Constrained Focusing of Vector Fields Intensity in Near Zone and/or Complex Scenarios as a Low-Dimensional Global Optimization

G. M. Battaglia, A. F. Morabito, R. Palmeri, and T. Isernia

A hybrid effective approach is proposed to focus the intensity of a vector field generated by an arbitrary fixed-geometry array antenna into a target point and keep it bounded elsewhere. To overcome the complexity of the underlying nonconvex problem involving a possibly large number of unknowns, we show how the space of possible polarizations can be regarded as a 5-sphere and introduce a nested procedure which jointly relies on an (external) global optimization of the field polarizations. The approach can deal with both the cases of near-field (NF) and far-field (FF) focusing as well as with complex inhomogeneous 3-D media. The high-performance results achieved through full-wave simulations of realistic scenarios confirm the actual feasibility of tackling the problem as a low-dimensional global optimization, so that the best possible focusing can be hopefully realized.

Keywords: array synthesis; convex programming; electromagnetic focusing; hybrid optimization.

1. Introduction

The canonical problem of focusing a field at a given location while controlling its intensity level elsewhere has a considerable interest in many applications, including array antenna synthesis [1]-[7], targets localization [8], and deep tissue hyperthermia treatments planning [9]. In particular, driven by recent emerging applications such as radio frequency identification (RFID) systems [10]-[12], medical systems for microwave imaging [13] or superficial hyperthermia [14], gateway control system [15]

or wireless power transmission [16], a considerable body of work has been developed also for the problem of focusing in the NF region.

The general purpose of the aforementioned applications is to maximize the field intensity in a size-limited spot belonging to a region of space close to or far from the antennas. Notably, opposite to more classical FF focusing problems, a full 3-D vector treatment of the field is required for the NF focusing problem.

In a first instance, one can consider two different cases, i.e., the case where one is interested in focusing a scalar field (or a single component of a vector field) and the case where one pursues instead the focusing of the overall intensity of the vector field at hand. A further element leading to different focusing problems is whether or not one is interested in controlling the level of the field intensity outside of the target region (which leads to a constrained optimization problem). Last, but not least, in an eventual classification of the different problems, one also has to consider the degrees of freedom at disposal for synthesis. In fact, one can consider the amplitude and the phase of the excitations of an array [17], or just the phases of these latter [18], or the aperture-field distribution [19], and many other cases including the case where locations of the array are unknown.

In the following, we focus on the case where a constrained optimization is of interest, and the degrees of freedom of the field are the amplitude and the phase of the excitations of a fixed-geometry array¹. In order to effectively discuss its advantages, in the following we briefly recall the main features of some available techniques.

A recent approach is the one in [16], where the focused field at a target point located in the NF zone is obtained by an iterative algorithm ensuring that the radiated field lies in a specific mask. In case focusing of a single component is of interest, the technique allows full control of the depth of focus, spot diameter, and side lobe level (SLL).

¹ As continuous aperture sources can be properly discretized [3], the presented approach is of interest also for that case.

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An elder overlooked approach dealing with the focusing of scalar fields with constrained SLL is the one in $[1]^2$. This latter has been then generalized and applied to 3D complex scenarios and hyperthermia problems [20] for the case where one component of the field is dominant. However, this strategy cannot be trivially extended to the constrained focusing of the intensity of a vector field. In fact, when dealing with vector fields, the cost function to be optimized cannot be reduced anymore to a linear function of the unknowns and therefore suitable strategies are required to achieve the globally optimal solution of the arising non-convex problem [21].

An attempt to solve the field intensity focusing problem has been made in [22] where, however, the formulated optimization problem (acting directly on excitations) results in a non-convex optimization on many variables, so that solution procedures may still get stuck at local optima.

A recent general approach partially overcoming the above difficulties has been introduced in [23]. In the latter, exploiting [1], the overall problem is tackled as the solution of a sequence of convex problems. In fact, one can solve the convex problem corresponding to any possible field polarization into the target point and then select the solution which guarantees the best focusing performances. Unfortunately, the overall optimization procedure is very time consuming and/or affected by errors deriving from an insufficiently-fine discretization of the space of polarizations.

In the attempt of overcoming the above difficulties, we propose here a hybrid approach, for a generic fixed-geometry set of sources, based on nested optimizations where the inner (convex) problem looks for the excitations corresponding to the optimal polarization which, in turn, is externally pursued through global optimization (GO). Consequently, the optimal focusing problem is reduced to a GO of a cost functional which just depends on the field polarization.

The exploitation of GO algorithms is obviously not a novelty in the antenna synthesis community. However, differently from [24] (where other antenna problems

² Although written for the FF case, which is the reason while it has been overlooked in the NF literature, [1] explicitly notes the applicability to the cases of NF constraints and NF focusing.

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are considered), our GO-based problem only has to deal with a very reduced number of parameters (i.e., the five parameters identifying the field polarization at the target point – see Appendix). Such a feature plays a decisive role in order to get actually optimal solutions to the problem at hand. In fact, although still dealing with a non-convex problem, the approach is much more effective than all the ones where the excitations of the array (which can have very many elements) are directly looked for.

Finally, it is worth to underline that the approach is completely general, so it is able to optimally deal with NF and FF targets, NF and FF constraints, as well as with (known) inhomogeneous scenarios (which is the case of hyperthermia).

In the remainder of the paper, numerical examples in Section III corroborate the method proposed and discussed in Section II. Conclusions follow.

2. Intensity Focusing of Vector Fields via Nested Constrained Optimizations (IN-FOCO)

Given an arbitrary set of *N* sources with given locations, the total radiated field can be written as:

$$\boldsymbol{E}(\underline{\boldsymbol{r}}) = \sum_{n=1}^{N} I_n \boldsymbol{\Phi}_n(\underline{\boldsymbol{r}})$$
(1)

wherein \underline{r} and I_n denote the coordinate spanning the observation space and the *n*-th complex excitation coefficient, respectively, while $\boldsymbol{\Phi}_n(\underline{r})$ is the complex vector field induced by the unitary-excited *n*-th antenna in the region of interest Ω when all the other antennas are off. As such, the function $\boldsymbol{\Phi}_n(\underline{r})$ represents the *n*-th Active Element Field (AEF) and includes possible mutual-coupling and mounting-platform effects [25].

If we denote by $\underline{r}_t \in \Omega$ the target point, i.e., the point in which we want to focus the field intensity, the constrained focusing problem can be formulated as follows:

Determine the complex excitations set such to maximize $|\mathbf{E}(\underline{r}_t)|^2$ while enforcing elsewhere [i.e., $\forall \underline{r} \notin B(\underline{r}_t)$, $B(\underline{r}_t)$ indicating a given neighborhood of \underline{r}_t and identifying

the 'target' region] precise upper bounds on the field amplitude according to the particular application at hand³.

Unfortunately, the cost function, i.e., $|E(\underline{r}_t)|^2$, is a non-negative quadratic polynomial with respect to the unknowns. As such, the overall optimization problem is non-convex [21], so that one can be eventually trapped into sub-optimal solutions when solving it.

In order to develop a method robust with respect to such an issue, we will first consider an auxiliary preparatory case, and then we will turn back to the original general problem.

For the first (and auxiliary) case, let us initially consider the (still unknown) polarization plane of the field at the target point. In such a point the total vector field can be written as [26]:

$$E(\underline{r}_{t}) = \sum_{n=1}^{N} I_{n} \langle \boldsymbol{\Phi}_{n}(\underline{r}_{t}), \boldsymbol{p} \rangle \boldsymbol{p} + \sum_{n=1}^{N} I_{n} \langle \boldsymbol{\Phi}_{n}(\underline{r}_{t}), \boldsymbol{q} \rangle \boldsymbol{q}$$
$$= \sum_{n=1}^{N} I_{n} \varphi_{pn}(\underline{r}_{t}) \boldsymbol{p} + \sum_{n=1}^{N} I_{n} \varphi_{qn}(\underline{r}_{t}) \boldsymbol{q} \quad (2)$$

where p and q are the unit vectors associated to two generic orthogonal polarizations of the field.

If the polarization which guarantees the most convenient $|\mathbf{E}(\underline{r}_t)|^2$ value (say $\hat{\mathbf{p}}$) was apriori known, then the corresponding field into the target point would be equal to:

$$\boldsymbol{E}(\underline{r}_{t}) = \sum_{n=1}^{N} I_{n} \langle \boldsymbol{\Phi}_{n}(\underline{r}_{t}), \widehat{\boldsymbol{p}} \rangle \widehat{\boldsymbol{p}} = \sum_{n=1}^{N} I_{n} \varphi_{\widehat{\boldsymbol{p}}n}(\underline{r}_{t}) \widehat{\boldsymbol{p}} \quad (3)$$

Therefore, the focusing problem could be formulated as a Convex Programming (CP) one (as in [1]), i.e.:

Find the complex excitations I_n (n=1,...,N) such to:

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³ For instance, in case of oncological hyperthermia treatments, the upper bounds will be enforced in such a way to guarantee a safe field value on the healthy tissues surrounding the tumor [9].

$$\max_{I_1,\dots,I_N} \Re\left\{\sum_{n=1}^N I_n \varphi_{\hat{p}n}(\underline{r}_t)\right\}$$
(4)

subject to:

$$\Im\left\{\sum_{n=1}^{N} I_n \varphi_{\hat{p}n}(\underline{r}_t)\right\} = 0$$
(5)

$$|\boldsymbol{E}(\underline{r})|^2 \le UB(\underline{r}) \qquad \forall \underline{r} \notin B(\underline{r})$$
 (6)

where $\Re\{\cdot\}$ and $\Im\{\cdot\}$ respectively denote the real and imaginary parts of the complex arguments and $UB(\underline{r})$ is a non-negative arbitrary function (say the *mask* function) enforcing the upper bound constraint on the field intensity outside $B(\underline{r}_t)$.

Unfortunately, in actual cases, one does not know a priori the optimal polarization \hat{p} . On the other side, one can exploit the above result for the auxiliary problem in order to find an effective strategy for the full (and original) problem.

As a matter of fact, we know from the preparatory problem (4)-(6) that, for any fixed polarization, the problem reduces to a CP one, and we also know how to solve it. Therefore, since the polarization of a field is determined by only five parameters (see Appendix), our very simple idea is to introduce an optimization procedure looking for the best polarization \hat{p} . To this aim, we formulate the problem as a nested optimization wherein one looks externally (by means of a GO procedure) for the most convenient polarization while the internal optimization looks for the optimal excitations set corresponding to the polarization at hand. Accordingly, the new formulation (which we call IN-FOCO) will be as follows:

Find **p** and the corresponding I_n (n=1,...,N) in such a way that:

$$\max_{p} \left\{ \max_{I_1, \dots, I_N} \Re \left\{ \sum_{n=1}^N I_n \varphi_{pn}(\underline{r}_t) \right\} \right\}$$
(7)

subject to:

$$\Im\left\{\sum_{n=1}^{N}I_{n}\varphi_{pn}(\underline{r}_{t})\right\}=0$$
(8)

$$|\boldsymbol{E}(\underline{r})|^{2} \leq UB(\underline{r}) \qquad \forall \underline{r} \notin B(\underline{r}_{t}) \qquad (9)$$
$$\|\boldsymbol{p}\|_{2}^{2} = 1 \qquad (10)$$

wherein p is the unknown encoding the specific polarization [see the Appendix, where a discussion of constraint (10) is also included]. A flowchart describing the proposed nested approach is depicted in Figure 1.



Figure 1. Flowchart of the proposed nested optimization for the focusing of field intensity.

By virtue of the above discussion, in (7) the external minimization will require a GO tool, whereas the internal minimization, by virtue of its CP nature, can be conveniently solved by means of a fast local-optimization. As the external GO acts on polarization, while the internal one identifies the value of the cost function along with the corresponding optimal excitations for the trial polarization at hand, we are indeed optimizing simultaneously polarization and excitations in a nested fashion.

From a physical point of view, we can state that the original problem can be interpreted as a global optimization in the space of the possible polarizations. This is a

key point of the proposed method if we compare it with the method in [24] or many other ones where 'brute force' GO-based strategies are generally adopted and one has to deal with a number of real unknowns at least twice the number of antennas. In those cases, since the computational complexity of GO procedures is expected to exponentially grow with the number of unknowns [27], the global optimality of the solutions is very difficult to guarantee in the (finite) available time. Conversely, the present approach deals with a GO procedure having only five unknowns (more precisely, they belong to a zero-measure set of a five-dimensional space) and, although the convergence to the global optimum is not ensured in a deterministic sense, this problem is generally believed to be effectively solvable by the state-of-the-art computers.

By virtue of the adopted formulation, the proposed approach guarantees the fulfillment of constraints (9) (provided that they lead to a feasible optimization problem), while the optimization algorithm only stops once the field intensity is maximum at the target point. In this way, the user can always have the assurance of achieving the best focusing performance for the particular scenario and the particular array antenna at hand.

If the co-polarization (i.e., the desired polarization) is a-priori known, then the proposed approach automatically adopts it, i.e., it just finds the optimal array excitations associated to that polarization. Otherwise, if the most convenient polarization cannot be a-priori identified (as it happened for instance in the cases reported in Section 3), then the proposed method actually determines it, and then use it in order to find the optimal excitations.

Finally, it is also worth noting that, while in the near-field case or in the case of non-homogeneous region of space enclosed amongst the antennas (see for instance [29]) the problem requires to determine five parameters in order to find the globally-optimal polarization, in the far-field case the search space for the 'optimal' polarization can be restricted to the transverse plane, which further simplifies the determination of the optimal solution.

3. Numerical Results

In order to prove the actual feasibility and generality of the proposed IN-FOCO technique, as well as to test its performance in actual cases, we report two numerical examples dealing with the problem of focusing the radiation pattern of a planar array in the NF zone of the antenna and within a complex inhomogeneous scenario, respectively. In both cases, the achieved results are compared with the FOCO (FOcusing by Constrained Optimization) approach [1] which optimizes just a single (dominant) component of the field.

As far as the synthesis procedure is concerned, we first assigned the array layout and elements, computed the active element patterns, and assigned the target point \underline{r}_t and region $B(\underline{r}_t)$. Then, we assigned the upper-bound value for the field intensity outside the target region $B(\underline{r}_t)$. Finally, the numerical problem (7)-(10) has been solved. In particular, the internal and external parts of the minimization (7) have been respectively performed through the *fmincon* and *ga* routines of MATLAB (version R2016B)⁴, while the AEF were obtained through a FDTD full-wave commercial software.

In the first example, we tested the proposed focusing approach in the NF. The volume Ω is $4.5\lambda_{bg} \times 4.5\lambda_{bg} \times 2\lambda_{bg}$ large (λ_{bg} being the wavelength in the background medium) and located in the NF zone of a $\lambda_{bg}/2$ spaced array of 9×9 microstrip patch antennas working at 2.4 GHz and printed on a FR4 substrate with a relative permittivity equal to 4.4 and a thickness equal to 1.6mm. The dimensions of the patch have been set as in [28]. In particular, by referring to Figure 2, it is: W_p=29mm, L_p=29mm, W_g= $\lambda_{bg}/2$, L_g= $\lambda_{bg}/2$, W_f=3mm, L_f=L_g/2-L_p/2. The target region is an ellipsoidal volume $2\lambda_{bg}$ far

⁴ The *ga* algorithm is a stochastic, population-based algorithm that searches randomly by mutation and crossover among population members. Its parameters configuration has been performed as follows (see [30] for more details concerning the different variables): population size=50; elite count: 0.05*Population Size; number of variables=5; number of generations=100*number of variables; constraint tolerance: 1e-3; function tolerance: 1e-6; crossover fraction: 0.8.

On the other side, the *fmincon* algorithm setup has been performed as follows (see [31] for more details concerning the different variables): Algorithm: 'active-set'; Constraint tolerance: 1e-6; Max function evaluations: 3000; Max iterations: 1000; Optimality tolerance: 1e-6; Step tolerance: 1e-10.

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from the array aperture and whose semi-axes are $0.8\lambda_{bq} \times 0.8\lambda_{bq} \times \lambda_{bq}$.



Figure 2. Single radiating element for array considered in the first numerical example (see also Fig. 12 of [28]): (a) top view; (b) side view.

The outcomes of the proposed IN-FOCO approach are reported in Figure 3(b). By comparing these results with those achieved by the standard FOCO procedure [see Fig. 3(a)] optimizing just the y (dominant) component of the field, it can be seen that better focusing performance are granted by the novel approach. As a matter of fact, undesired 'hot spots', i.e., high field intensity outside the target region, are suppressed with the present approach, as also witnessed by the field cuts in Figure 3(c). Moreover, IN-FOCO provided a narrower beam. Notably, we were not able to achieve similar performance by means of the enumerative approach in [23].



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Figure 3. NF focusing - Power distribution synthesized according to the standard FOCO technique and the new IN-FOCO approach [\underline{r}_t =(-0.3051,-0.3051,-4.9888) λ_{ba}]: (a) normalized to maximum power deposition when only the y-component of the field is optimized; (b) normalized to maximum power deposition granted by the proposed technique; (c) superposition of the x-cuts of the two power distributions normalized to their maximum sidelobe level.

In the second example, we dealt with the complex non-homogeneous 3D scenario depicted in Figure 4, in which the region of interest Ω is a sphere of radius about $2\lambda_{ba}$ filled with air and embedding two dielectric objects, i.e., a cube and a sphere. The cube has a side of $\lambda_{bg}/2$ and a dielectric permittivity equal to 4, whereas the sphere has a radius of $\lambda_{bg}/4$ and a dielectric permittivity equal to 3. A cylindrical array of radius $4\lambda_{bg}$ and made up by 65 unitary-excited infinitesimal dipoles working at 1.5 GHz surrounds Ω . The array elements are arranged (with a random tilt angle) over 5 equallyspaced circumferences along the z-direction. Finally, the considered focusing target area is a sphere with a radius equal to $\lambda_{bg}/3$.



Figure 4. Reference inhomogeneous 3-D scenario adopted in the second numerical example.

As in the previous test case, the results achieved by the new IN-FOCO approach have been compared with the ones attained by the original FOCO one optimizing just the z (dominant) component of the field. As it can be observed from Figure 5, the proposed approach led to a better-focused (narrower) field granting a higher separation amongst the focused beam and sidelobes.

The achieved results are coherent with expectations as when a single given component of the field is taken into account the other components do not cooperate for the field intensity-focusing phenomenon. They also prove the capability of the approach to take actual advantage of the vector nature of the field.



Figure 5. Focusing in a complex scenario - Power distribution synthesized by the standard FOCO technique and the new IN-FOCO approach [\underline{r}_t =(-0.0687,- 0.0687,- 0.0687) λ_{bg}]: (a) normalized to maximum power deposition when only the z-component of the field is optimized; (b) normalized to maximum power deposition granted by the proposed technique; (c) superposition of the x-cuts of the two power distributions normalized to their maximum sidelobe level.

4. Conclusions

An innovative and effective approach has been proposed for focusing the intensity of a vector field into a target point while keeping it bounded elsewhere. In particular, it has been shown that, when dealing with vector fields, the non-convex focusing problem

at hand can be re-interpreted as the optimization of the polarization of the field at the target point.

Accordingly, a very general problem (including situations as different as FF, NF, and 3-D non-homogeneous regions of space, mutual coupling, mounting platform effects) has been reduced to an external global optimization in the space of polarizations nested with an internal convex programming optimization of the excitations guaranteeing, for any fixed polarization, the achievement of the unique (and hence globally-optimal) solution.

Although the approach still requires the exploitation of GO techniques, such an optimization just deals with a zero-measure set of a five-dimensional space, with the inherent advantages with respect to the cases where GO directly looks for the array excitations.

Appendix

This Appendix is aimed at illustrating the meaning of the five parameters we have used to encode polarization.

The total complex vector field in (1) is expressed by:

$$\boldsymbol{E}(\underline{r}) = E_x(\underline{r})i_x + E_y(\underline{r})i_y + E_z(\underline{r})i_z \qquad (A.1)$$

 i_x, i_y, i_z denoting the unit vectors of the Cartesian coordinate system. As a common phase constant does not alter polarization, a convenient choice is to assume that one component of the field, say $E_x(\underline{r}_t)$, is purely real in the target point. By so doing, a field having a generic polarization and amplitude can be represented by five real quantities, i.e.:

$$\Re\{E_x(\underline{r}_t)\}, \Re\{E_y(\underline{r}_t)\}, \Im\{E_y(\underline{r}_t)\}, \Re\{E_z(\underline{r}_t)\}, \Im\{E_z(\underline{r}_t)\}$$
(A.2)

In such a space, every point lying on a single half-line departing from the origin has the same identical polarization, so that the unit vector along the half line univocally identifies the polarization. Hence, if

$$p_{1} = \frac{\Re\{E_{x}(\underline{r}_{t})\}}{|E|}, p_{2} = \frac{\Re\{E_{y}(\underline{r}_{t})\}}{|E|}, p_{3} = \frac{\Im\{E_{y}(\underline{r}_{t})\}}{|E|}$$
$$p_{4} = \frac{\Re\{E_{z}(\underline{r}_{t})\}}{|E|}, \qquad p_{5} = \frac{\Im\{E_{z}(\underline{r}_{t})\}}{|E|} \qquad (A.3)$$

then the unit vector associated to the polarization which is needed in (7) can be finally written as:

$$\boldsymbol{p} = p_1 i_x + \{p_2 + jp_3\} i_y + \{p_4 + jp_5\} i_z \qquad (A.4)$$

so that E = |E|p. Note that, in view of definition and expected properties, the parameters p_1, \ldots, p_5 (have to) satisfy the constraint:

$$p_1^2 + p_2^2 + p_3^2 + p_4^2 + p_5^2 = 1$$
 (A.5)

This latter property implies that the space of possible polarizations can be regarded as a 5-sphere.

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