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A LIMIT ANALYSIS APPROACH IN THE CONTEXT OF NONLOCAL MATERIALS

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Abstract. *The paper proposes a limit analysis approach for the evaluation of a lower bound to the collapse load multiplier of structural elements made of metal matrix nanocomposites. Taking into account both the microstructure and the ductile behavior of such materials, a nonlocal-elastic perfectly-plastic constitutive model is assumed for their description. In particular, for the elastic phase an enhanced version of the Eringen nonlocal integral model is adopted, while the plastic phase is modeled by means of a von Mises type flow rule.*

1 INTRODUCTION

This paper investigates the applicability of a limit analysis procedure in the context of non-local materials. The motivation of such a study arises from the will to provide a reliable numerical tool for the evaluation of the load bearing capacity of structural elements made up with complex materials whose constitutive behavior can be appropriately described only by a non-local approach. Is this, for instance, the case of some innovative materials, as the metal matrix nanocomposites (MMNCs), to which reference is made hereafter. The MMNCs are indeed very light and possess good performances in terms of mechanical characteristics so that they are nowadays employed in structural applications of noticeable industrial fields such as aeronautical and automotive. For this reason a considerable research effort has been recently devoted to the study of these type of materials. However, the constitutive complexity of such materials makes difficult the evaluation of their behavior in post elastic regime as well as of collapse for structures employing them. In this context even well consolidated numerical tools do not give reliable results and it may result more effective a limit analysis approach thanks to which it is possible to directly estimate the bearing capacity of the structural elements of interest.

Grounding on the above arguments and having also in mind the overall ductile behavior of the MMNCs, this paper proposes a *plastic collapse limit analysis* approach of *nonlocal type*. In particular, it deals with the reformulation, in a nonlocal context, of a procedure known in the relevant literature as elastic compensation method which allows to evaluate a lower bound to the plastic collapse load multiplier of structural elements made up of MMNCs. The procedure, already utilized by the authors in different (local) contexts [1], makes use of sequences of elastic analyses in order to mimic the behavior of the structure at a limit state of incipient collapse. In the nonlocal framework, here considered, the above elastic analyses are nonlocal and performed by means of a nonlocal finite element code, implemented by the authors in [2]. Moreover, a nonlocal elastic perfectly plastic constitutive model is assumed with the hypothesis that the nonlocal behavior pertains only to the elastic phase, described in the follow by an enhanced version of the Eringen model [3].
The proposed formulation is tested by means of a case study and the obtained results are critically discussed.

2 THE CONSTITUTIVE MODEL FOR MMNCs

MMNCs materials are characterized by the presence of nanoparticles within a metal matrix. The presence of nanoparticles can be appropriately described by a nonlocal constitutive model, while the ductility induced to the overall material by the metal matrix can be caught by a plastic model. These considerations justifies the assumption of the nonlocal-elastic perfectly-plastic constitutive model expressed by the Eqs.(1)-(3) reported below:

$$\boldsymbol{\sigma}(\boldsymbol{x}) = \boldsymbol{D}(\boldsymbol{x}) : \mathcal{R}(\boldsymbol{\varepsilon}(\boldsymbol{x}) - \boldsymbol{\varepsilon}^{\boldsymbol{p}}(\boldsymbol{x})) = \boldsymbol{D}(\boldsymbol{x}) : \mathcal{R}(\boldsymbol{\varepsilon}^{e}(\boldsymbol{x})) \tag{1}$$

$$\dot{\boldsymbol{\varepsilon}}^{\boldsymbol{p}}(\boldsymbol{x}) = \dot{\lambda}\frac{\partial f(\boldsymbol{\sigma}(\boldsymbol{x}))}{\partial \boldsymbol{\sigma}(\boldsymbol{x})} \tag{2}$$

$$f(\boldsymbol{\sigma}) \leq 0; \qquad \dot{\lambda} \geq 0; \qquad \dot{\lambda} f(\boldsymbol{\sigma}) = 0. \tag{3}$$

The above postulated constitutive model has been already utilized by the authors in [5] and considers a material in which the nonlocal behavior pertains only to the elastic phase, while the plastic phase is local and obeys the associative von Mises flow rule (Eqs. 2-3). In the above equations: $\boldsymbol{D}(\boldsymbol{x})$ is the elastic moduli tensor, $\boldsymbol{\sigma}(\boldsymbol{x})$ and $\boldsymbol{\varepsilon}(\boldsymbol{x})$ are the stress and the strain tensors, the apices *p* and *e* stand for *plastic* and *nonlocal elastic*, respectively; finally, the regularization operator $\mathcal{R}$ is defined as:

$$\mathcal{R}(\boldsymbol{\varepsilon}^e(\boldsymbol{x})) = \boldsymbol{\varepsilon}^e(\boldsymbol{x}) - \alpha \int_V \boldsymbol{\mathcal{J}}(\boldsymbol{x}, \boldsymbol{x}') : [\boldsymbol{\varepsilon}^e(\boldsymbol{x}') - \boldsymbol{\varepsilon}^e(\boldsymbol{x})] \, \mathrm{d}\, V' \quad \forall\, (\boldsymbol{x}, \boldsymbol{x}') \in V. \tag{4}$$

Equation (4) is known in literature as strain-difference based nonlocal elasticity model [3] and has been widely applied by the authors to solve different nonlocal problems (see e.g. [2] and references therein).

The nonlocal elastic model of Eq.(4) is characterized by the presence of two nonlocal elastic parameters: α, which furnishes the proportion of the nonlocal phase within the model, and ℓ, an internal length material scale entering the nonlocal tensor $\boldsymbol{\mathcal{J}}(\boldsymbol{x}, \boldsymbol{x}')$ through an attenuation function, say $\mathrm{g}(\boldsymbol{x}, \boldsymbol{x}')$. Details on the non local tensor $\boldsymbol{\mathcal{J}}$ and on the scalar function g are discussed in the above quoted papers, here it is important to highlight that the shape of $\mathrm{g}(\boldsymbol{x}, \boldsymbol{x}')$, beside the value assumed for ℓ, determines the so called influence distance, say L_R, i.e. the length denoting the wideness of the continuum region, neighbor of $\boldsymbol{x}$, within which long distance nonlocal interactions act. In the follow a bi-exponential form of the attenuation function is employed.

3 EVALUATION OF A P_{LB} BY A NONLOCAL LIMIT ANALYSIS APPROACH

This section proposes the evaluation of a lower bound to the collapse load multiplier of a structural element made of MMNCs. In particular, a method known in literature as Elastic Compensation Method (ECM) is here rephrased in a nonlocal context. The ECM is an iterative procedure aimed to the construction of a plastically admissible stress field. The procedure involves many sequences of linear FE-based analyses and, being the MMNCs governed by the nonlocal constitutive model furnished by Eqs.(1)-(3), the above analyses are here performed by means of a nonlocal finite element (NL-FEM) tool [2]. In the rationale of the ECM highly loaded regions of the structure are systematically weakened by reduction of the modulus of elasticity and this in order to simulate the effects of stress redistribution arising within the structure approaching its limit state. Precisely, at each sequence of linear nolocal elastic analyses the elastic moduli are modified only within "critical regions" identified by the elements with stress values greater than the yield one. This allows one to define the maximum plastically admissible stress field for the structure and then to lead to a P_{LB} load multiplier. The nonlocal ECM is formulated in the deviatoric π-plane where the von Mises yield surface is a circle of equation $\rho^2 - \rho_y = 0$, ρ denoting the deviatoric stress invariant with ρ_y the yield threshold. At the first sequence, $s = 1$, the structure is subjected to a load $P^{(s)}\bar{\mathbf{p}}$, where $P^{(s)}$ denotes the load multiplier of the current sequence while $\bar{\mathbf{p}}$ is the reference load. The structure is discretized in NL-FEs and, at the current iteration say $(k-1)$, the nonlocal stress value $\rho_{\#e}^{(k-1)}$, averaged within the GPs of the element $\#e$, is computed. Among all the $\rho_{\#e}^{(k-1)}$ in the mesh, the maximum stress, say $\rho_{max}^{(k-1)}$, is detected, such a stress point is the one located furthest away from the von Mises circle in the π-plane. If $\rho_{max}^{(k-1)} > \rho_y$ the method tries to redistribute the current load $P^{(s)}\bar{\mathbf{p}}$ by updating all the elastic moduli in the elements where $\rho_{\#e}^{(k-1)} > \rho_y$ by applying the updating rescaling formula, [5]:

$$E_{\#e}^{(k)} = E_{\#e}^{(k-1)} \left[\frac{\rho_y}{\rho_{\#\, e}^{(k-1)}} \right]^2 . \tag{5}$$

If $\rho_{max}^{(k-1)} \leq \rho_y$ all the stress points lay inside or onto the yield surface, individuating a state of admissible stress field. At this stage it is possible to increase the intensity of the acting loads setting $P^{(s+1)} = P^{(s)} + \triangle P$ and perform a new sequence of nonlocal elastic analyses in the attempt to redistribute a greater load. The sequences stop when the load increase does not allow the maximum stress to be brought below, or at, yield by redistribution. The P_{LB} is then the last computed value corresponding to the last admissible stress field, say at iteration K, attained for the maximum (redistributed) load, say the one obtained at sequence $(\mathcal{S})$, i.e. $P^{(\mathcal{S})}\bar{\mathbf{p}}$. The P_{LB} is eventually given by:

$$P_{LB} = \rho_y \frac{P^{(\mathcal{S})}}{\rho_{max}^{(K)}}. \tag{6}$$

4 NUMERICAL EXAMPLE AND CONCLUDING REMARKS

The procedure briefly expounded in the previous sections is implemented for the analysis of a square plate subjected to tensile uniform stress. The mechanical sketch of the plate is reported in Fig.1 which also gives the geometrical dimensions, boundary and loading conditions. The material the plate is made with is the one addressed in [4], that is 1.5% CNTs/AZ91D. The numerical example, even if simple, is aimed to verify the numerical implementation of

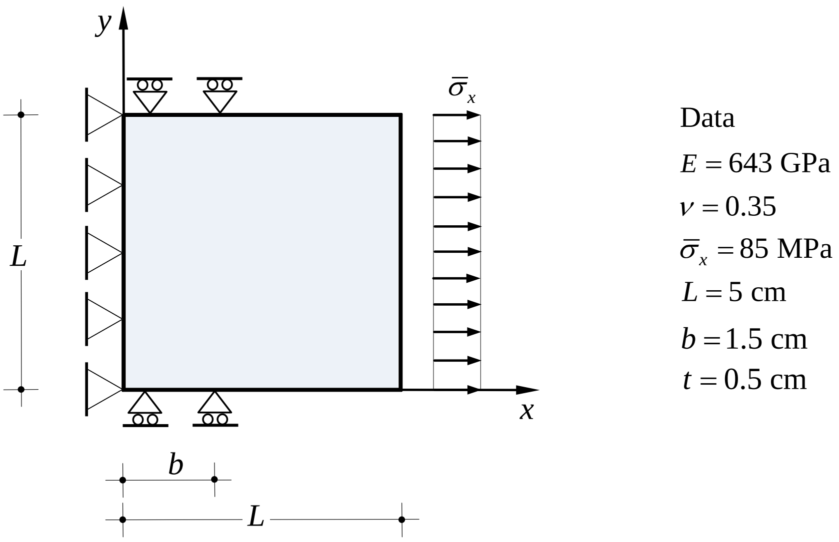

Figure 1: Mechanical sketch of the analysed nonlocal square plate.

the ECM within a nonlocal context as well as to test the real capability of it in capturing the nonlocal effects in the prediction of the load bearing capacity of the analyzed structural elements. For the considered example the value of the nonlocal parameter, ℓ, is varied in a range from 0 (local case) to 5, while the nonlocal parameter α is set equal to 50. The procedure has been implemented in a fortran code which performs the sequences of nonlocal elastic analyses of the structure by a NL-FE code. To this purpose, a uniform discretization of the plate is used through 100 isoparametric, 8-nodes, plane stress nonlocal elements. The results of the performed analyses are reported in Fig.2 by the contour plots, over the entire mesh and at last converged sequence of the NL-ECM procedure, of the stress scalar measure ρ. The distribution of such invariant furnishes information on the admissible stress field; as in fact, the zones of the structure in which the value of ρ is close to its threshold limit (ρ_y) locate the zones of the structure in which plastic mechanisms are at an incipient activation stage. By observing Fig.2

it can be affirmed that the differences in ρ distributions for four different value of ℓ are quite significative. Moreover, for small value of ℓ (when $\mathrm{g}(\boldsymbol{x}, \boldsymbol{x}')$ tends to a Dirac delta) or for high value of ℓ (when $\mathrm{g}(\boldsymbol{x}, \boldsymbol{x}')$ flattens), i.e. for ℓ values resulting in a local behavior, the ρ (local) distributions equal each other, as it has to be. The same qualitative results can be read through Fig.3 which reports the P_{LB} values versus the four values assumed for the internal length. The P_{LB} values in the nonlocal cases appear more conservative and deviates significantly from the ones evaluated in the local cases.

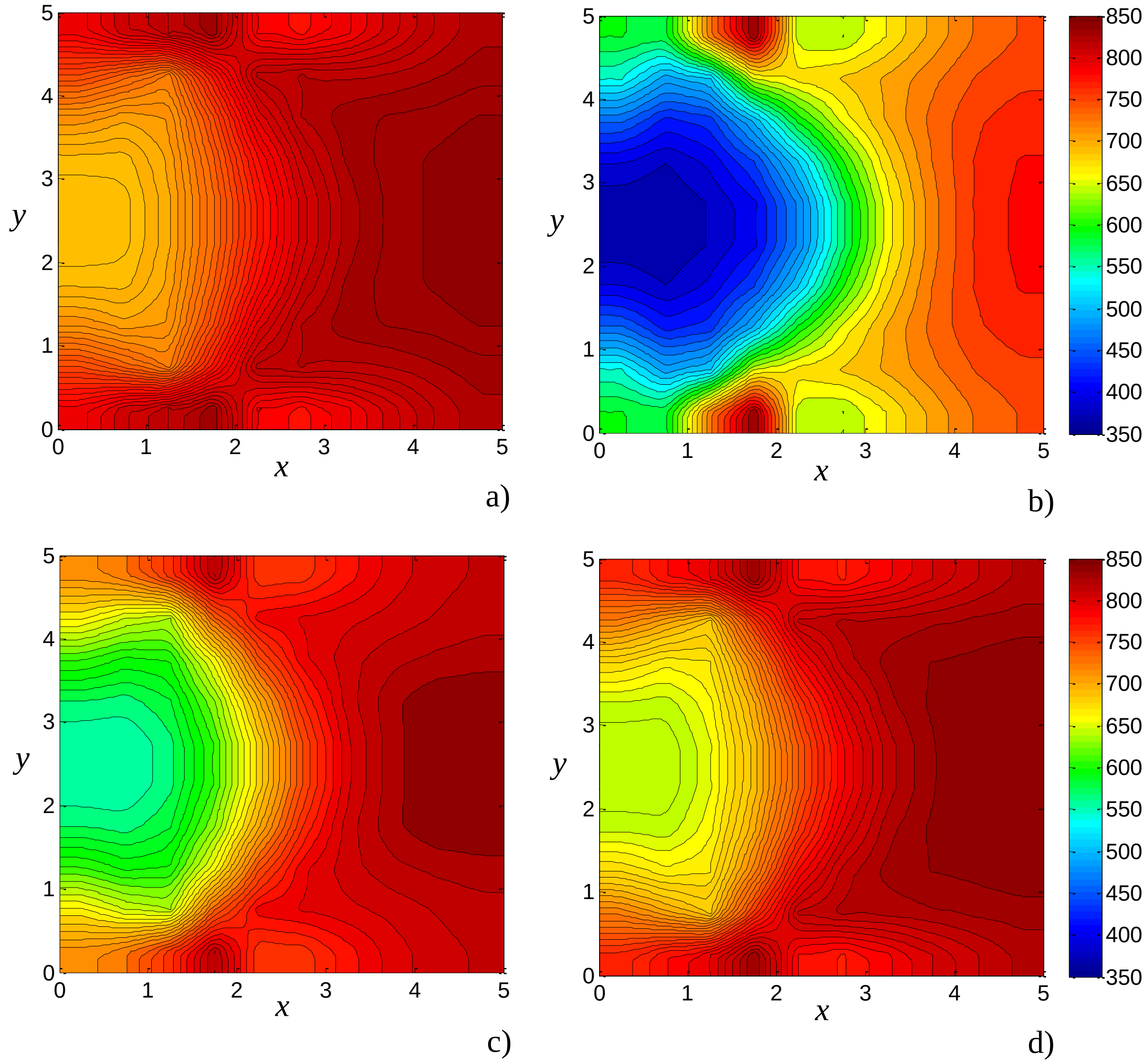

Figure 2: Contour plots of the stress invariant ρ $[daN/cm^2]$ at last sequence of the NL-ECM and for: a) $\ell = 0$ (local case); b) $\ell = 1$; c) $\ell = 3$; d) $\ell = 5$.

In conclusion it can be observed that:

- The differences in terms of P_{LB}, with respect to the local case, present a wide range of variability (from 1.6% to 25%) showing a *significant dependence* of the plastic collapse load from the nonlocal features of the constituent material the structure is made with.
- The differences in the results also applies to the distribution of plastically admissible stresses at collapse inside the structure.

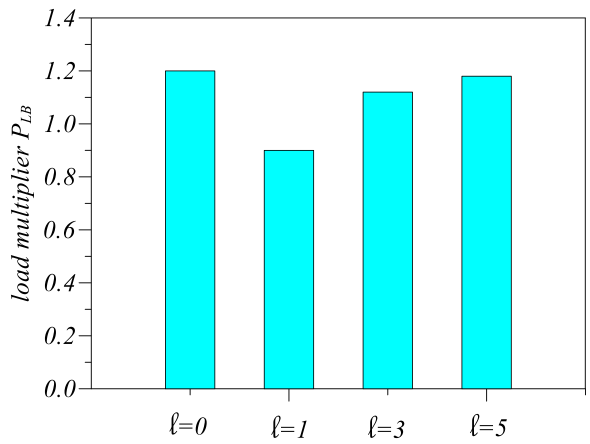

Figure 3: Lower bound multiplier P_{LB} versus internal length ℓ.

- In the authors' opinion, the promoted nonlocal limit analysis procedure offers a new perspective for a classical design tool in the realm of advanced innovative materials as, for example, the metal matrix nanocomposites here addressed.

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