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a cura di Marina Mistretta, Bruno Mussari, Adolfo Santini

Modello energetico con “ritardo” per una crescita economica sostenibile: alcune osservazioni nello spirito di Horizon 2020

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La ricerca dei principi di base per i modelli della crescita economica è oggi una nuova e ardua sfida che è stata oggetto di numerose indagini. Dal 1862 Spencer ha sostenuto che la crescita economica delle società dipende dalla loro capacità di sfruttare le crescenti quantità di energia. Di conseguenza, la quantità di energia che una società consuma diventa uno strumento economico per misurare il suo progresso e quindi l'accumulazione di capitale rappresenta un'importante strategia per il processo di crescita. In particolare, il modello di Solow, che riguarda la funzione di produzione aggregata, ha dato un contributo importante alla teoria della crescita economica, soprattutto perché è stato dimostrato che è in grado di spiegare le differenze tra paesi del PIL per lavoratore. Tuttavia, come discusso in Dalgaard e Strulik, potrebbe essere più appropriata la derivazione di una legge del moto per il capitale senza ricorrere all'esistenza di una funzione di produzione aggregata. L'obiettivo principale di questo studio è di fare luce su quali effetti possono produrre i ritardi in un'economia, considerata come una rete di trasporto per l'energia, in cui la legge del movimento del capitale si verifica con un ritardo distribuito. Con questo lavoro si individuano condizioni necessarie e sufficienti in base alle quali uno stato stazionario perde stabilità e le dinamiche oscillatorie emergono attraverso le biforcazioni Hopf.

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STUDIES AND RESEARCH ON HISTORICAL HERITAGE AND ANTHROPIC LANDSCAPES, CONSERVATION AND REGENERATION
www.archistor.unirc.it

ArchHistoR EXTRA 6 (2019) ISSN 2384-8898
Supplemento di ArchHistoR 12/2019 ISBN 978-88-85479-08-1
DOI: 10.14633/AHR188

Distributed Time Delay Energy Model for Sustainable Economic Growth: Some Remarks in the Spirit of Horizon 2020

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In the last decade, many researchers applied certain principles of physics and biology in order to create a model of the law of motion for capital per worker[1]. In the West[2] the authors introduced a growth model for living tissue which was derived by assuming that energy is required by cells for their survival and reproduction (thermodynamics conservation principle), assuming that the capital stock increases if total energy expenditure exceeds the energy costs. One of the fundamental principles is Kleiber's law[3], which collates the correlation between the energy consumption of biological organisms (basal metabolism) and their energy requirements (body mass). Following these principles, biological systems are viewed as energy transporting networks and Kleiber's law models the diffusion and absorption of energy. All of the previous fundamental laws refer to biological networks that were developed through natural selection, which produced more efficient networks. In Bettencourt[4] the authors have applied these principles to artificial networks with the aim of discovering universal

1. See West, Brown, Enquist 1997, pp. 122-126; Banavar, Maritan, Rinaldo 1999, pp. 130-132; West, Brown, Enquist 1999, pp. 1677-1679; Banavar, Maritan, Rinaldo 2002, pp. 10506-10509.

2. West Brown, Enquist 2001, pp. 628-631.

3. Kleiber 1932, pp. 315-332.

4. Bettencourt *et alii* 2007, pp. 7301-7306.

laws with applications to human societies. In Banavar, Dalgaard and Strulik[5] mathematical models have been developed for an economy viewed as a transport network for energy. In these models, the energy consumption per worker is seen as the counterpart to metabolism, and capital per worker as the counterpart to body size. In order to describe a distributed time delay energy model for sustainable economic growth, it needs to be attributed to geographically dispersed sites where investments are deemed profitable and the sustainable aspects are more relevant. In this research, we present a model inspired by Dalgaard and Strulik[6].

This research field was pioneered by Domar and Solow[7], who analysed the growth process where physical capital accumulation is seen as a key growth engine. The authors discovered fundamental structural characteristics which improve labour productivity, in the long run, focusing their attention on: savings, population growth, technological change and more. The main result in this study is that the previous factors importantly affect the ability of an economy to mobilise resources for the purpose of capital accumulation. These facts are far from basic neoclassical growth theory abstracts which declare that it makes little sense to acquire a piece of machinery, at a particular time and place, unless the machine can be supplied with electricity and put it to use. In fact, these applications underline that an economy is able to distribute electricity across the economy to the sites where investments are deemed profitable.

Aim of the Research

Energy and food security, health and transport are the main objectives of Horizon 2020, the European Union's new funding programme, launched on 2014, and the way to attain these goals is through focusing on research and innovation. "Secure, clean and efficient energy" is one of the objectives set out by Horizon 2020 in order to face up to "social challenges" and achieve "a reliable, sustainable and competitive energy system in the face of increasingly scarce resources, increasing energy needs and climate change". Energy is the core element at the heart of a number of different objectives, with the programme also focusing on "smart, green and integrated transport" and "climate action, resource efficiency and raw materials". These aims are considered crucial for sustainable

5. Banavar, Maritan, Rinaldo 1999; Dalgaard, Strulik 2011.

6. See at note 5.

7. Domar 1946, pp. 137-147; Solow 1956, pp. 65-94.

Figure 1. Economic Growth, https://www.filodiritto.com/innovative-entrepreneurship-economic-growth-european-union (accessed 15 March 2019).

development in Europe and intrinsically connected with innovation and research in energy, and in particular in electricity. Innovation and research have been two of many actors with distinguishing traits for many years now, complementing their activities across the globe with initiatives and projects in which sustainability and energy efficiency are brought together with competitiveness and the fight against climate change. In this paper, we are going to introduce a modified growth model by considering some new element of analysis towards a new vision including tools relevant in the optic of Horizon 2020: energy and in particular electricity. The challenge of a sustainable approach towards Horizon 2020 is to elaborate a strategic and profound development plan for the low innovator (following EU terminology) and low developed regions, such as Calabria. The aim of our study is to promote a theoretical model by which we will be able to extend successively in a computational and experimental way.

The Model

Research in the natural sciences by West and Banavar[8] modelled biological organisms as an energy distributing networks to verify a statistical finding in biology, known as Kleiber's law[9], which states the correlation between the energy consumption of biological organisms (basal metabolism) and their energy requirements (body mass). Dalgaard and Strulik[10] used this concept and include an electricity distribution network into a macroeconomic model by taking electricity consumption per capita to be equivalent to metabolism and capital per capita to be equivalent to body mass. Given the relationship between capital and energy, they were able to obtain the following metabolic-energetic founded law of motion,

$$\dot{k}(t) = \frac{\epsilon}{\upsilon}\,[k(t)]^{a} - \frac{\mu}{\upsilon}\,k(t) \tag{1}$$

where $k(t)$ denotes capital stock per capita at any given time t, μ is the energy required to operate and maintain the generic capital good, υ is the energy cost to create a new capital good, a is a

8. See at note 1.

9. See at note 3.

10. Dalgaard, Strulik 2011, pp. 782-797.

real constant, $0 < a < 1$, proportional to the dimension and efficiency of the network, and $\varepsilon > 0$ is a real constant. Eq. (1) coincides with the structural form of the economist's core model of economic growth, the neoclassical model or Solow growth model[11]. Accordingly, there exists a unique stable equilibrium to which the economy adjusts.

Bianca[12] generalised the Dalgaard and Strulik[13] model by assuming the energy conservation equation to be affected by a time delay. As a result, the dynamics of the model became characterised by the following delay differential equation

$$\dot{k}(t) = \frac{\epsilon}{v}\,[k(t-\tau)]^{a} - \frac{\mu}{v}\,k(t-\tau) \tag{2}$$

where $\tau > 0$ represents a time delay. Notice that Eqs. (1) and (2) have the same unique equilibrium. Although they look similar, the technique used for their dynamical analysis is different. For (1) the characteristic equation is a polynomial and the fundamental theorem of algebra tells us how many roots (eigenvalues) to expect. For (2) the characteristic equation is transcendental, so there is no theorem about the number of roots, which could be countably infinite. As τ increases, they showed that the stable equilibrium of the neoclassical model becomes a saddle point and endogenous cycles may emerge. Time delays in economic situations can be modelled in two different ways: fixed time delays, suitable when an institutionally or socially determined fixed period of time delay is present for the agents involved, and continuously distributed time delays, applicable when the delay is uncertain or different lengths of delays are distributed over the different agents. Consequently, it is relevant to investigate how the choice of modelling delays and its economic consequences affect the long-run dynamics of the economic variables. Thus, we now reconsider the Dalgaard and Strulik[14] model under the assumption of continuously distributed time delays. In doing so, we arrive at the following model

11. Solow 1956.

12. Bianca, Ferrara, Guerrini 2013, pp. 139-143.

13. See at note 10.

14. *Ibidem*.

$$\dot{k}(t) = \frac{\epsilon}{\upsilon}\left[\int_{-\infty}^{t} g(t-r,S,m)\,k(r)dr\right]^{a} - \frac{\mu}{\upsilon}\int_{-\infty}^{t} g(t-r,S,m)\,k(r)dr \qquad (3)$$

where *m* and *n* are nonnegative integers, *S* and *T* are positive real parameters which represent average delays, and the delay kernel *g* in (3) is given by a gamma-type distribution[15]. Parameter *ζ = S, T* is associated with the average length of the continuous delay and *l = m,n* determines the shape of the weighting function. The two special cases and are called weak delay kernel and strong delay kernel, respectively. If *ζ → 0* the gamma-type distribution function approaches the Dirac function. As the shape parameter tends to infinity, one recovers the fixed delay case (2). The dynamic equation (3) is a Volterra type integro-differential equation (i.e. an equation that involves both integrals and derivatives of a function), where the characteristic equation is a polynomial equation with finitely many eigenvalues. Since this law of motion of capital is mathematically isomorphic to the one considered by Guerrini[16], with $\alpha = a$, $s = \epsilon/\upsilon$ and $\delta = \mu/\upsilon$, the model shares the technical properties with their model. Therefore, we derive that the model with weaker kernel is more stable than the one with stronger kernel in the sense that the stability region of the former is larger. Moreover, in the limiting case in which the distributed delay converges to the fixed delay, the model with the fixed delay has the smallest stability region. It also proved analytically the existence of a threshold value of the delay such that the model is stable for the delay less than this threshold, and becomes unstable for any delay larger than the threshold. Furthermore, it is numerically demonstrated that a cycle emerges for this threshold value and stability loss and gain may repeat alternatively, implying that the delay has a destabilising effect as well as a stabilising effect. Such duality of the delay does not appear in the fixed delay model.

15. See Macdonald 1978.

16. Guerrini, Matsumoto, F. Szidarovszky 2019, pp. 234-247.

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