UNA PROCEDURA NUMERICA PER LA VALUTAZIONE DEL CARICO DI COLLASSO PLASTICO DI COLLEGAMENTI IN ACCIAIO TRAVE-COLONNA

A NUMERICAL PROCEDURE FOR THE PLASTIC COLLAPSE LOAD EVALUATION OF WELDED BEAM-TO-COLUMN STEEL CONNECTIONS

Raffaele Pucinotti,
Aurora Angela Pisano,
Paolo Fuschi
University *Mediterranea* of Reggio Calabria
Department PAU - Patrimonio, Architettura, Urbanistica
Reggio Calabria, Italy
raffaele.pucinotti@unirc.it; aurora.pisano@unirc.it; paolo.fuschi@unirc.it

ABSTRACT

The paper proposes a limit analysis FE-based numerical procedure for the plastic collapse load evaluation of welded beam-to-column steel joints. This work can be considered as a first step of an ongoing research project whose final goal is indeed the set up of a reliable and simple numerical procedure oriented to seismic retrofitting of masonry structures. It is well known that the restoration as well as the seismic upgrading of a masonry building often requires the use of steel components, mutually aggregated. Plastic collapse limit design can be the right tool to handle the complex system of masonry, whose post-elastic behavior is not well defined, and steel members whose *weaknesses* points rely on the connections. In this context, two different numerical techniques, based on the static and kinematic approach of limit analysis respectively, are simultaneously employed to define the actual plastic collapse limit load of the addressed steel connections. The procedure and the numerical findings are validated by comparison with experimental outputs on real scale prototypes.

SOMMARIO

L'articolo propone una procedura numerica di analisi limite, che fa uso degli elementi finiti, per il calcolo del carico di collasso plastico di giunti saldati trave-colonna. Questo lavoro può essere considerato come il primo passo di una ricerca in fieri il cui scopo finale è quello di mettere a punto una procedura numerica, affidabile, e al contempo semplice, orientata al restauro di edifici in muratura. È infatti ben noto che sia il restauro che l'adeguamento sismico di un edifico murario spesso richiedono l'utilizzo di elementi metallici collegati tra loro. Progettare allo stato limite di collasso può essere lo strumento adatto nel caso di sistemi complessi costituiti da murature, il cui comportamento post elastico non è ben definito, e da elementi in acciaio che hanno i loro punti di *debolezza* nei collegamenti. In questo contesto si propone di utilizzare simultaneamente due metodi numerici, basati rispettivamente su un approccio statico e uno cinematico dell'analisi limite e volti alla definizione dell'effettivo carico di collasso plastico di giunti in acciaio. La procedura e i risultati numerici ricavati dalla sua applicazione sono validati attraverso un confronto con risultati sperimentali ottenuti su un provino a scala reale.

1 INTRODUCTION

The restoration or the seismic upgrading of existing buildings is often pursued with the adoption or insertion of steel components. This design choice is, as known, very useful to improve the strength capabilities of the original parts of the building and, in seismic zones, to create parts able to absorb and/or to dissipate the energy of the earthquake actions.

However, when dealing with masonry constructions, a conflict in the adoptable design methodologies arises. From one side, steel members, steel frames or steel connections are configured and designed with sophisticated design codes based on a complete and experimented knowledge of the post-elastic behaviour of this material. Step-by-step procedures, in statics or in dynamics, able to reproduce the steel sub-system response up to collapse are normally offered by all the commercial codes. From the other side, masonry components or masonry organisms do not possess a precise and reliable behaviour especially in the post-elastic phase. The constitutive assumptions and the adopted models for masonry structures are often inappropriate because the real behaviour is often related to circumstances which depends not only on the material but also on the constructive techniques.

In this context, simplified methods of analysis such as direct methods, [1], can result more effective and reliable. One of the most popular is, with no doubt, limit analysis that, renouncing to a detailed description of the post-elastic behaviour of a structure, focuses on a different goal, namely the prediction of the ultimate load the structure can bear. Thinking to a structural system made by steel and masonry members, limit analysis should be able to predict a limit load of plastic collapse for steel members and of rupture for masonry ones. Limit analysis in a real engineering practical context should also be performed numerically, i.e. based on a finite element friendly procedure.

The present work belongs to the above outlined research line and, as a first step of the study, presents a limit analysis FE-based numerical procedure applied to steel connections oriented to masonry building restoration actions. The method is here applied to welded beam-to-column connections to predict their plastic collapse load. Two different numerical techniques, based on the static and kinematic approach of limit analysis respectively, are simultaneously applied to detect eventually the plastic collapse limit load of the analysed connection [2], [3]. The procedure and the related numerical findings are validated by comparison with experimental outputs on real scale prototypes, [4].

2 LIMIT ANALYSIS PROCEDURE: BASIC CONCEPTS

In its classical formulation the theory of limit analysis refers to perfectly plastic structures, made up of standard materials, and it is based on a lower and an upper bound theorem.

More precisely, the upper bound theorem states that an upper bound, say P_{UB}, to the collapse load multiplier, for a given body of volume V, is given by:

$$P_{UB} = \frac{\int_V \sigma_j^y \dot{\varepsilon}_j^c \mathrm{d}V}{\int_{S_t} \overline{p}_i \dot{u}_i^c \mathrm{d}S_t}, \tag{1}$$

where $\dot{\varepsilon}_j^c = \dot{\lambda}\, \partial f / \partial \sigma_j$ are the components of the outward normal to the yield surface $f(\sigma_j) = 0$ (with $\dot{\lambda} > 0$ a positive scalar multiplier); σ_j^y are the stresses at yield associated to given compatible strain rates $\dot{\varepsilon}_j^c$; $\dot{u}_i^c$ are the related displacement rates. $\overline{p}_i$ are the surface force components of the reference load vector $\overline{\boldsymbol{p}}$ acting on the external portion S_t of the body surface. The set ($\dot{\varepsilon}_j^c$, $\dot{u}_i^c$) defines a collapse mechanism.
On the other hand, the lower bound theorem states that if for every point within V exists a stress field, say $\tilde{\sigma}_j$, which satisfies the condition $f(\tilde{\sigma}_j) \leq 0$ and in equilibrium with the applied load $P\,\overline{\boldsymbol{p}}$ for a value of P, say P_{LB}, then P_{LB} is a lower bound to the collapse limit load multiplier.
The numerical procedure promoted hereafter grounds on the two above limit analysis theorems and the relevant literature refers to them as *Linear Matching Method* (LMM) and *Elastic Compensation Method* (ECM), respectively.
A detailed description of the two methods is here omitted for sake of brevity; the Reader can refer to [1, 3, 5] for further theoretical and operative information.
However, the key ideas on which they are based on are briefly reported in the follow.

2.1 The Linear Matching Method

The LMM is an iterative procedure involving one sequence of linear FE-based analyses and it is aimed at constructing a collapse mechanism for the evaluation of a P_{UB}. The linear analyses are carried on a structure made, by hypothesis, of a fictitious, viscous incompressible material with spatially varying moduli. In particular, at each iteration, the fictitious moduli are *adjusted* so that the computed fictitious stresses are brought on the yield surface at a fixed strain rate distribution. This allows one to define a collapse mechanism (strain and displacement rates), the related stress at yield and then, by applying equation (1), a P_{UB} load multiplier.

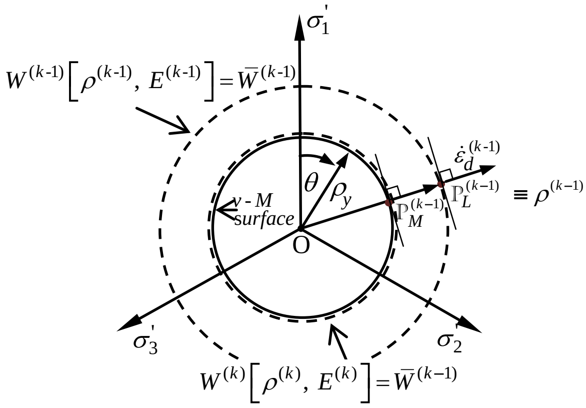

Fig. 1. Geometrical sketch of the LMM procedure in the deviatoric plane.

An easy understanding of the method can be achieved by looking at its geometrical interpretation in the deviatoric plane, as in Fig.1. In such plane, the von Mises surface is a circle of radius ρ_y and the fictitious solution can be represented on a complementary energy dissipation rate equipotential surface homothetic, due to the assumed incompressibility, to the von Mises circle. At the current iteration of the procedure, say at the (*k-1*)*th* FE analysis, the structure made with the fictitious material is analyzed under loads $P^{(k-1)}\overline{p}_i$. The fictitious linear solutions, computed at each Gauss point (GPs) of the FE mesh, can be represented by a point $\mathrm{P}_L^{(k-1)}$ lying on the complementary dissipation rate equipotential surface $W^{(k\text{-}1)} = \overline{W}^{(k\text{-}1)}$. The latter, as said, is a circle concentric to the von Mises one of equation:

$$W(\rho, E^{(k-1)}) = \frac{3\rho^2}{4E^{(k-1)}}. \tag{2}$$

At this stage, it can be shown that to interpret the fictitious solution as real solution at collapse it suffices to find the matching point $\mathrm{P}_M^{(k-1)}$, i.e. the point on the yield surface, having outward normal $\dot{\varepsilon}_d^{(k\text{-}1)}$. To do so, it is sufficient to rescale the radius of the circle $W^{(k\text{-}1)} = \overline{W}^{(k\text{-}1)}$ and this by modifying the elastic modulus $E^{(k\text{-}1)}$ from which, as evident by Eq.(2), the dimension of this radius depends on. The updating formula for the Young modulus E it can be proven to be [1]:

$$E^{(k)} = E^{(k-1)} \frac{\rho_y}{\rho^{(k-1)}}. \tag{3}$$

When the point $\mathrm{P}_L^{(k-1)} = \rho^{(k-1)}$ is brought to coincide with $\mathrm{P}_M^{(k-1)} = \rho_y$ the corresponding fictitious strain $\dot{\varepsilon}_d^{(k\text{-}1)}$ can be interpreted, [1], as the strain rates solutions $\dot{\varepsilon}_d^{c(k-1)}$ at collapse compatible with the displacement $\dot{u}_i^{c(k-1)} = \dot{u}_i^{(k-1)}$. If the expounded rationale is repeated at all the GPs of the mesh, for the current iteration (*k-1*), an upper bound to the collapse load multiplier can be evaluated reinterpreting Eq.(1) in the deviatoric plane as:

$$P_{UB}^{(k-1)} = \frac{\int_V \rho_y^{(k-1)} \dot{\varepsilon}_d^{c(k-1)} \mathrm{d}V}{\int_{S_t} \overline{p}_i \dot{u}_i^{c(k-1)} \mathrm{d}S_t}. \tag{4}$$

However, the stress at yield computed through the matching do not meet the equilibrium conditions with the acting load $P^{(k-1)}\overline{p}_i$ inducing an iterative procedure that ends when the difference between two subsequent P_{UB} values is less than a fixed tolerance.

2.2 The Elastic Compensation Method

The ECM is aimed to constructing a plastically admissible stress field suitable for the evaluation of a P_{LB}. It is an iterative procedure involving more sequences of linear FE-based analyses in which highly loaded regions of the structure are systematically weakened by reduction of the modulus of elasticity and this in order to simulate the effects of stress redistribution arising within the structure approaching its limit state. Precisely, at each sequence of linear elastic FE analyses the elastic moduli are modified only within “critical regions” identified by the elements with stress values greater than the yield one. This allows one to define the maximum plastically admissible stress state for the structure and then to lead to a P_{LB} load multiplier. Once again a geometric interpretation of the method can help for its easy understanding, see Fig.2. Let us consider, at the (*k-1*)*th* FE analysis within the current sequence, the computed elastic solution in terms of deviatoric stress within each FE evaluated as mean value over all the GPs in the element. Among all the $\rho_{\#e}^{(k-1)}$ in the mesh, the maximum stress, say $\rho_{max}^{(k-1)}$, is detected, such a stress point is the one located furthest away from the von Mises circle. If, like in Fig.2, $\rho_{max}^{(k-1)} > \rho_y$ the method tries to redistribute the current load $P^{(S)}\overline{p}_i$ by updating all the elastic moduli in the elements where $\rho_{\#e}^{(k-1)} > \rho_y$ by applying the updating formula [1]:

$$E^{(k)} = E^{(k-1)} \left[\frac{\rho_y}{\rho_{\#e}^{(k-1)}} \right]^2, \tag{5}$$

and performing iterations (FE analyses) within the sequence. When $\rho_{max}^{(k-1)} \le \rho_y$ all the stress points lay inside or onto the yield surface, individuating a state of admissible stress filed. A lower bound to the collapse load multiplier can then be computed as:

$$P_{LB}^{(k-1)} = \frac{\rho_y P^{(S)}}{\rho^{(k-1)}}. \tag{6}$$

At this stage of the procedure it is possible to increase the intensity of the acting loads and perform a new sequence of elastic analyses in the attempt to redistribute a great load. The P_{LB} is then the last computed value corresponding to the maximum redistributed admissible stress filed.

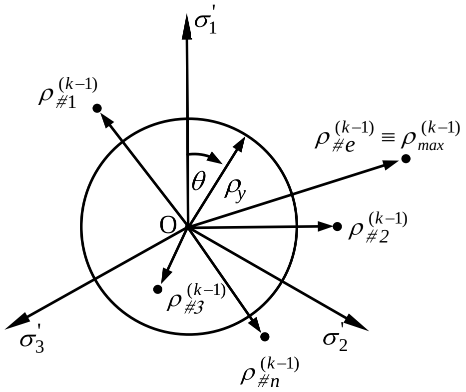

Fig. 2. Geometrical sketch of the ECM procedure in the deviatoric plane.

3 NUMERICAL VERSUS EXPERIMENTAL FINDIGS

3.2 The beam-to-column connection problem

The mechanical problem under study concerns a beam-to-column steal connection, like the one sketched in Fig.3, where geometrical quantities (in mm), loading and boundary conditions are also reported. Figure 3 is indeed the schematic representation of a laboratory experimental test on a real prototype, namely the one reported in [4] as BCC5-E. The prototype is subjected to a monotonic load, which at collapse has been estimated in about 228kN. Figure 4 reports the picture of the experimental device utilized, while table 1 lists all the information related to geometry and material parameters of the two metal profiles elements of the connection. It is worth noting that the values reported in Tab.1 refer to the actual quantities measured during the experimental campaign that deviate, even in a significant way, from the theoretical ones. In order to validate the discussed limit analysis procedure, the above experiment has been utilized as benchmark and numerically reproduced.

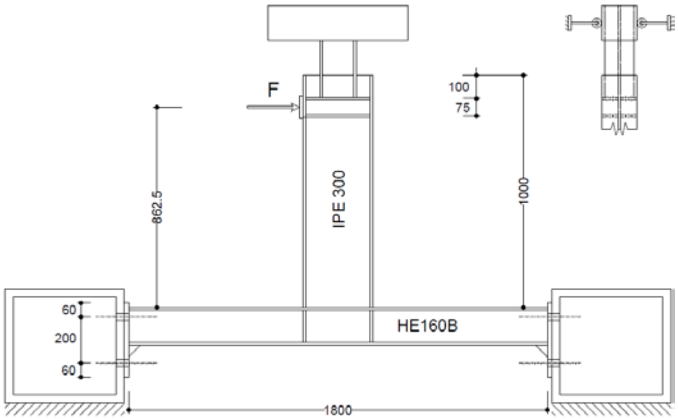

Fig. 3. Schematic representation of the beam-to-column steal connection.

Fig. 4. Picture of the experimental device, after Castiglioni, Pucinotti 2009, [4].

To this purpose both LMM and ECM have been implemented in a FORTRAN main code which drives a commercial FE code, ADINA, used to perform all the elastic analyses involved in the procedure. In particular, a first main code drives the sequence of fictitious elastic analysis to build, in the LMM procedure, a plastic collapse mechanism of the real structure. This is pursued by updating iteratively (till convergence) the Young elastic moduli values (originally equal to 210 GPa) computed by Eq.(3) of the matching procedure, operating, as already said, at each GP of each element and at each iteration within the sequence. A second FORTRAN main code governs the ECM procedure, that drives the sequences of FE elastic analyses increasing, for each sequence, the acting load. Moreover, within each sequence, as described in section 2.2 the Young moduli are appropriately reduced in order to redistribute the acting load. A P_{LB} at the end of each sequence is evaluated till the redistribution of the load is possible. To reproduce the mechanical problem a FE mesh of 160, 8 node, shell elements has been adopted. Despite the mesh is quite coarse, for the considered example, the results in term of P_{UB} and P_{LB} are quite good, identifying a narrow band in which the collapse load multiplier is located. Figure 5, reports the evaluated P_{UB} and P_{LB}, versus the number of iterations and sequences, respectively. Such figure confirms the robustness and reliability of the applied procedure related to the registered monotonic convergence of the iterative methods and to the few number of iterations or sequences required.

Table 1. Geometrical and mechanical quantities of the Specimen.

IPE 300		**HE 160 B**	
Geometry [mm]		Geometry [mm]	
Height	299	Height	162.5
Base	151	Base	162
Flange thick.	10.3	Flange thick.	13
Web thick.	7.2	Web thick.	8.9
Yield stress σ_y [MPa]		Yield stress σ_y [MPa]	
Flange	274.78	Flange	323.13
Web	305.54	Web	395.56

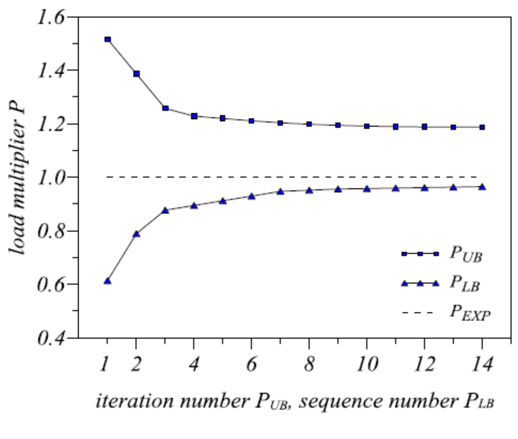

Fig. 5. P_{UB} and P_{LB} load multipliers versus iteration number and sequence number, respectively.

A further aspect to be highlighted is the capacity, shown by the LMM procedure also in other contexts [1, 5], to predict the collapse mechanism of the structure. Figure 6 reports the picture of the BCC5-E prototypes at collapse, while figure 7 reports the strains counterplots in the load direction at the last FE analysis of the LMM procedure. Figure 7 clearly individuate the plasticized zones which are reasonably close to the ones experimentally detected. Also the deformed shape of the numerical model is similar to the one experimentally detected showing how around the plasticized zones the remainder parts of the structure rotate rigidly, exhibiting, even if qualitatively, the peculiar trend of a plastic collapse mechanism. This kind of results is really interesting being useful to localize critical zones or weaker members within structures of larger dimensions.

Fig. 6. Specimen BCC5-E at collapse.

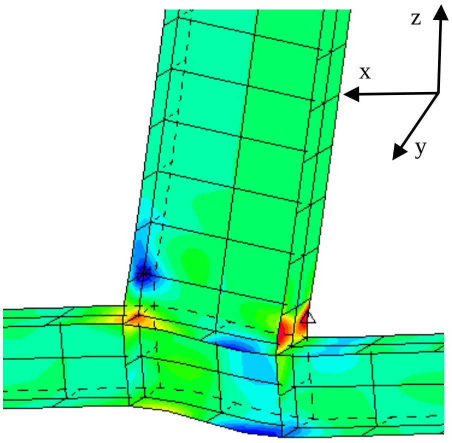

Fig. 7. Counterplots of the x strains component at last iteration of the LMM procedure.

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KEYWORDS

Steel Connections, Limit analysis, Numerical Modelling, FE-based procedure, Experimental test.