Ultimate load prediction of MMNCs structures

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Abstract

The paper proposes a *nonlocal limit analysis procedure* to evaluate the load bearing capacity of metal matrix nanocomposites (MMNCs) structural elements. The promoted procedure rephrases and extends, in a nonlocal context, a static limit analysis numerical method, known in literature as elastic compensation method. Due to the *ductility* and the *intrinsic nonlocal behaviour* of MMNCs materials, a nonlocal elastic-perfectly plastic constitutive behaviour is hypothesized with the further hypothesis that nonlocality belongs only to the elastic phase. An associative von Mises type yield condition together with a nonlocal elasticity model of integral type are adopted. A nonlocal formulation of the finite element method is used throughout and a numerical application is presented and critically discussed with the aim to test the capability of the method in capturing the main features of the novel nonlocal limit analysis approach.

Keywords: Nonlocal limit analysis, Nonlocal FEM, Metal matrix nanocomposites, Computational modelling.

1. Introduction

In recent years, a great research effort has been devoted to the production and the analysis of new composite materials designed for structural and industrial applications. To be competitive, such materials have to possess, beside the desirable lightweight, an improved mechanical behavior that means higher stiffness and strength properties, appropriate damping capacity, wear resistance etc. Metal matrix nanocomposites (MMNCs) which are obtained by adding, to the metal matrix (usually steel, aluminium or magnesium alloys), stronger nanoparticles, such as aluminium and yttrium oxide, carbon nanotubes etc., seem to provide, among others, the research answer to the industrial needs (see e.g. Casati and Vedani [1] or Mirza and Chen [2] and references therein). Notwithstanding the foregoing affirmed, the widespread use of these materials is still limited in practical applications due to the difficulty in modeling their constitutive behavior mainly for what concern the post-elastic phase and consequently in using commercial numerical codes for the analysis of structural elements consisting thereof. It should be further noted that, by their own nature, the MMNCs are materials, in which the diffusive processes arising at a nano-scale level influence the behavior at the macroscopic level, [5], so that a proper modeling of these materials cannot apart from use of the so called *nonlocal approaches* (see e.g. Bažant and Jirásek [3] or Bažant and Cedolin [4]).

To overcome the drawbacks related to a reliable description of the post-elastic behavior, taking also into account the nonlocal nature of structural elements made of MMNCs and in the final perspective of providing useful information about the load carrying capacity of such elements, this work proposes a *plastic limit analysis* approach of *nonlocal type*. This kind of approach appears plausible due to an overall ductile behavior shown by MMNCs, [6]. In particular, a limit analysis procedure, known as *elastic compensation method*, [7] is considered *to evaluate a lower bound* to the plastic collapse load multiplier of MMNCs structural elements. The procedure, which makes use of sequences of elastic analyses in order to mimic the behavior of the structure at a state of incipient collapse, is here formulated *in a nonlocal elastic context*. To this purpose, a nonlocal elastic-perfectly plastic constitutive model is assumed with the hypothesis that the nonlocal behavior pertains only to the elastic phase. The nonlocal elastic description is pursued by an enhanced version of the nonlocal integral Eringen model,[10] , while the nonlocal elastic computations, required by the promoted limit analysis numerical procedure, are performed with a nonlocal finite element method (NL-FEM) implemented by the authors in [8]. A numerical example is presented and the obtained results are critically discussed to highlight limits and potentials of the novel approach.

2. Theoretical framework

2.1. Constitutive assumptions

Structural elements made of MMNCs are considered in their elastic regime, under the hypotheses of small strains, plane stress conditions and loads acting in a quasi-static manner. To take into account the presence of nanoparticles within the metal matrix, the elastic behavior is modelled as nonlocal, i.e. a constitutive relation of integral type and containing two internal material parameters is assumed. By hypothesis the nonlocal behavior pertains only to the elastic phase, the post elastic phase is assumed to be (local) perfectly-plastic and obeying an associative flow rule of von-Mises type (see e.g. Ref.[9]). The postulated constitutive equations read:

$$\boldsymbol{\sigma}(\boldsymbol{x}) = \boldsymbol{D}(\boldsymbol{x}) : \mathcal{R}(\boldsymbol{\varepsilon}(\boldsymbol{x}) - \boldsymbol{\varepsilon}^{\boldsymbol{p}}(\boldsymbol{x})) = \boldsymbol{D}(\boldsymbol{x}) : \mathcal{R}(\boldsymbol{\varepsilon}^{\boldsymbol{e}}(\boldsymbol{x})) \tag{1}$$

$$\dot{\boldsymbol{\varepsilon}}^{\boldsymbol{p}}(\boldsymbol{x}) = \dot{\lambda}\frac{\partial f(\boldsymbol{\sigma}(\boldsymbol{x}))}{\partial \boldsymbol{\sigma}(\boldsymbol{x})} \tag{2}$$

$$f(\boldsymbol{\sigma}) \leq 0; \qquad \dot{\lambda} \geq 0; \qquad \dot{\lambda} f(\boldsymbol{\sigma}) = 0. \tag{3}$$

In the above equations: $\boldsymbol{\sigma}(\boldsymbol{x})$ and $\boldsymbol{\varepsilon}(\boldsymbol{x})$ are the stress and the strain tensors, the apices p and e stand for *plastic* and *nonlocal elastic*, respectively; $\boldsymbol{D}(\boldsymbol{x})$ is the elastic moduli tensor, $f(\boldsymbol{\sigma})$ is the assumed von-Mises yield function and plastic potential so that Eqs.(2) and (3) are the classical flow laws of standard J_2 plasticity. Finally, $\mathcal{R}$ is a *regularization operator* obtainable from the following explicit form of Eq.(1):

$$\boldsymbol{\sigma}(\boldsymbol{x}) = \boldsymbol{D}(\boldsymbol{x}) : \boldsymbol{\varepsilon}^{\boldsymbol{e}}(\boldsymbol{x}) - \alpha \int_V \boldsymbol{\mathcal{J}}(\boldsymbol{x}, \boldsymbol{x}') : [\boldsymbol{\varepsilon}^{\boldsymbol{e}}(\boldsymbol{x}') - \boldsymbol{\varepsilon}^{\boldsymbol{e}}(\boldsymbol{x})] \,\mathrm{d}\,V' \quad \forall\, (\boldsymbol{x},\, \boldsymbol{x}') \in V. \tag{4}$$

The novelty, with respect to a local treatment, is given by Eq.(4), which is a nonlocal elastic relation of Eringen-type already experienced by the authors in ([8], [10]). More precisely, equation (4) represents a two-phases local-nonlocal material model, the proportion of the nonlocal phase being governed by the nonlocal material parameter α. The nonlocal tensor $\boldsymbol{\mathcal{J}}(\boldsymbol{x}, \boldsymbol{x}')$, entering Eq.(4), is defined in terms of: the elastic moduli tensor evaluated at $\boldsymbol{x}$ and $\boldsymbol{x}'$; a scalar *attenuation function*, say $\mathrm{g}(\boldsymbol{x}, \boldsymbol{x}')$, together with a weighted form of it. By hypothesis, $\mathrm{g}(\boldsymbol{x}, \boldsymbol{x}')$ depends on an *internal length* material scale, say ℓ, as well as on the Euclidean distance, $|\boldsymbol{x} - \boldsymbol{x}'|$, between points $\boldsymbol{x}$ and $\boldsymbol{x}'$ in

V. The attenuation function plays a crucial role in nonlocal theory (see again [8]), assigning a weight to the nonlocal effects induced at the field point $\boldsymbol{x}$ by a phenomenon acting at the source point $\boldsymbol{x}'$. The shape of $\mathrm{g}(\boldsymbol{x}, \boldsymbol{x}')$, beside the value assumed for ℓ, determines the so called influence distance, say L_R, i.e. the length denoting the wideness of the continuum region, neighbor of $\boldsymbol{x}$, within which long distance nonlocal interactions act. Skipping the analytical details, given in the previously quoted papers, it is worth noting that the adopted nonlocal elastic model (4) belongs to the general class of microcontinuum field theories which try to model diffusive processes arising at microscopic molecular level by macroscopic material parameters, as α and ℓ in (4). The need for doing so is related to the necessity of overcome some deficiencies of local classical elasticity which is unable to take into account phenomena arising within the microstructure of the material that, however, have a relevant influence on the material/ structural behavior (see e.g. stress description at a tip of a crack, size effects, material containing nano-particles, and so on).

Approaches based on nolocal integral theories can be traced back to the late fifties and even before (see e.g. Refs. [13], [12], [14], [11]); more recently, gradient theories (see e.g. Aifantis and co-workes [15], [16]); peridynamic theories (see e.g. Silling [17], [18]), continualization theories [19] moved in the same direction and basically with the same goal of solving problems the classical elasticity cannot solve. A part from the very different analytical and conceptual structure of the above theories, they all share a common feature, namely the assumption of an internal length material scale. As suggested in [20], [21], just to quote one of the most attractive nonlocal approach, if the

analysis is carried on at micro– or molecular–level the value of ℓ should be related to the intermolecular distance, but if, as desired, the analysis is carried on at a macroscopical level, ℓ, or a quantity related to ℓ defining at which extent nonlocal interaction act, should be defined according to convenience. In practice the value of ℓ has to be chosen taking into account the element-size within a FE approach or the grid-size within a meshless approach.
Focusing the attention on the integral nonlocal approach here assumed, if, from one side, an abundant recent literature deals with the analytical shape of the kernel $\mathrm{g}(\boldsymbol{x}, \boldsymbol{x}')$ (see e.g. Ref. [22]), on the other side, to the authors' knowledge, no experimental studies have solved the problem of determining a reliable value of ℓ for a given nonlocal material. The identification of ℓ, as well as of α, the second material parameter entering the adopted model, remains a point, not tackled here, left open to discussion being the weak point of the here adopted, but perhaps of all, nonlocal approaches when applied to real problems. Different values of ℓ will be assumed in the sequel to highlight the differences between a local and a nonlocal treatment of a FE-based limit analysis procedure applied to a MMNCs structural element.

2.2. Nonlocal finite elements

As described in details in [8], to which we refer for sake of brevity, on the base of Eq.(4) and of a *nonlocal total potential energy principle*, first conceived in [11], a nonlocal formulation of the well known finite element method can be developed. The main peculiarity of NL-FEM is the circumstance that each element, beside the standard local stiffness matrix, is endowed with a set of nonlocal element stiffness matrices whose entries derive from the mutual interactions with the other elements in the mesh. Such set of element matrices can be given the shape:

$$\boldsymbol{k}_n^{nonloc} := \int_{V_n} \boldsymbol{B}_n^T(\boldsymbol{x})\, \gamma^2(\boldsymbol{x})\, \boldsymbol{D}\,(\boldsymbol{x})\, \boldsymbol{B}_n(\boldsymbol{x})\, \mathrm{d}\, V_n, \tag{5}$$

$$\boldsymbol{k}_{nm}^{nonloc} := \int_{V_n}\int_{V_m} \boldsymbol{B}_n^T(\boldsymbol{x})\, \boldsymbol{\mathcal{J}}\,(\boldsymbol{x}, \boldsymbol{x}')\, \boldsymbol{B}_m(\boldsymbol{x}')\, \mathrm{d}\, V_m\, \mathrm{d}\, V_n, \tag{6}$$

where: $\boldsymbol{B}_n(\boldsymbol{x})$ and $\boldsymbol{B}_m(\boldsymbol{x})$ are the usual notations for the element matrices containing the shape functions derivatives pertaining to elements $\#n$ and $\#m$, respectively; $\boldsymbol{\mathcal{J}}(\boldsymbol{x}, \boldsymbol{x}')$ is the nonlocal operator(in matrix form) entering model (4) and $\gamma(\boldsymbol{x})$ is a scalar-valued nonlocal operator equal to the integral on the domain volume V of the function $\mathrm{g}(\boldsymbol{x}, \boldsymbol{x}')$, (see again [8] for further details). The nonlocal nature of matrices (5) and (6), pertaining to element $\#n$, is witnessed both by the presence of $\gamma(\boldsymbol{x})$ and the double integration carried on element $\#n$ and its neighbor $\#m$, respectively. The structure of this matrices, all derived through a variationally consistent rationale, renders each NL-FE able to catch the (nonlocal) interactions exerted on it by the neighbors NL-FEs.

It is worth noting that the NL-FEs mutual interactions are confined to within a certain number of neighbors elements, i.e. the ones belonging to an influence zone whose wideness depends from the material internal length ℓ. The assembling procedure, yielding a global nonlocal stiffness matrix and a (standard) global nodal force vector, follows the standard procedure of FEM. Obviously, the global nonlocal stiffness matrix will collect many contributions for each NL-FE so resulting with a bandwidth larger with respect to the standard (local) one. All the integrations are performed with standard rules, standard it is also the way the boundary conditions are enforced. Once the global system of algebraic equations is solved, stresses, strains and related quantities, are easily output at the sampling points and/or at nodes. A recent contribution, [23], explored the correct way to handle structural symmetric nonlocal problems, but this is out of the scope of the present work.

3. Nonlocal limit analysis

A *plastic limit analysis approach* of *nonlocal type* is proposed for the addressed MMNCs structural elements. The approach grounds on two main remarks: *i)* MMNCs structural elements exhibit a *ductile* behavior, [6]; so to search for a plastic collapse load, with reference to an associative von Mises-type yield condition, seems appropriate in this context; *ii)* the limit analysis here performed is aimed to the *evaluation of a lower bound* (LB) to the plastic collapse load multiplier. Such LB is evaluated through a FE-based procedure, already experienced by the authors in different contexts, ([25], [26], [27], [28]) and requires *only sequences of elastic analyses.*

The key-idea of the approach promoted in this paper is then to rephrase the LB-FE-based procedure performing *sequences of nonlocal elastic FE analyses.* The NL-FEM of section 2.2 will be the numerical tool. The constitutive assumptions of section 2.1, confining the nonlocality of the material only to the elastic phase, will be also consistently met.

The procedure to be rephrased is known, in the local context, as elastic compensation method (ECM) and belongs to the static approach of limit analysis aimed at evaluate the maximum load value, in equilibrium with a plastically admissible stress field, at which the structure find itself at a state of incipient collapse. *The nonlocal ECM* (NL-ECM) will mimic the *redistribution* of nonlocal stresses arising within the structure, discretized in NL-FEs,

and subjected to load increasing up to collapse. To this aim, as in the local version, the redistribution is operated performing many sequences of nonlocal elastic FE analyses. At each sequence, the applied loads, say $P^{(s)}\bar{\mathbf{p}}$ with $P^{(s)}$ = load multiplier of the current sequence (s) and $\bar{\mathbf{p}}$ = reference loads, are increased of a fixed increment, say $\triangle P$, so that $P^{(s+1)} = P^{(s)} + \triangle P$. Within the current sequence a NL-FE analysis of the structure is carried on *iteratively* redistributing the computed nonlocal elastic stresses by *reducing* the elastic modulus of each NL-FE in which a scalar (von Mises) measure of the nonlocal elastic stress attains or is beyond an admissible threshold. The sequences stop when, for the current applied load, the redistribution fails, that is when a fixed maximum value of iterations (NL-FE analyses) is attained without succeeds in bringing all the stresses below or at the admissible threshold.
The NL-ECM can be easily explained with reference to the sketch of Fig.1 where the assumed von Mises surface is represented in the deviatoric π-plane as:

$$\rho^2 - \rho_y = 0, \tag{7}$$

which is a circle of radius $\rho_y := \sqrt{\frac{2}{3}}\sigma_y$ (σ_y being the uniaxial yield stress) with $\rho := \sqrt{2J_2}$ (J_2 being the second deviatoric stress invariant).
At start, i.e. for $s = 1$ (first sequence), a load $P^{(s)}\bar{\mathbf{p}}$ is applied to the structure discretized in NL-FEs. $P^{(s)}$ denotes the load multiplier of the current sequence, $\bar{\mathbf{p}}$ the reference loads. A first ($k = 1$) NL-FE analysis is carried out and averaged (within the GPs of the element) nonlocal stress value $\rho_{\#e}^{(k-1)}$ is computed for each NL-FE#e. Within the elements where $\rho_{\#e}^{(k-1)}$ is greater than ρ_y (see again Fig.1) the elastic modulus is reduced to *bring* the not admissible stress onto the yield surface according to:

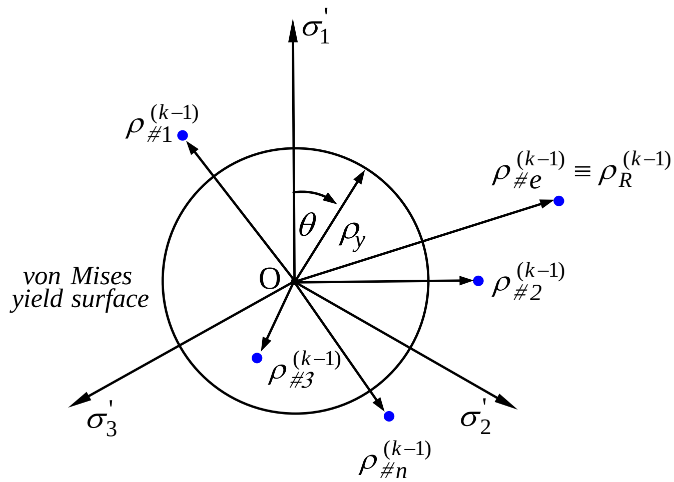

Figure 1: Geometrical sketch of the limit analysis procedure in the deviatoric plane: averaged scalar-valued stresses measured within the elements $\#1$, $\#2$, $\#3, ..., \#e, ..., \#n$ to apply NL-ECM, $\rho_{max}^{(k-1)}$ denoting the "maximum stress" among all the elements in the mesh.

$$E_{\#e}^{(k)} = E_{\#e}^{(k-1)} \left[\frac{\rho_y}{\rho_{\#\,e}^{(k-1)}} \right]^2 . \tag{8}$$

Such values are sored to be used at next iteration (analysis). The "maximum stress" in the whole mesh is also detected, i.e. the stress point farthest away from the von Mises circle, $\rho_{max}^{(k-1)}$ in Fig.1. If such maximum value is greater than ρ_y you try to redistribute the current load $P^{(s)}\bar{\mathbf{p}}$ performing a new NL-FE analysis with the updated $E_{\#e}^{(k)}$ moduli; if $\rho_{max}^{(k-1)}$ is less than ρ_y compute a lower bound multiplier as:

$$P_{LB}^{(k-1)} = \rho_y \frac{P^{(s)}}{\rho_{max}^{(k-1)}}, \tag{9}$$

increase the intensity of the acting loads setting $P^{(s+1)} = P^{(s)} + \triangle P$ ($\triangle P$

= fixed increment of the load multiplier) and perform a new sequence of elastic analyses in the attempt to redistribute a greater load. The sequences stop when the load increase does not allow the maximum stress to be brought below, or at, yield by redistribution. The P_{LB} is then the last computed value corresponding to the last admissible stress field, say at iteration K, attained for the maximum (redistributed) load, say the one obtained at sequence $(\mathcal{S})$, i.e. $P^{(\mathcal{S})}\bar{\mathbf{p}}$. The P_{LB} is eventually given by:

$$P_{LB} = \rho_y \frac{P^{(s)}}{\rho_{max}^{(K)}}. \tag{10}$$

4. Validation of the proposed procedure

The nonlocal limit analysis approach presented in the previous sections is employed to solve a simple numerical example. Beside to verify the numerical implementation of the whole procedure, which is not trivial, the numerical example is aimed to test the capability of the method to capture the effects of nonlocality in the ultimate load value predicted for the structural element made of MMNCs, precisely: $1.5\% CNTs/AZ91D (after [29])$. A square plate of length L, thickness t and subjected to tensile uniform stress at the right side is considered. On the opposite left side the plate is constrained so to prevent displacements in both horizontal and vertical directions. In the left part of the upper and lower edges and for a length equal to b the vertical displacements are also impeded. The mechanical configuration of the plate is drawn in Fig.2; the latter reports the geometrical data, the sketch and the intensity of the applied load as well as the mechanical parameters of the considered MMNCs, which is assumed to obey the constitutive relation given

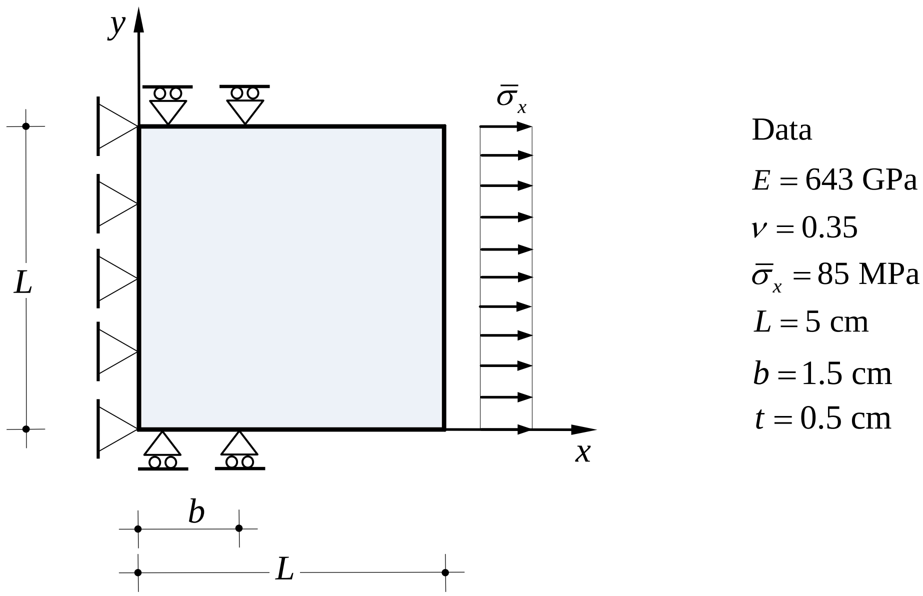

Figure 2: Mechanical sketch of the analysed nonlocal square plate.

in (4). In the run example, and in absence of appropriate experimental data, the nonlocal parameter α is set equal to 50, while the value of the nonlocal parameter, ℓ, is varied in a range from 0 (local case) to 5. As explicitly declared in Section 2.1, the purpose of the application is not so much to perform a sensitivity analysis varying the nonlocal parameters, but rather to highlight the differences between a local and a nonlocal limit analysis procedure both in terms of lower bound to the plastic collapse limit load multiplier and distribution of plastically admissible stresses at collapse. The procedure has been implemented by the authors in a fortran code which performs the sequences of nonlocal elastic analyses of the structure by the NL-FEM code. To this purpose, the plate of Fig.2 is discretized using a uniform mesh consisting in 100 isoparametric, 8-nodes, plane stress nonlocal elements.

ℓ	P_{LB}	*Num Seq*	*Diff %*
0	1.20	12	--
1	0.90	14	25
3	1.12	9	6.6
5	1.18	15	1.6

Table 1: Lower bound multipliers, P_{LB}, for the analyzed plates; number of sequences performed, $Num\ Sequ$; differences, in percentage, between local and nonlocal cases, *Diff* %

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The result of the analyses, in terms of P_{LB}, are reported in Table 1 for the four examined ℓ-value cases. By inspection of the predicted values appears evident that, if a nonlocal behavior of the plate is assumed, the model returns a *more conservative* lower bound. The differences, with respect to the local case, present a wide range of variability (from 1.6% to 25%) showing a *significant dependence* of the predictable lower bound of the plastic collapse limit load of the structure from the nonlocal features of its constituent material. It has also to be noted that, for all the examined cases, the P_{LB} value is reached in few sequences, so confirming the robustness of the NL-ECM method. Further considerations can arise by observing the contour plots of the stress scalar measure ρ, reported in Fig.3, over the entire mesh and at last considered sequence of the NL-ECM procedure. Indeed, the distribution of the stress invariant ρ furnishes information about the admissible stress field related to the maximum redistributable load; in particular, the zones of

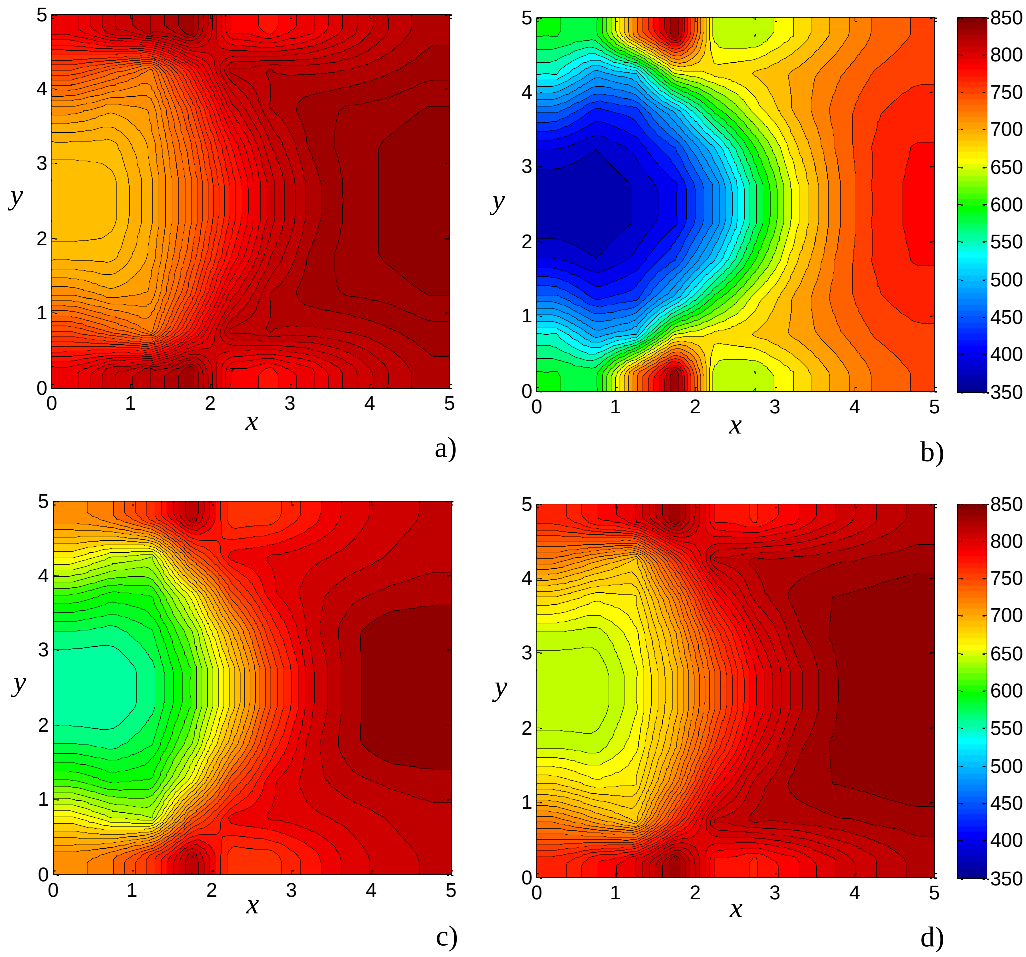

Figure 3: Contour plot of the stress invariant ρ $[daN/cm^2]$ at last sequence of the NL-ECM procedure and for $\ell = 0$ (local case) a); $\ell = 1$ b); $\ell = 3$ c); $\ell = 5$ d).

the structure in which the value of ρ is close to its threshold limit (ρ_y) locate the zones of the structure in which plastic mechanisms are at an incipient activation stage. The differences in ρ distributions, being equal all the other parameters, are greater the smaller are the values of ℓ. Comparing Figures 3 a) and b), corresponding to the cases $\ell = 0$ and $\ell = 1$, respectively, it can be observed as in the first case (local) there is a sort of uniformity in the values of ρ (and therefore of the stress state) being the area with values of ρ near to its threshold close to the loaded edge. Quite different appears the situation in the second case, (for $\ell = 1$), where the values of ρ near to the limit threshold are registered in the two narrow areas of wideness b at the upper and lower edges on the left. Figure 3 b), (for $\ell = 1$), shows also a less redistribution of the stresses within the structure which justifies the lower value of the P_{LB} in this case with respect to the local one. The same qualitative behavior can be observed for the cases $\ell = 3$ and $\ell = 5$, Fig 3c) and 3d) respectively, showing this time that more ℓ is larger more the "nonlocal" result is close again to the local one. This last assertion is understandable looking at the adopted constitutive model (4) and considering the way the attenuation function acts. Indeed, model (4) returns results very close to the local ones either for values of ℓ approaching *zero* and for values of ℓ being *large* with reference to the size of the structural element and this by the flattering at the weighting (attenuation) function $\mathrm{g}(\boldsymbol{x}, \boldsymbol{x}')$ for *large* ℓ values.

5. Conclusions

The paper has presented a nonlocal formulation of the elastic compensation method, a limit analysis numeriaca procedure aimed to evaluate a lower bound to the plastic collapse load multiplier of structural elements made up of metal matrix nanocomposites. Precisely, materials whose constitutive description requires a nonlocal treatment. The promoted approach has been developed assuming for the MMNCs a nonlocal elastic-perfectly plastic behaviour in which the nonlocality, of integral type, has been confined only to the elastic phase. An associative von Mises-type yield condition has also been hypothesized. The nonlocal elastic analyses required by the presented nonlocal limit analysis procedure have been performed by means of a NL-FE code already developed by the authors. The procedure has been tested with the aid of a simple numerical example which has highlighted the differences between the local and the nonlocal approach both in terms of lower bound to the elastic collapse load multiplier and in terms of distribution of stresses at a state of incipient collapse. In particular, it has been evidenced that the nonlocal limit analysis returns a more conservative lower bound and that also the redistribution of the stresses at collapse is different. Eventually, the differences in the numerical findings have shown a significative dependence from the nonlocal features of the constituent material. It is worth noting that very attractive "non-conventional" limit analysis formulations have been proposed in recent contributions in the realm of strain gradient plasticity (see e.g.[30], [31] and references therein). The present contribution is obviously much different from those above, not only because it takes advantage from a numerical procedure which, in the nonlocal context here addressed, requires only nonlocal elastic FE analyses. This task is easier from a computational point of view possessing an apposite NL-FEM code already experienced.

From a wider point of view, the quoted nonconventional limit analysis theories and the present nonlocal limit analysis theories and the present nonlocal limit analysis procedure move in the same direction, that is they offer a new perspective for a classical design tool, as it was limit analysis for steel structures many decades ago, in the realm of advanced innovative materials as, for example, the metal matrix nanocomposites have addressed. The authors, avoiding enthusiastic conclusions, are conscious that the presented results are at embryonic stage however they give a sufficient confidence on the novel method here proposed.

References

[1] Casati R., Vedani M. Metal Matrix Composites Reinforced by Nano-Particles - A Review. Metals 2014; 4: 65-83.

[2] Mirza F.A., Chen D.L. A Unified Model for the Prediction of Yield Strength in Particulate-Reinforced Metal Matrix Nanocomposites. Materials 2015; 8: 5138-5153.

[3] Bažant Z.P., Jirásek M. Nonlocal Integral Formulations of Plasticity and Damage: Survey of Progress. Journal of Engineering Mechanics 2002; 128(11): 1119-1149.

[4] Bažant Z.P., Cedolin L. Stability of Structures: elastic, inelastic, fracture and damage theories. World Scientific Publishing Company 2010.

[5] Zhang Z., Chen D.L. Consideration of Orowan strengthening effect in particulate-reinforced metal matrix nanocomposites: A model for predicting their yield strength. Scripta Materialia 2006; 54: 1321-1326.

[6] Goh C.S., Wei J., Lee L.C., Gupta M. Simultaneous enhancement in strength and ductility by reinforcing magnesium with carbon nanotubes. Materials Science and Engineering A 2006; 423: 153-156.

[7] Pisano A.A., Fuschi P., De Domenico D. Limit analysis on RC-structures by a multi-yield-criteria numerical approach. Direct methods for limit and shakedown analysis of structures: advanced computational algorithms and material modelling. Springer International Publishing 2015.

[8] Fuschi P., Pisano A.A., De Domenico D. Plane stress problems in nonlocal elasticity: finite element solutions with a strain-difference-based formulation. Journal of Mathematical Analysis and Applications 2015; 431: 714-736.

[9] Polizzotto C., Borino G., Fuschi P. Shakedown of cracked bodies with nonlocal elasticity. Proceeding of ECCOMAS2000. Barcelona 11-14 September 2000.

[10] Polizzotto C., Fuschi P., Pisano A.A. A nonhomogeneous nonlocal elasticity model. European Journal of Mechanics A Solids 2006; 25: 308-333.

[11] Polizzotto C. Nonlocal elasticity and related variational principles. Int. Journal of Solids and Structures 2001;38: 7359-7380.

[12] Edelen, D.G.B., Laws, N. Thermodynamics with internal variables. Journal of Chemical Physics 1971; 47: 597-613.

[13] Eringen, A.C. Nonlocal polar elastic continua. Int. Journal of Engineering Science 1972; 10: 1-16.

[14] Rogula D. Introduction to nonlocal theory of material media. In: Nonlocal theory of material media, D. Rogula (Ed.), Springer-Verlag, Berlin 1982; 125-222.

[15] Aifantis E.C. Update on a class of gradient theories. Mechanica Materialia 2003; 35: 259-280.

[16] Askes H., Aifantis E.C. Gradient elasticity in statics and dynamics: an overview of formulations, length scale identification procedures, finite element implementations and new results. International Journal of Solids and Structures 2011; 48: 1962-1990.

[17] Silling S.A. Reformulation of elasticity theory for discontinuities and long-range forces. Journal of Mechanics and Physics of Solids 2000; 48(1): 175-209.

[18] Silling S.A., Epton M., Weckner O., XuJ., Askari E. Peridynamic States and Constitutive Modeling. Journal of Elasticity 2007; 88: 151-184.

[19] Andrianov I.V., Awrejcewicz J., Weichert D. Review Article. Improved Continuous Models for Discrete Media. Hindawi Publishing Corporation Mathematical Problems in Engineering 2010. Article ID 986242.

[20] Macek R.W., Silling S.A. Peridynamics via finite element analysis. Finite Elements in Analysis and Design 2007; 43: 1169 - 1178.

[21] Chowdhury S.R., Roy P., Roy D.,Reddy J.N. A peridynamic theory for linear elastic shells. International Journal of Solids and Structures 2016; 84:110132.

[22] Ghosh S., Sundararaghavan V., Waas A.M. Construction of multi-dimensional isotropic kernels for nonlocal elasticity based on phonon dispersion data. International Journal of Solids and Structures 2014; 51: 392-401.

[23] Pisano A.A., Fuschi P. Structural symmetry within nonlocal integral elasticity: theoretical issues and computational strategies. Curved and Layered Structures 2017; 4: 17.

[24] Mackenzie D., Boyle J.T. A method of estimating limit loads by iterative elastic analysis, Parts I, II, III. International Journal of Pressure Vessels and Piping 1993; 77: 77-142.

[25] Pisano, A.A., Fuschi, P. Mechanically fastened joints in composite laminates: Evaluation of load bearing capacity. Composites Part B: Engineering 2011; 42(4): 949-961.

[26] Pisano AA, Fuschi P, De Domenico D. A layered limit analysis of pinned-joints composite laminates: Numerical versus experimental findings. Composites: Part B 2012; 43: 940-952.

[27] Pisano A.A., Fuschi P., De Domenico D. Failure modes prediction of multi-pin joints FRP laminates by limit analysis. Composites Part B: Engineering 2013; 46: 197-206.

[28] De Domenico D., Pisano A.A., Fuschi P. A FE-based limit analysis approach for concrete elements reinforced with FRP bars. Composite Structures 2014; 107: 594-603.

[29] Liu S., Gao F., Zhang Q., Zhu X., Li W. Fabrication of carbon nanotubes reinforced AZ91D composites by ultrasonic processing. Transactions of Nonferrous Metals Society of China 2010; 20(7):1222-1227.

[30] Polizzotto C. Strain gradient plasticity, strengthening effects and plastic limit analysis. International Journal of Solids and Structures 2010; 47: 100-112.

[31] Polizzotto C. Size effects on the plastic collapse limit load of thin foils in bending and thin wires in torsion. European Journal of Mechanics - A/Solids 2011; 30(6): 854-864.