coefficients μ_R and μ_L for right and left track respectively. A kinematic relation between $\tilde{w}$ and the controlled tracks sprockets angular velocities $w = \begin{bmatrix} w_R \, w_L \end{bmatrix}^T$ is

$$\tilde{w} = H(\mu) \cdot w \tag{5}$$

where

$$H(\mu) = \begin{bmatrix} \mu_R & 0 \\ 0 & \mu_L \end{bmatrix} \tag{6}$$

and $\mu = \begin{bmatrix} \mu_R \, \mu_L \end{bmatrix}^T$. Finally, by assembling (1)-(5) the following nonlinear kinematic model representing the motion of a skid-steered tracked robot in presence of skidding and slipping effects is considered

$$\dot{q} = G(q) \cdot J \cdot H(\mu) \cdot J^{-1} \cdot u = f(q, \mu, u) \tag{7}$$

being $u = \begin{bmatrix} V \ \omega \end{bmatrix}^T$ the vector where V (ω) denotes the forward (rotational) control velocity. Consider a local reference system **L**, as shown in Fig.1. Be $q_0 = \begin{bmatrix} x_0 & y_0 & \theta_0 \end{bmatrix}^T$. The following transformation from **E** to **L** holds

$$q_L = R_E^L(\theta_0) \cdot (q - q_0) \tag{8}$$

being

$$R_E^L(\theta_0) = \begin{bmatrix} cos(\theta_0) & sin(\theta_0) & 0 \\ -sin(\theta_0) & cos(\theta_0) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Let

$$T^D(\cdot) = \begin{bmatrix} q^D(\cdot) & u^D(\cdot) \end{bmatrix}$$

be a desired trajectory expressed in **L** reference system. $T^D(\cdot)$ is defined in terms of feasible couples of state and control inputs compliant with non-holonomic constraints (1) over the time window $[0, \hat{t}]$ such that $q^D(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$ and $u^D(t) = \begin{bmatrix} V^D & 0 \end{bmatrix}$. V^D represents the mobile robot forward velocity. The desired state trajectory at the time $t \in [0, \hat{t}]$ is denoted as follows: $q^D(t) = \begin{bmatrix} V^D \cdot t & 0 & 0 \end{bmatrix}^T$. By recurring to classical linearization and discretization arguments the following discrete linear time invariant system, representing trajectory tracking error dynamic, is obtained:

$$e(t_{k+1}) = Ae(t_k) + B\delta u(t_k) + B_D d(t_k) \tag{9}$$

where A, B and B_D are matrices of appropriate dimensions, $e = q - q_D$, $\delta u = u - u_D$, $d = \mu - \mu_D$, being μ_D the nominal value of friction coefficients and

$$T_s = t_{k+1} - t_k \ \forall k \geqslant 0$$

the sampling time.

III. Trajectory tracking and control design

Given the discrete linear time invariant representation of tracking error (9), the following ellipsoidal constraints are considered:

$$e(t) \in \Omega_e, \ \Omega_e \triangleq \{e \in \mathcal{R}^3 : e^T e \leqslant e_{max}^2\} \tag{10}$$

$$\delta u(t) \in \Omega_u, \ \Omega_u \triangleq \{\delta u \in \mathcal{R}^2 : \delta u^T \delta u \leqslant u_{max}^2\} \tag{11}$$

Consider the following problem:
Constrained Control Problem (CCP) - *given (9) find the state feedback control action $\delta u(\cdot) = K \cdot e(\cdot)$ fulfilling the*

prescribed constraints (10)-(11) for all the realizations of external disturbance $d \in \Omega_D$ being

$$\Omega_D \triangleq \{d \in \mathcal{R}^2 : d^T d \leqslant d_{max}^2\} \tag{12}$$

Solution of **CCP** can be tackled by the following double step procedure:

1 according to [23], compute a stabilizing state-feedback control law $\delta u(\cdot) = Ke(\cdot)$ accomplishing (10) and (11) within the ellipsoidal positively invariant region

$$\Gamma_0 = \{e \in \mathcal{R}^3 : e^T P_0 e \leqslant 1 \;\; P_0 \geqslant 0\} \tag{13}$$

2 according to [24], by recurring to Pontryagin difference arguments, define the maximal ellipsoidal d-invariant subset

$$\Gamma_\infty = \{e \in \mathcal{R}^3 : e^T P_\infty e \leqslant 1 \;\; P_\infty \geqslant 0\} \subseteq \Gamma_0 \tag{14}$$

such that

$$e(t_k) = \Phi^k e(t_0) + \sum_{h=0}^{k-1} \Phi^{k-1-h} B_D d_h \in \Gamma_0 \quad \forall t_k \geqslant 0$$

$\forall e(t_0) \in \Gamma_\infty$ and $\forall d(t_j) \in \Omega_D$, $t_j < t_k$ being

$$e(t_{k+1}) = \Phi e(t_k) + B_D d(t_k) \tag{15}$$

the closed loop tracking error dynamic with $\Phi = (A + B \cdot K)$.

The above two step procedure provides a solution (K, Γ_∞) of **CCP** in terms of feedback control action $\delta u(\cdot) = Ke(\cdot)$ within Γ_∞ fulfilling the prescribed constraints (10), (11) for all admissible $d \in \Omega_D$.

IV. Motion planning algorithm

Assume the trajectory tracking problem was tackled as shown in Section III and a solution of *CCP* was provided in terms of a couple (K, Γ_∞).

Solving a trajectory planning problem implies the solution of a path planning problem with a prescribed timing law. Assume a 2D operational scenario $\Delta \subseteq \mathcal{R}^2$.

Δ is firstly discretized with a finite dimensional grid of feasible positions. An undirected weighted graph $\mathcal{G}$ is then defined whose nodes $V \in R^2$ are the above mentioned grid points.

Def. Δ-compatibility. Two nodes $E_1 = (x_1, y_1) \in V$ and $E_2 = (x_2, y_2) \in V$ are Δ-compatible if $\forall \alpha \in [0, 1]$

$$P_\alpha = (x_\alpha, y_\alpha) \in \Delta$$

with $x_\alpha = (1 - \alpha)x_1 + \alpha x_2$ and $y_\alpha = \eta_\alpha x_\alpha + \tau_\alpha$ being

$$\eta_\alpha = \frac{y_1 - y_2}{x_1 - x_2}$$

and

$$\tau_\alpha = \frac{x_1 y_2 - x_2 y_1}{x_1 - x_2}$$

Arcs of graph $\mathcal{G}$ are defined as follows: be $A, B \in V$ two Δ-compatible nodes. Arc connecting A and B is denoted as

$$T^{AB}(\cdot) = \begin{bmatrix} q^{AB}(\cdot) & \tilde{u}^{AB} \end{bmatrix}$$

It represents, in a local reference frame centred in A and such that B belongs to robot positive x-axis, the trajectory crossing the segment $\bar{AB}$ at the constant velocity V_{AB} for N_{AB} time steps with a null angular velocity being $q^{AB}(t) = \begin{bmatrix} V_{AB} \cdot t & 0 & 0 \end{bmatrix}$ and $\tilde{u}^{AB} = [V_{AB} \; 0]^T$ solution of (1) over the time horizon $t \in [0, T_s \cdot N_{AB}]$. N_{AB} is the maximum positive integer such that

$$N_{AB} \leqslant \frac{d_{AB}}{V_{AB} \cdot T_s}$$

Arc cost is assumed to be d_{AB} the length of segment $\bar{AB}$.

In this paper an $A* - like$ algorithm is adopted to compute a shortest path (if any) in terms of a succession of arcs connecting starting $V_0 \in V$ and ending $V_F \in V$ nodes. $A*$ algorithm is a classical approach in searching the best path in a graph [25]. It relies upon exploration of the most promising arc according to an heuristic function estimating the lower-bound of the cost of path including arcs to be explored. As shown in [26], $A*$ guarantees the exploration of fewer nodes than any other algorithm using the same heuristic if the heuristic function never overestimates the cost of path. Hereinafter, we will assume as heuristic function the sum of two terms: a) the cost of computed path; b) the euclidean distance between the last explored node and destination node. If a succession of arcs W_{OF} connecting V_O and V_F is explored, cost of path γ_{OF} becomes an upper bound of the heuristic function. Thus, a path stops being explored when its heuristic exceeds the current upper bound. Obviously, upper bound must be updated if a shorter path reaching the destination node is found. Given three nodes $A, B, C \in V$ suppose the segments A, B and B, C are both Δ-compatibles. Consider a path including the two adjacent arcs T^{AB} and T^{BC}, see Fig.2. If the robot has to track planned trajectory, a switch from T^{AB} to T^{BC} at switching time $t_{N_{AB}} = T_s \cdot N_{AB}$ is required. Switching is considered admissible if the following condition is fulfilled:

$$(e(t_{N_{AB}}) + \Pi) \in \Gamma_\infty \tag{16}$$

where $e(t_{N_{AB}})$ is the tracking error at switching time and

$$\Pi = \begin{bmatrix} x_B - x^D(t_{N_{AB}}) & y_B - y^D(t_{N_{AB}}) & \delta_\theta \end{bmatrix}^T$$

A trajectory is feasible if every switch between trajectory segments is admissible, feasibility is guaranteed by a conservative check involving admissibility of switches between arcs. Firstly consider the following result:

Lemma 1: Given the closed loop dynamic (15). Suppose the tracking error at the time instant $e(t_0) \in S_A$ being

$$S_A = \{e \in \Gamma_\infty : e^T P_A e \leqslant 1, \, P_A \geqslant 0\} \tag{17}$$

The ellipsoidal set

$$S_{N_{AB}} = \{e \in \Gamma_\infty : e^T \Psi e \leqslant 1, \, \Psi \geqslant 0\} \tag{18}$$

representing the set of allowable tracking error $e(t_{N_{AB}})$ for every admissible realization of the disturbance $d(t_h) \in \Omega_D$ with $h = 0 \cdots (N_{AB} - 1)$ can be obtained by solving the following SDP minimization problem:

$$\min_{\Psi, \tau_0, \cdots \tau_{N_{AB}}} \quad logdet \Psi^{-1} \tag{19}$$

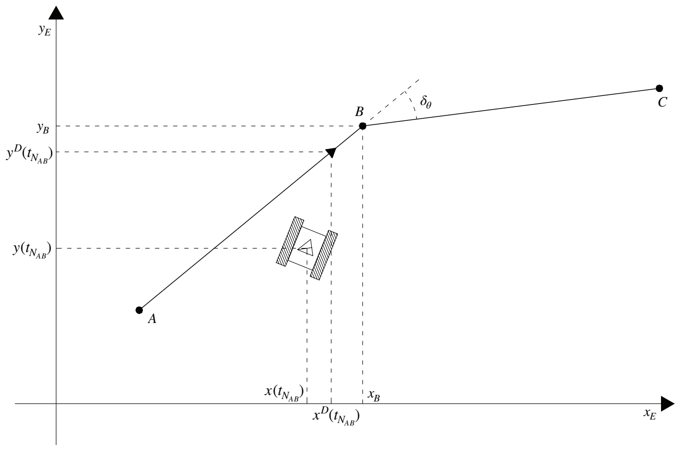

Fig. 2. Trajectory switch.

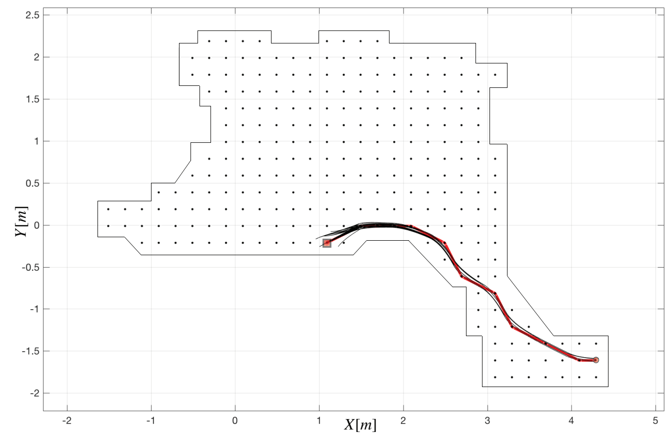

Fig. 3. 2D operating environment and approximating grid points. Red line denotes the optimal feasible trajectory connecting starting (square) and ending (circle) points. Black solid lines represent simulated tracked mobile robot trajectories at varying initial conditions and friction coefficients.

s.t.

$$1 - \sum_{h=0}^{N_{AB}-1} \tau_h d_{max}^2 - \tau_{N_{AB}} \geqslant 0 \quad (20)$$

$$\begin{bmatrix} -\hat{\Phi}^T \Psi \hat{\Phi} + \tau_{N_{AB}} P_A & -\hat{\Phi}^T \Psi H \\ * & -H^T \Psi H + \sum_{h=0}^{N_{AB}-1} \tau_h J_h^T J_h \end{bmatrix} \geqslant 0 \quad (21)$$

being τ_h with $h = 0 \cdots N_{AB}$ positive scalars, $\Psi \geqslant 0$, $\hat{\Phi} = \Phi^{N_{AB}}$, $H = \begin{bmatrix} \Phi^{N_{AB}-1} B_D & \Phi^{N_{AB}-2} B_D & \cdots & B_D \end{bmatrix}$, J_h the matrix of proper dimension such that $J_h \underline{d} = d(t_h)$ being $d = \begin{bmatrix} d(t_0)^T & d(t_1)^T & \cdots & d(t_{N_{AB}})^T \end{bmatrix}^T$

Proof - Condition

$$e(t_{N_{AB}})^T \Psi e(t_{N_{AB}}) \leqslant 1$$

with $e(t_{N_{AB}}) = \hat{\Phi} e(t_0) + H \underline{d}$ and $\Psi \geqslant 0$ can be recast in the following form

$$\begin{bmatrix} 1 & e(t_0)^T & \underline{d}^T \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ * & -\hat{\Phi}^T \Psi \hat{\Phi} & -\hat{\Phi} \Psi H \\ * & * & -H^T \Psi H \end{bmatrix} \begin{bmatrix} 1 \\ e(t_0) \\ \underline{d} \end{bmatrix} \geqslant 0 \quad (22)$$

Condition $e(t_0)^T P_A e(t_0) \leqslant 1$ with $P_A \geqslant 0$ can be recast in the following form

$$\begin{bmatrix} 1 & e(t_0)^T & \underline{d}^T \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ * & -P_A & 0 \\ * & * & 0 \end{bmatrix} \begin{bmatrix} 1 \\ e(t_0) \\ \underline{d} \end{bmatrix} \geqslant 0 \quad (23)$$

Condition

$$(J_h \underline{d})^T (J_h \underline{d}) \leqslant d_{max}^2$$

with $h = 0 \cdots (N_{AB} - 1)$ can be recast in the following form

$$\begin{bmatrix} 1 & e(t_0)^T & \underline{d}^T \end{bmatrix} \begin{bmatrix} d_{max}^2 & 0 & 0 \\ * & 0 & 0 \\ * & * & -J_h^T J_h \end{bmatrix} \begin{bmatrix} 1 \\ e(t_0) \\ \underline{d} \end{bmatrix} \geqslant 0 \quad (24)$$

By recurring to classical *S-procedure* arguments, LMIs (20)-(21) can be readily obtained.

□

Given a solution of Lemma 1, tracking error switch Π is assumed admissible

$$S_{N_{AB}} \bigoplus \Pi \subseteq \Gamma_\infty$$

and then T^{BC} can follow T^{AB} in a feasible trajectory if there exists $\tau > 0$ such that the following LMI holds

$$\begin{bmatrix} 1 - \tau - \Pi^T P_\infty \Pi & * \\ -P_\infty \Pi & -P_\infty + \tau P_A \end{bmatrix} \geqslant 0 \quad (25)$$

Notice that (25) can be stated by exploiting the same arguments of Lemma 1. All the above considerations can be readily recast into a recursive fashion algorithm.

V. Numerical results

Consider a skid-steered tracked mobile robot with a radius of track sprocket $R = 8cm$ and a distance between the two tracks $d = 50cm$. The following constraints are considered:

a the terrain-dependent friction coefficients are supposed bounded

$$\mu_R, \mu_L \in (0.7, 1.2) \quad (26)$$

b the following constraints on the control velocities are considered

$$0 \leqslant V \leqslant 1.4 \left[m/s \right] \quad (27)$$

$$-2.15 \leqslant \omega \leqslant 2.15 \left[rad/s \right] \quad (28)$$

The two-dimensional space domain Δ is shown in Fig.3. It was discretized by recurring to a regular grid of $0.2m$ with about 270 resulting nodes. In order to construct the graph $\mathcal{G}$ a maximum length of $0.5m$ is assumed for trajectory segment connecting two $\Delta - compatibles$ nodes. Finally, a nominal forward velocity is assumed constant along the trajectory segments $V^D = 0.7$ [*m/s*].

According to Section III, be (K, Γ_∞) the solution of the *Constrained Control Problem (CCP)* .

First simulation scenario is shown in Fig.3 Red solid line represents the optimal feasible trajectory resulting from the

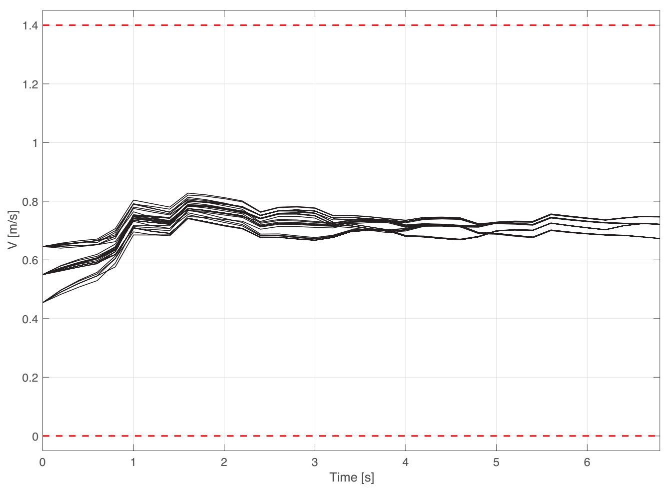

Fig. 4. Robot forward velocity.

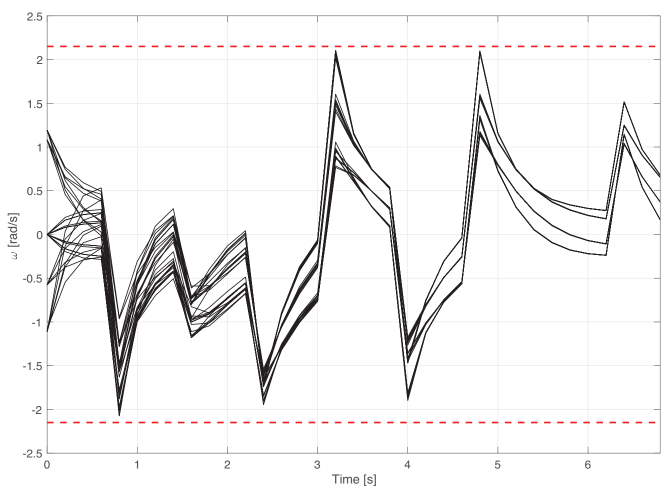

Fig. 5. Robot angular velocity.

proposed algorithm connecting starting (square) and ending (circle) nodes. The whole trajectory includes 10 segments with an overall length of about $3.93m$. Black solid lines denote the robot trajectories obtained by means of numerical simulations at varying robot initial pose and constant friction coefficients according to (26). Figs.4-5 show the control velocities.

A second simulation scenario is shown in Fig.6. Three additional obstacles (black boxes) are considered in the operating environment. Square denotes the starting point while the circle is the ending point. Red line denotes an optimal feasible trajectory including 14 segments with a full length of about $6.5m$. Notice that the proposed approach does not exclude trajectory loops. In such a scenario all the shortest trajectories are non-admissible and the optimal one requires a loop to perform required $90deg$ turn. Black solid lines represents the mobile robot trajectory. Friction coefficients assumed in this simulation are shown in Fig.7 whereas Figs.8 and 9 show the control velocities.

In both considered scenarios, as expected, the tracked mobile robot is able to follow the whole trajectory fulfilling all the prescribed constraints.

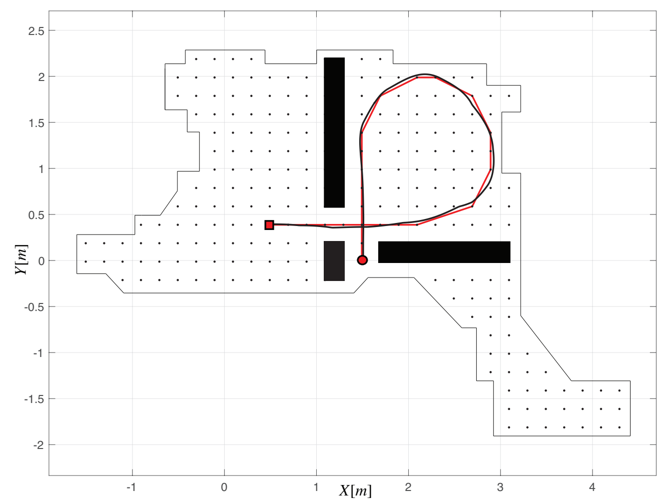

Fig. 6. 2D operating environment with obstacles (black boxes). Red line denotes the optimal feasible trajectory connecting starting (square) and ending (circle) points. Black solid line represents tracked mobile robot trajectory.

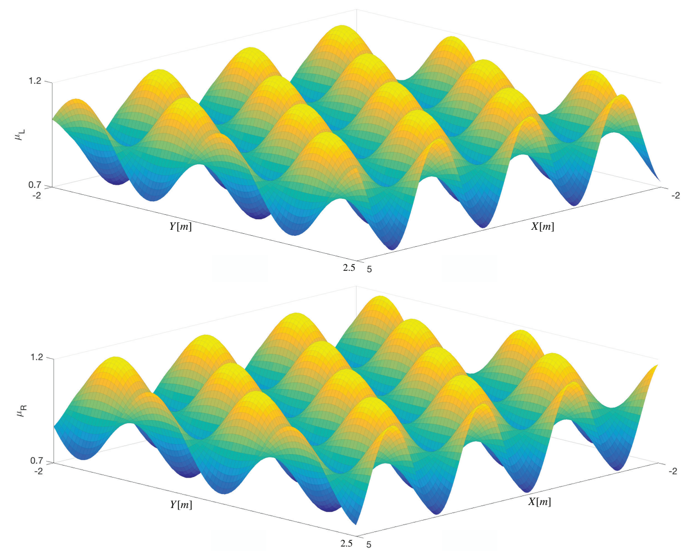

Fig. 7. Friction coefficients.

VI. Conclusions

In this paper the problem of motion planning of a skid-steered tracked mobile robot subject to skid and slip phenomena is tackled. A feedback control action is firstly designed to achieve trajectory tracking of mobile robot accounting for constraints on trajectory tracking error and robot control velocities. In order to find optimal feasible trajectory in terms of succession of segments to cross with an assigned nominal velocity, a procedure based on shortest trajectory search algorithm is proposed. Trajectory feasibility is guaranteed by recurring to set based arguments involving the solution of SDP minimization problems. Finally, some numerical simulations are discussed to show effectiveness of the proposed approach.

Fig. 8. Robot forward velocity.

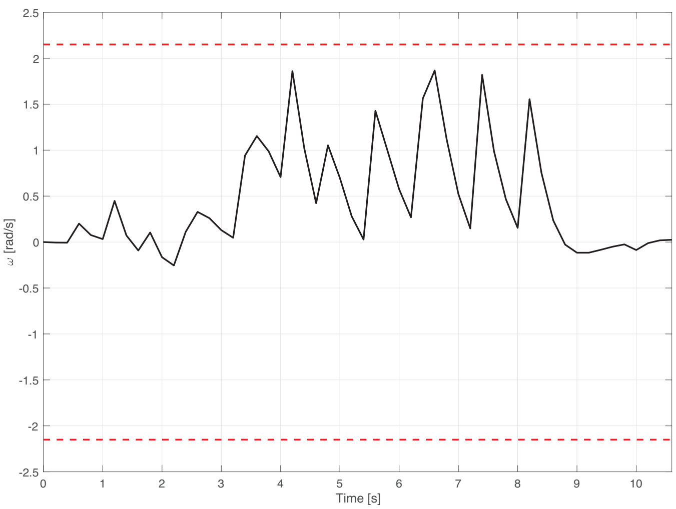

Fig. 9. Robot angular velocity.

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