A feasible trajectory planning algorithm for a network controlled robot subject to skid and slip phenomena

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***Abstract*—This paper aims to propose a feasible solution to the trajectory planning problem for a constrained skid-steering mobile robot whose control system, sensors and actuators are connected through a communication network. Operating environment is firstly discretized by a finite dimensional grid. Then a weighted graph, whose nodes are the above mentioned points and whose arcs denote the trajectory segments connecting points is defined. Finally an algorithm to obtain the shortest feasible succession of segments connecting given starting and ending points is proposed. Trajectory feasibility is guaranteed in terms of sufficient conditions involving the solution of semi-definite programming (SDP) problems. In order to show the effectiveness of the proposed approach, some numerical simulations are proposed.**

I. Introduction

Nowadays, in several interesting applications involving mobile robots, it is appealing to consider the network implementation of control systems in which feedback control loop is closed through a communication network [1], [2], [3] [4], [5]. The integration of control systems with computer networks, also called networked control system (NCS), offers valuable opportunities, including higher system testability and resource utilization, cost reduction, flexibility in control architecture design and simplified installation and maintenance. At the same time, building a control system supported by a communication network is a challenging task because the communication network introduces non negligible effects that may cause closed loop performance degradation [6] [7] [8]. These effects, including data losses, variable delays and data corruption are not commonly dealt with classical control methods. For such reasons, the past decades have witnessed an explosive growth of the number of research contributions in the area of networked control systems [9], [10], [11], [12], [13].

This paper will address the task of developing a trajectory planning algorithm capable to provide a feasible trajectory for a tracked mobile skid-steering robot where the trajectory tracking control loop is closed through a wireless communication network.

A trajectory planning algorithm finds a path on which a timing law is assigned so that the mobile robot is able to reach the target point in a given time. The complexity of trajectory planning problem depends on the assumptions related to the operating environment (i.e. a-priori known obstacles, static or dynamic environment). Moreover, in a skid-steering mobile robot, the steering action is performed by controlling the velocities of the left and the right side of vehicle respectively [14]. Skid-steering drive scheme is affected by several problems from motion control point of view since the presence of non-negligible skidding and slipping phenomena makes the prediction of robot motion a non-trivial task. [15].

In literature solutions of motion planning problem can be classified in two main categories [16]: planning by construction and planning by modification. First method extends a trajectory by attaching new waypoints until the target is reached [17]. Obtained trajectories are typically non-smooth due to the discretization of operating environment, see [18], [19], [20] and references therein. Planning by modification method, instead, perturbs a given trajectory such that a set of prescribed properties is fulfilled. Such kind of approach can provide smoother trajectories but typically it requires the solution of highly non-convex optimization problems [16], [21].

This paper tackles the problem of computing an optimal feasible trajectory in a-priori known environment for a constrained network-controlled mobile robot in presence of bounded skid and slip phenomena. The proposed solution requires a preliminary discretization of operating environment by a finite dimensional grid points. Then a weighted graph is defined. Graph nodes are the above mentioned points while arcs denote trajectory segments connecting points. An algorithm to find optimal trajectory is finally introduced being optimal trajectory obtained in terms of a succession of arcs, connecting starting and ending points. Trajectory feasibility is guaranteed by recurring to set-based arguments taking into account constraints on control inputs, trajectory tracking error and network delay.

In Section II mathematical modelling of skid-steering network-controlled mobile robot is proposed. In Section III the trajectory tracking control design for a network controlled

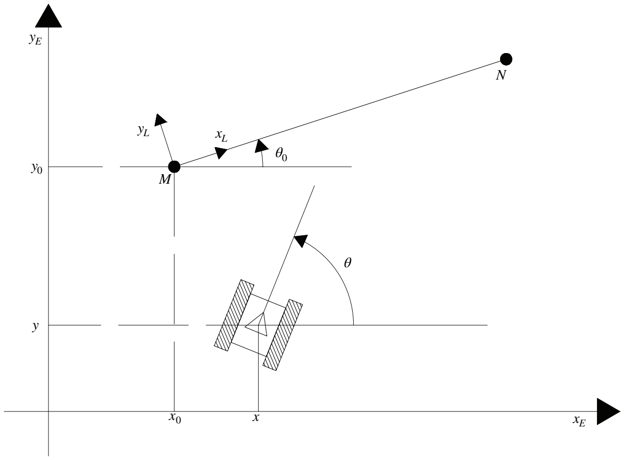

Fig. 1. Coordinate Frames for the skid-steering tracked robot.

robot is tackled. In Section IV the feasible motion planning algorithm is introduced. Finally, in Section V numerical simulations are discussed in order to show the effectiveness of the proposed algorithm.

II. Mathematical modelling

Given the inertial reference frame **E**, see Fig.1, let

$$q = \begin{bmatrix} x \, y \, \theta \end{bmatrix}^T$$

be the robot pose vector. The nominal kinematic first-order model is

$$\dot{q} = G(q) \cdot \tilde{u} \tag{1}$$

with

$$G(q) = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \tag{2}$$

being $\tilde{u} = \begin{bmatrix} \tilde{V} \, \tilde{\omega} \end{bmatrix}^T$ where $\tilde{V}$ and $\tilde{\omega}$ are the *effective* forward and rotational robot velocities respectively. The relationship between $\tilde{u}$ and the vector of effective angular velocities of the track sprockets $\tilde{w} = \begin{bmatrix} \tilde{w}_R \, \tilde{w}_L \end{bmatrix}^T$ can be written in the following form

$$\tilde{u} = J \cdot \tilde{w} \tag{3}$$

being

$$J = \begin{bmatrix} R/2 & R/2 \\ R/D & -R/D \end{bmatrix} \tag{4}$$

where D is the distance between tracks and R is the radius of track sprockets.

While rotating, the vehicle experiences both skidding (inner track) and slipping (outer track) effects. These effects can be modelled as terrain-dependent possibly time-varying friction coefficients μ_R and μ_L for right and left track respectively [22]. The relationship between $\tilde{w}$ and the controlled track sprockets angular velocities $w = \begin{bmatrix} w_R \, w_L \end{bmatrix}^T$ is shown below

$$\tilde{w} = H \cdot w \tag{5}$$

where

$$H = \begin{bmatrix} \mu_R & 0 \\ 0 & \mu_L \end{bmatrix} \tag{6}$$

Equations (1)-(6) can be recast into the following nonlinear model

$$\dot{q} = G(q) \cdot J \cdot H \cdot J^{-1} \cdot u \tag{7}$$

being $u = \begin{bmatrix} V \, \omega \end{bmatrix}^T$ the vector where V and ω denote the forward and rotational control velocities respectively.

Let **L** be a reference system whose origin is located in M with x_L directed as the $\overline{MN}$ segment as shown in Fig.1. The roto-translation from **E** to **L** is expressed by the following equation

$$q_L = R_E^L(\theta_0) \cdot (q - q_0) \tag{8}$$

with

$$R_E^L(\theta_0) = \begin{bmatrix} cos(\theta_0) & sin(\theta_0) & 0 \\ -sin(\theta_0) & cos(\theta_0) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and $q_0 = \begin{bmatrix} x_0 & y_0 & \theta_0 \end{bmatrix}^T$.

Be

$$T_L^D(\cdot) = \{ q_L^D(\cdot) \, , \, u_L^D(\cdot) \}$$

the desired trajectory expressed in **L**. $T_L^D(\cdot)$ has the form of couples pose and control inputs such that $q_L^D(\cdot)$ is compliant with (7) over the horizon $[0, \hat{t}]$ by assuming

$$u_L^D(\cdot) = \begin{bmatrix} V_L^D & \omega_L^D \end{bmatrix}^T \tag{9}$$

being V_L^D and ω_L^D the nominal mobile robot forward and rotational velocities respectively where

$$\omega_L^D = \frac{2V_L^D}{D} \frac{\mu_R^D - \mu_L^D}{\mu_R^D + \mu_L^D}$$

being μ_R^D and μ_L^D nominal value of friction coefficients. In this paper the classical Networked Control System schema represented in Fig.2 is considered. The control input is assumed to be held to its previous value as long as a new packet from the controller is received. The sensors are regularly sampled with a constant time T_s.

The communication delays between sensors and controller τ_1 and between controller and actuators τ_2 along with computational delay τ_c can be embedded in a single term

$$\tau_k := \tau_1^k + \tau_2^k + \tau_c^k \tag{10}$$

hereinafter called network delay.

In this work a reliable connection-oriented and ordered communication protocol is assumed, namely the TCP/IP stack [23]. Under the hypothesis that τ_1^k and τ_2^k are both always shorter than the relevant TCP max retransmission window duration, packet loss can be assumed to be zero. Let $\bar{d}$ be the smallest integer such that the following inequality holds

$$\bar{d} \geqslant \frac{\tau_{max}}{T_s} \tag{11}$$

where τ_{max} is the maximum network delay.

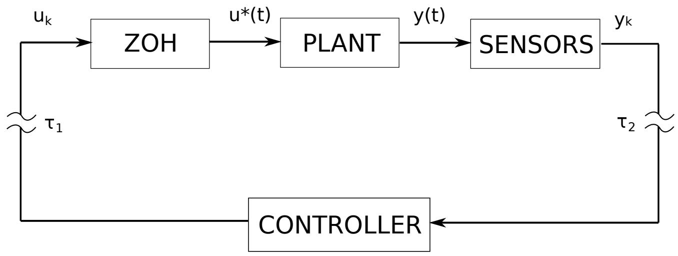

Fig. 2. Classical NCS control schema.

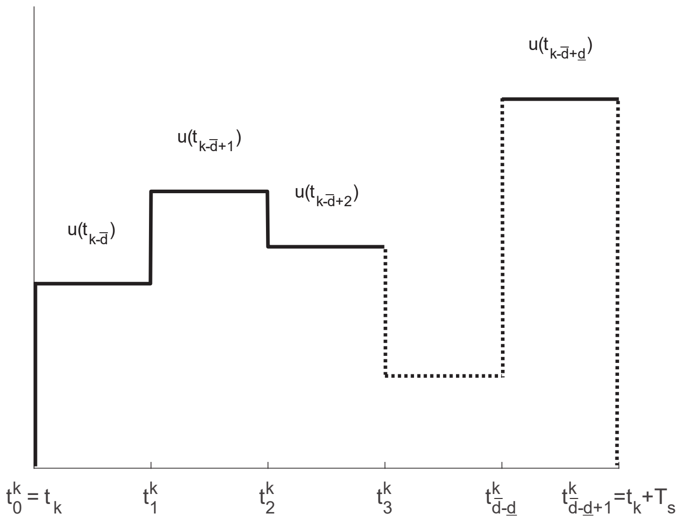

Fig. 3. Actuation update instants t_j^k

Let $\underline{d}$ be the largest integer such that

$$\underline{d} \leqslant \frac{\tau_{min}}{T_s} \tag{12}$$

where τ_{min} is the minimum network delay. The following discrete linear time invariant system representing trajectory tracking error dynamic is obtained [24]

$$x(t_{k+1}) = e^{AT_s}x(t_k) + \sum_{j=0}^{\bar{d}-\underline{d}} \int_{T_s - t_{j+1}^k}^{T_s - t_j^k} e^{As} ds B \delta u(t_{k-j-\bar{d}}) + \int_0^{T_s} e^{As} ds B_D d(t_k) \tag{13}$$

where A, B and B_D are matrices of appropriate dimensions obtained by applying the classical linearization and discretization arguments being $x(t_k) = q(t_k) - q_L^D(t_k)$, $\delta u(t_k) = u(t_k) - u_L^D(t_k)$ and

$$d(t_k) = \begin{bmatrix} \mu_R(t_k) - \mu_R^D(t_k) & \mu_L(t_k) - \mu_L^D(t_k) \end{bmatrix}^T \tag{14}$$

According to the Fig.3, be θ_k the vector of actuation update instants

$$\theta_k := \begin{bmatrix} t_1^k & \cdots & t_{\bar{d}-\underline{d}}^k \end{bmatrix}^T \tag{15}$$

where

$$0 = t_0^k \leqslant t_1^k \leqslant \cdots \leqslant t_{\bar{d}-\underline{d}+1} = T_s \tag{16}$$

Be recurring to NCS theory [24], delay-free linear discrete time model of (13) can be expressed as

$$\xi(t_{k+1}) = \tilde{A}(\theta_k)\xi(t_k) + \tilde{B}(\theta_k)u(t_k) + B_D d(t_k) \tag{17}$$

in terms of the lifted state vector

$$\xi(t_k) = \begin{bmatrix} x(t_k)^T & u(t_{k-1})^T & \cdots & u(t_{k-\bar{d}})^T \end{bmatrix}^T \tag{18}$$

with

$$\tilde{A}(\theta_k) = \begin{bmatrix} \Lambda & M_{\bar{d}-1}(\theta_k) & M_{\bar{d}-2}(\theta_k) & \cdots & M_1(\theta_k) & M_0(\theta_k) \\ 0 & 0 & \cdots & \cdots & 0 & 0 \\ 0 & I & \cdots & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 & 0 \\ 0 & 0 & \cdots & 0 & I & 0 \end{bmatrix} \tag{19}$$

$$\tilde{B}(\theta_k) = \begin{bmatrix} M_{\bar{d}}(\theta_k) \\ I \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{20}$$

with $\Lambda = e^{AT_s}$. Matrices $M_j(\theta_k)$ are computed as

$$M_{\bar{d}}(\theta_k) = \int_{T_s - t_{j+1}^k}^{T_s - t_j^k} e^{As} ds B \tag{21}$$

for $j \in \begin{bmatrix} 0 & \bar{d} - \underline{d} \end{bmatrix}$ while $M_{\bar{d}}(\theta_k) = 0$ for $j \in (\bar{d} - \underline{d} \quad \bar{d}]$.

The equation (17) can be embedded in a suitable structure by a polytopic overapproximation. In this work, polytopic overapproximation is performed using the real Jordan form technique as described in [24]. Thus the following polytopic discrete linear time invariant system representing trajectory tracking error dynamic is obtained

$$\xi(t_{k+1}) = A_j \xi(t_k) + B_j \delta u(t_k) + B_D d(t_k) \tag{22}$$

where A_j and B_j are matrices of appropriate dimensions with

$$j = 1, \cdots, N_v$$

being N_v the number of polytope vertices.

III. Trajectory tracking control design

In this section the trajectory tracking control design is tackled. Such a step is required to properly introduce the constraints for feasible trajectory planning task.

The following constraints are assumed

$$\xi(\cdot) \in \Omega_\xi, \ \Omega_\xi \triangleq \{\xi \in \mathcal{R}^n : \xi^T \xi \leqslant \xi_{max}^2\} \tag{23}$$

$$\delta u(\cdot) \in \Omega_u, \ \Omega_u \triangleq \{\delta u \in \mathcal{R}^2 : \delta u^T \delta u \leqslant \delta u_{max}^2\} \tag{24}$$

Let consider the **Constrained Control Problem (CCP)** - *given (22) find the state feedback control action*

$$\delta u(t) = K \cdot \xi(t) \tag{25}$$

fulfilling prescribed constraints (23)-(24) for any external disturbance realization $d(\cdot) \in \Omega_D$ *over the horizon* $[0, \bar{N}]$ *being*

$$\Omega_D \triangleq \{d \in \mathcal{R}^2 : d^T d \leqslant d_{max}^2\} \tag{26}$$

CCP can be solved by the means of a double step procedure:

1 according to [25], compute a stabilizing state-feedback control law $\delta u(t) = K \cdot \xi(t)$ fulfilling (23) and (24) within the ellipsoidal positively invariant region Γ_0

$$\Gamma_0 = \{\xi \in \mathcal{R}^n : \xi^T P_0 \xi \leqslant 1 \ \ P_0 \geqslant 0\} \tag{27}$$

2 according to [26], define the largest volume ellipsoidal subset

$$\Gamma_\infty = \{\xi \in \mathcal{R}^n : \xi^T P_0 \xi \leqslant \gamma_\infty \leqslant 1\} \subseteq \Gamma_0 \tag{28}$$

such that $\xi(t_k) \in \Gamma_0 \ \forall t_k \in [0, \bar{N}]$ being

$$\xi(t_k) = \{\Phi_j^k \xi(t_0) + \sum_{h=0}^{k-1} \Phi_j^{k-1-h} B_D d(t_h)\} \in \Gamma_0 \tag{29}$$

where $\Phi_j = (A_j + B_j \cdot K)$ with $j = 1 \cdots N_v$, $\xi(t_0) \in \Gamma_\infty$ and $d(\cdot) \in \Omega_D$.

IV. Trajectory planning algorithm

Consider the polytopic mathematical modelling of the trajectory tracking error dynamic (22) expressed in terms of the lifted state vector (18). Assume a solution of the Costrained Control Problem as shown in Section III. Let $\Delta \subseteq \mathcal{R}^2$ be the 2D operational scenario. By discretizing Δ with a finite dimensional grid of positions, it is possible to define an undirected weighted graph $\mathcal{G}$ whose nodes V represent grid points.

Def. Δ-compatibility. Two points $E_1 = (x_1, y_1)$ and $E_2 = (x_2, y_2)$ distant less than $\bar{L}$ are deemed Δ-compatible if $\forall \alpha \in [0, 1]$

$$P_\alpha = (x_\alpha, y_\alpha) \in \Delta$$

with $x_\alpha = (1 - \alpha)x_1 + \alpha x_2$ and $y_\alpha = \eta_\alpha x_\alpha + \rho_\alpha$ with

$$\eta_\alpha = \frac{y_1 - y_2}{x_1 - x_2}$$

and

$$\rho_\alpha = \frac{x_1 y_2 - x_2 y_1}{x_1 - x_2}$$

Arcs of graph $\mathcal{G}$ are defined as follows: be A and B two Δ-compatible points, arc T_L^{AB} connecting A and B represents the trajectory crossing the segment $\overline{AB}$ at the constant velocity V^{AB} for N_{AB} time steps according to Section II where N_{AB} is the highest positive integer such that

$$N_{AB} \leqslant \frac{l_{AB}}{V^{AB} \cdot T_s} \tag{30}$$

The cost of arc l_{AB} is the length of segment $\overline{AB}$.

This work exploits an $A^* - like$ algorithm in order to compute shortest path (if any) as a succession of arcs connecting starting node $V_0 \in V$ and ending node $V_F \in V$. A^* relies upon the exploration of the most promising arc according to an heuristic function estimating the lower-bound of the cost of path including arcs to be explored. If the heuristic function never overestimates path cost, as shown in [27], the algorithm guarantees the exploration of fewer nodes than any other algorithm using the same heuristic. This paper will assume as heuristic function the sum of two contributes: a) the euclidean distance between the last explored node and destination node; b) the cost of computed path to the last explored node. If a succession of arcs W_{0F} connecting V_0 and V_F is explored, cost of path β_{0F} becomes an upper bound of the heuristic function. Thus, a path stops being explored when its heuristic exceeds the current upper bound. Upper bound has to be updated if a shorter path reaching the destination node is found. Given three nodes $V_A, V_B, V_C \in V$ representing the points $A, B, C \in \Delta$ respectively, suppose that couples of points A, B and B, C are both Δ-compatibles. Consider a path including the two adjacent arcs T_L^{AB} and T_L^{BC}, see Fig.4. If the robot has to track considered trajectory, a switch from T_L^{AB} to T_L^{BC} at time $t_{N_{AB}} = T_s \cdot N_{AB}$ is required. Switch is admissible if the following condition is fulfilled:

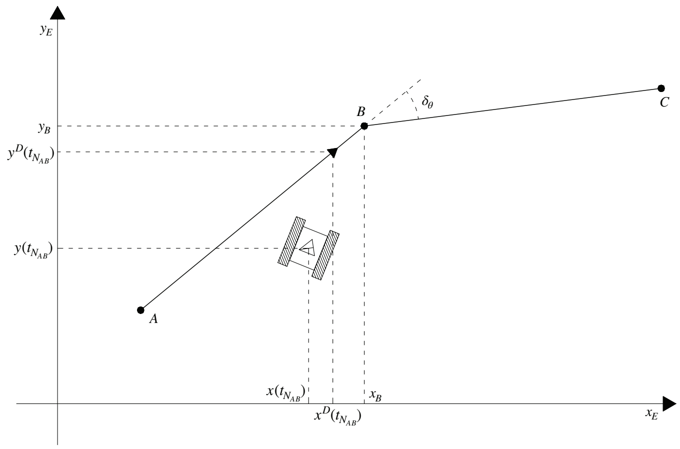

Fig. 4. Trajectory switch.

$$(\xi(t_{N_{AB}}) + \Pi) \in \Gamma_\infty \tag{31}$$

where $\xi(t_{N_{AB}})$ is the lifted state vector at the switching time and

$$\Pi = \begin{bmatrix} x_B - x^D(t_{N_{AB}}) & y_B - y^D(t_{N_{AB}}) & \delta_\theta & 0 & \cdots & 0 \end{bmatrix}^T$$

see Fig. 4 for details.

A trajectory is feasible if all switches between trajectory segments are admissible. In order to guarantee the admissibility of all switches consider the following result

Lemma 1: Given (III), suppose the initial tracking error $\xi(t_0) \in S_A$ where

$$S_A = \{\xi \in \Gamma_\infty : \xi^T P_A \xi \leqslant 1, \ P_A \geqslant 0\} \tag{32}$$

Be $S_{N_{AB}} \in \Gamma_\infty$ the smallest ellipsoidal set

$$S_{N_{AB}} = \{\xi \in \Gamma_\infty : \xi^T P_\infty \xi \leqslant \tilde{\gamma}\} \tag{33}$$

containing all allowable tracking errors $\xi(t_{N_{AB}})$ under the assumption that $d \in \Omega_D$ remains constant along the segment $\overline{AB}$ (see Remark). $S_{N_{AB}}$ can be obtained by solving the following SDP minimization problem

$$\min_{\tau_1^j, \tau_2^j, \tilde{\gamma}} \ \tilde{\gamma} \tag{34}$$

s. t.

$$0 \leqslant \tilde{\gamma} \leqslant 1 \tag{35}$$

$$\tau_1^j \geqslant 0 \tag{36}$$

$$\tau_2^j \geqslant 0 \tag{37}$$

$$\tilde{\gamma} - \tau_1^j d_{max}^2 - \tau_2^j \geqslant 0 \tag{38}$$

$$\begin{bmatrix} -\bar{\Phi}_j^T P_\infty \bar{\Phi}_j + \tau_2^j P_A & -\bar{\Phi}_j^T P_\infty \bar{\Phi}_j \\ * & -\bar{J}_j^T P_\infty J_j + \tau_1^j I \end{bmatrix} \geqslant 0 \tag{39}$$

where $\bar{\Phi}_j = \Phi_j^{N_{AB}}$,

$$\bar{J}_j = \begin{bmatrix} \Phi_j^{N_{AB}-1} B_D & \Phi_j^{N_{AB}-2} B_D & \cdots & B_D \end{bmatrix}$$

with $j = 1, \cdots, N_v$.

Proof - Condition

$$\xi(t_{N_{AB}})^T P_\infty \xi(t_{N_{AB}}) \leqslant \tilde{\gamma} \tag{40}$$

being $\xi(t_{N_{AB}}) = \bar{\Phi}_j \xi(t_0) + \bar{J}_j d$ with $j = 1, \cdots, N_v$ can be recast in the following form

$$\begin{bmatrix} 1 & \xi(t_0)^T & d^T \end{bmatrix} \begin{bmatrix} \tilde{\gamma} & 0 & 0 \\ * & -\bar{\Phi}_j{}^T P_\infty \bar{\Phi}_j & -\bar{\Phi}_j{}^T P_\infty \bar{J}_j \\ * & * & -\bar{J}_j^T P_\infty \bar{J}_j \end{bmatrix} \begin{bmatrix} 1 \\ \xi(t_0) \\ d \end{bmatrix} \geqslant 0 \tag{41}$$

with $j = 1, \cdots, N_v$.

Condition $\xi(t_0)^T P_A \xi(t_0) \leqslant 1$ with $P_A \geqslant 0$ can be recast in the following form

$$\begin{bmatrix} 1 & \xi(t_0)^T & d^T \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ * & -P_A & 0 \\ * & * & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \xi(t_0) \\ d \end{bmatrix} \geqslant 0 \tag{42}$$

Condition

$$d^T d \leqslant d_{max}^2$$

can be recast in the following form

$$\begin{bmatrix} 1 & \xi(t_0)^T & d^T \end{bmatrix} \begin{bmatrix} d_{max}^2 & 0 & 0 \\ * & 0 & 0 \\ * & * & -I \end{bmatrix} \begin{bmatrix} 1 \\ \xi(t_0) \\ d \end{bmatrix} \geqslant 0 \tag{43}$$

By recurring to classical *S-procedure* arguments, LMIs (34)-(39) can be readily obtained. □

Given a solution of Lemma 1, (31) is fulfilled if the following direct sum holds

$$S_{N_{AB}} \bigoplus \Pi \subseteq \Gamma_\infty \tag{44}$$

Thus, T_L^{BC} can follow T_L^{AB} in a feasible trajectory if there exists $\tau > 0$ such that the following LMI holds

$$\begin{bmatrix} 1 - \tau\tilde{\gamma} - \Pi^T P_\infty \Pi & * \\ -P_\infty \Pi & -P_\infty + \tau P_\infty \end{bmatrix} \geqslant 0 \tag{45}$$

Remark Notice that the assumption of constant friction coefficients along a generic trajectory segment is reasonable if $\bar{L}$ is small enough.

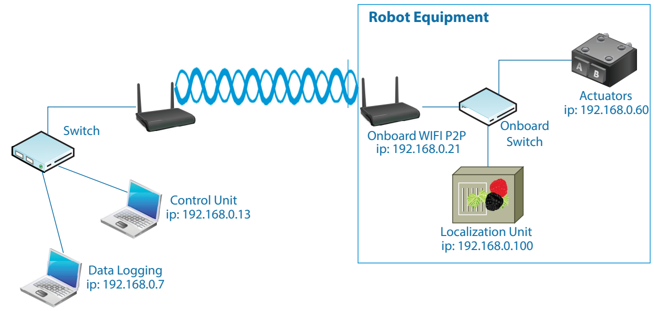

Fig. 5. Control framework.

TABLE I
MODEL PARAMETERS

Symbol	Value	Unit	
T_s	200	ms	Sample Time
τ_{max}	285.96	ms	Maximum network delay
τ_{min}	104	ms	Minimum network delay
V_L^D	0.2	m/s	Robot nominal forward velocity
ω_L^D	0	rad/s	Robot nominal rotational velocity
μ_R^D	1	–	Nominal right friction coefficient
μ_L^D	1	–	Nominal left friction coefficient

V. NUMERICAL RESULTS

Consider a skid-steering tracked mobile robot with a radius of track sprocket $R = 8\,cm$ and a distance between tracks $D = 50\,cm$. Fig.5 depicts the control framework taken into account in this work. Localization task and control action computation are supposed to be performed on two different remote computers connected through a wireless 802.11n network using the TCP/IP stack.

According to Section II, a polytopic mathematical model with 7 states and 48 vertices is carried out. The required model parameters are introduced in Table I. Considered constraints are reported in Table II. According to Section III, a solution of **CCP**-problem has been obtained in terms of couple (K, Γ_∞) taking into account the prescribed constraints.

The space domain Δ shown in Figs.6 and 7 was discretized by recurring to a regular grid of $0.2\,m$ with about 270 resulting points. In order to construct the graph $\mathcal{G}$ a maximum length of $\bar{L} = 0.5\,m$ is assumed for trajectory segment connecting two $\Delta - compatibles$ points.

In Fig. 6 a sensitivity analysis for the trajectory planning algorithm is proposed. Three different trajectories are obtained at varying of bounds of friction coefficients. Robot initial and final poses are denoted by arrows. Red line represents

TABLE II
PRESCRIBED CONSTRAINTS

Symbol	Min Value	Max Value	Unit	
V	0	0.4	m/s	Forward Velocity
ω	-0.45	0.45	rad/s	Rotational Velocity
μ_R	0.8	1.2	–	Right friction coefficient
μ_L	0.8	1.2	–	Left friction coefficient

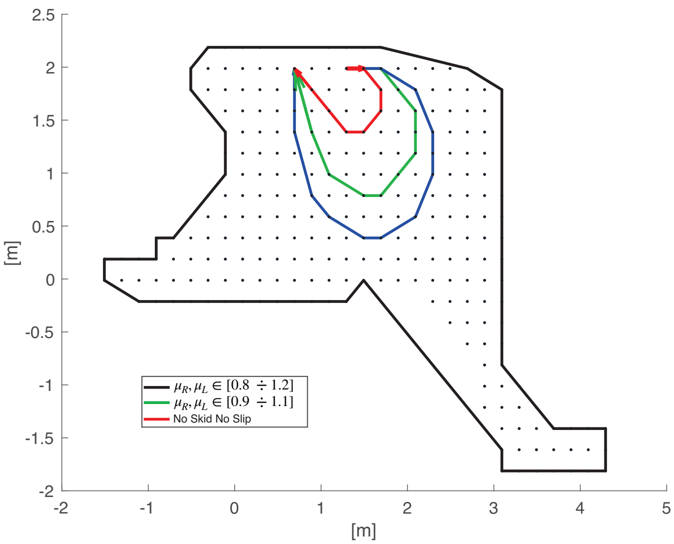

Fig. 6. Sensitivity analysis against the allowable range of friction coefficients

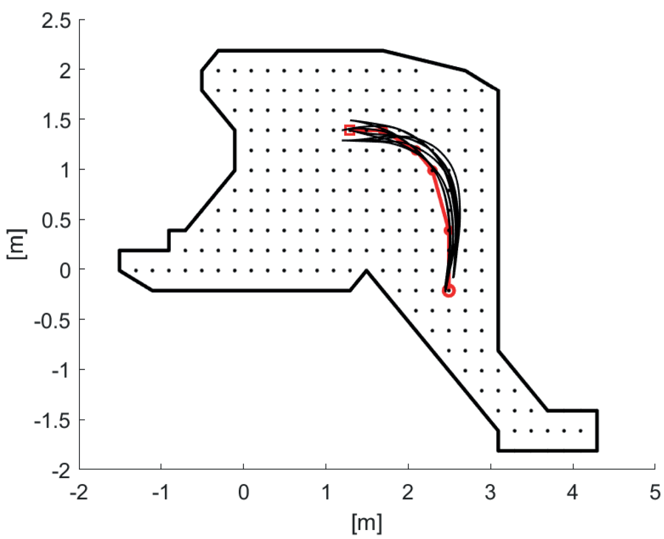

Fig. 7. Red line denotes the optimal feasible trajectory connecting starting (square) and ending (circle) points. Black solid lines represent simulated mobile robot trajectories.

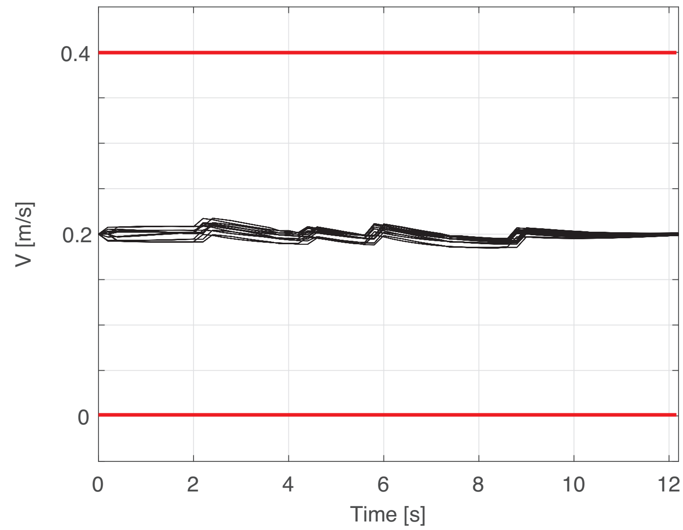

Fig. 8. Black lines represent robot forward velocities. Red lines represent considered constraints.

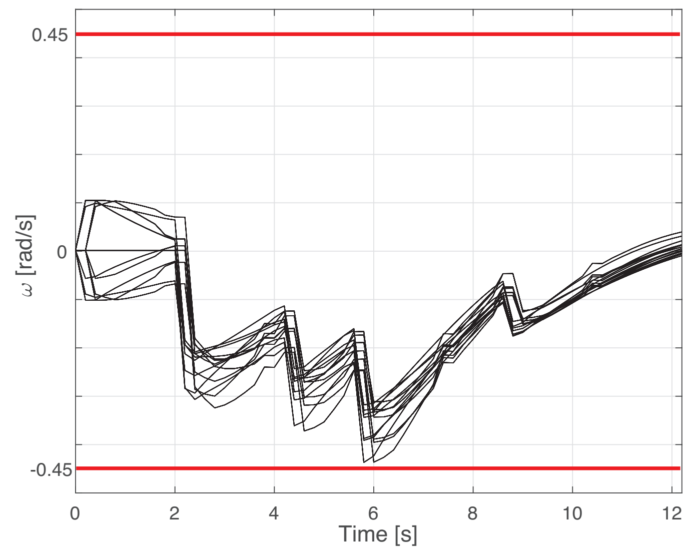

Fig. 9. Black lines represent robot angular velocities. Red lines represent considered constraints.

the optimal feasible trajectory in absence of skid and slip phenomena whereas green and blue lines represent optimal ones obtained by varying the allowable bounds of friction coefficients.

In Figs.7,8 and 9 a campaign of numerical simulations is proposed. In Fig.7 red line denotes the optimal feasible trajectory connecting starting (square) and ending (circle) points. Black solid lines represent simulated robot trajectories at varying of initial conditions, friction coefficients and network delay. As shown in Figs.8 and 9 considered actuation constraints are fulfilled.

In Fig. 10 the following scenario has been considered. Two additional obstacles (black box) are introduced in Δ. Red solid line represents the feasible optimal trajectory connecting starting (red square) and ending (red circle) points. Due to the presence of the additional obstacles a loop is required. In order to show the effectiveness of the proposed solution, the result of a numerical simulation is shown in Figs. 10-12. In particular, in Fig. 10 black solid line represents the simulated robot trajectory obtained by considering the friction coefficients shown in Fig. 13. It's worth to denote that trajectory feasibility guarantees that the prescribed constraints on control velocities are fulfilled along the whole trajectory as shown in Figs. 11 and 12.

VI. Conclusions

In this paper the optimal trajectory planning problem for a network-controlled skid-steering mobile robot is tackled. A feedback control action capable to achieve a solution of

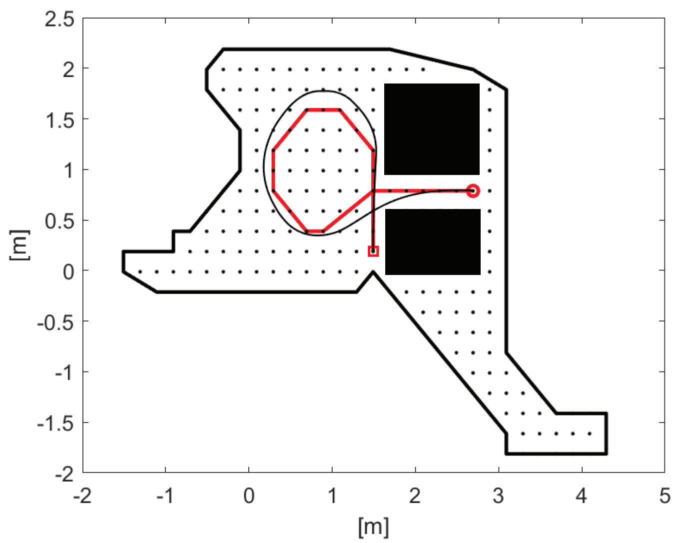

Fig. 10. Red line represent the optimal trajectory between starting (square) and ending points (circle). Black line denotes robot simulated trajectory.

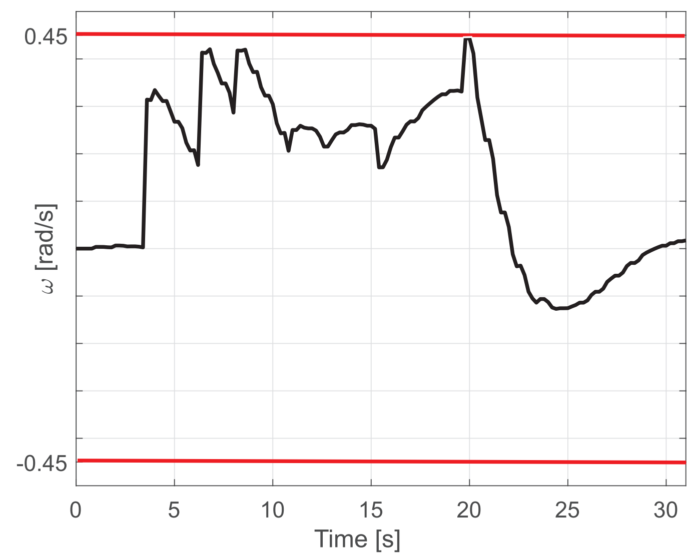

Fig. 12. Rotational robot velocity. Constraints are represented by red solid lines.

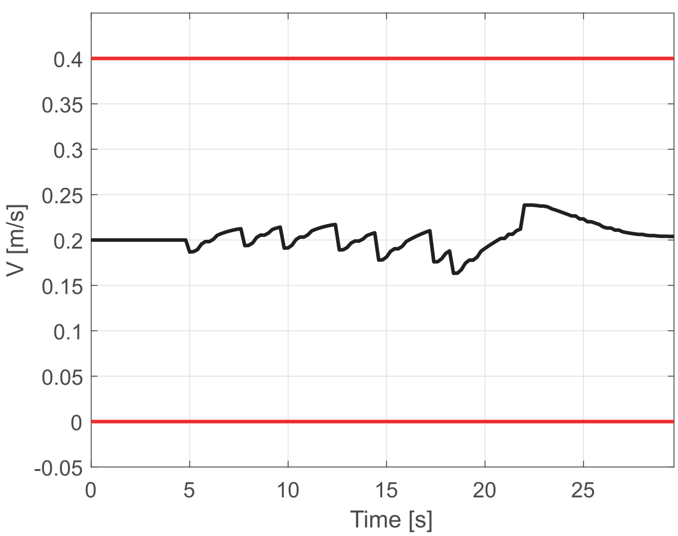

Fig. 11. Forward robot velocity. Constraints are represented by red solid lines.

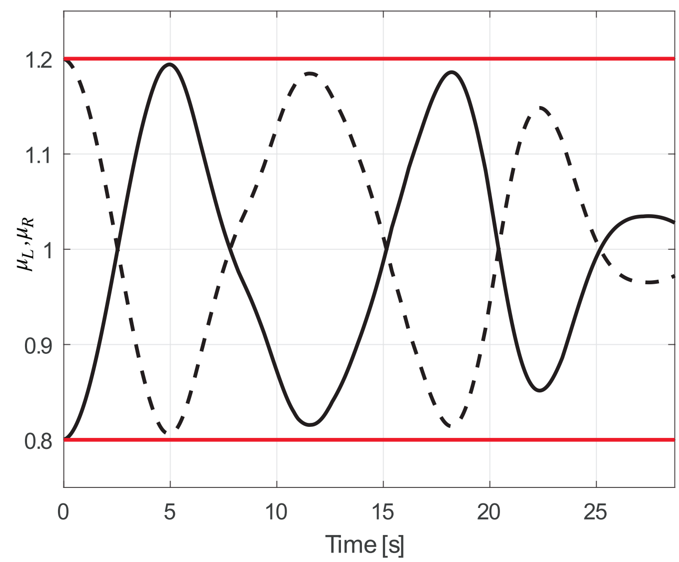

Fig. 13. μ_R (black solid line) and μ_L (black dashed line) values experienced by the robot during simulation. Bounds of the friction coefficients are denoted by red line.

the trajectory tracking problem accounting for network delay, tracking error constraints and control velocities bounds is firstly considered. Then a recursive algorithm is proposed to find optimal trajectory compliant with prescribed control constraints. Optimal trajectory is defined in terms of succession of segments connecting starting and ending points. Trajectory feasibility is guaranteed by exploiting the solution of optimization problems involving LMIs constraints. In order to show the effectiveness of the proposed approach some numerical simulations are discussed.

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