

Markovian modeling of seismic damage accumulation

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SUMMARY

Analysis of civil structures at the scale of life-cycle requires stochastic modeling of degradation. Phenomena causing structures to degrade are typically categorized as aging and point-in-time overloads. Earthquake effects are the members of the latter category this study deals with in the framework of performance-based earthquake engineering (PBEE). The focus is structural seismic reliability, which requires modeling of the stochastic process describing damage progression, due to subsequent events, over time. The presented study explicitly addresses this issue via a Markov-chain-based approach, which is able to account for the change in seismic response of damaged structures (i.e., state-dependent seismic fragility) as well as uncertainty in occurrence and intensity of earthquakes (i.e., seismic hazard). The state-dependent vulnerability issue arises when the seismic hysteretic response is evolutionary and/or when the damage measure employed is such that the degradation increment probabilistically depends on the conditions of the structure at the time of the shock. The framework set up takes advantage also of the hypotheses of classical probabilistic seismic hazard analysis, allowing to separate the modeling of the process of occurrence of seismic shocks and the effect they produce on the structure. It is also discussed how the reliability assessment, which is in closed-form, may be virtually extended to describe a generic age- and state-dependent degradation process (e.g., including aging and/or when aftershock risk is of interest). Illustrative applications show the options to calibrate the model and its potential in the context of PBEE.

Keywords: state-dependent fragility, damage, stochastic process, reliability, performance-based earthquake engineering.

1. INTRODUCTION

Mainly because of the importance of sustainability-related issues, there is an increasing interest in the life-cycle analysis of civil constructions; i.e., at the time-scale of years in which multiple seismic events, and or aging, may affect the structure leading to deterioration; e.g., [1-3]. Aging may be related to an aggressive environment, which worsens the mechanical features of structural elements (e.g., corrosion of reinforcing steel due to chloride attack, carbonation in concrete, etc.) or shocks the occurrence of which may be difficult to observe (e.g., ambient vibrations, traffic loads, fatigue, etc.). Degradation due to aging is typically described assuming that it takes place gradually over time. Earthquake shocks potentially accumulate damage on the hit structure during its lifetime. In general, mainly because earthquake occurrences can be treated as instantaneous with respect to structural life, that is safety-threatening point-in-time events, it is advantageous to model the cumulative seismic damage process separately from aging (i.e., gradual deterioration); see [4] for example. This study focuses on this latter issue; i.e., stochastic modeling seismic damage accumulation, as one of the component of life-cycle analysis of structures. A sketch of the issues tackled is given in Figure 1.

The motivation stays in the classical formulation of performance-based earthquake engineering (PBEE) framework [5] that features a mathematically convenient time-invariant representation of failure probability, which however may be not the best option to tackle issues related to degradation of structural performance. Indeed, PBEE does not explicitly consider the case in which, for example, partially-damaging events lead seismic losses to accumulate. On the other hand, most of the studies

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attempting to address stochastic modeling of degrading structures (e.g., [6]) relies on hypotheses sometimes believed to be strong in the civil engineering context, for example independent damage increments from one shock to another. In fact, it may be that the response of the structure and/or its susceptibility to damage change through shocks, an issue considered only in a few cases; e.g., [7,8].

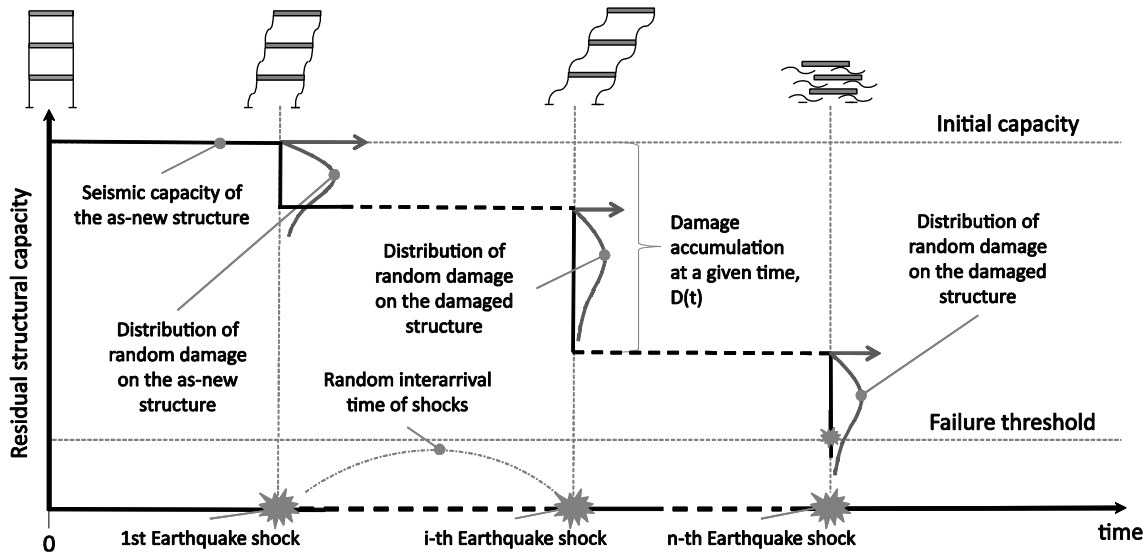


Figure 1. Sketch of the deterioration process for a structure subject to cumulative earthquake damage.

The simplest form of dependency between damage increments assumes that structural vulnerability (i.e., susceptibility to increase damage in one earthquake), given the features of the earthquake, depends (only) on the state of the structure at the time of the shock. On these premises, the study presented in the following tackles stochastic modeling of structures accumulating seismic damage when the structural fragility is state-dependent, and hazard is represented by means of random earthquake occurrence and random ground motion intensity given the occurrence of one earthquake.

To probabilistically account for state-dependent vulnerability a Markov-chain-based description of degradation is developed; i.e., a discrete-time homogenous Markovian process. Also the state of the process is treated as a discrete variable within the developed model. In particular, the domain of the structural performance is partitioned in a series of damage states, and transition probabilities between these states, given the occurrence of an earthquake, are derived. Random occurrence of seismic shocks is described via a homogenous¹ Poisson process (HPP), a classical assumption in seismic hazard analysis [9]. Combining the probabilistic description of the earthquake occurrence and its intensity, and the possible transitions between damage states in one earthquake event, the stationary matrix, which collects unit-time transition probabilities between states, and completely characterizes the damage process, is obtained. This leads to a closed-form solution for the reliability problem, easily allowing probabilistic predictions of the structural lifetime.

The paper is structured such that the formulation of the considered reliability problem is given first. In particular, it is discussed, from the structural engineering point-of-view, when a state-dependent [10] representation of damage accumulation is required, that is: (1) when *evolutionary* hysteretic behaviors are employed, but also (2) when non-evolutionary hysteretic rules are considered, yet the considered damage measure requires to keep track of the state of the structure. Then, the damage accumulation is addressed modeling both the process of occurrence of seismic shocks and the effect they produce. Subsequently, the Markov-chain-based solution for the reliability problem is derived. Then, some illustrative applications, referring to evolutionary and non-evolutionary hysteretic behavior structures, consistent with principles of PBEE, are exploited to describe the structural reliability issues the model allows to address and to show some options for

¹ It should be noted, however, that the model is capable of accounting for others earthquake occurrence process featuring neither independent nor stationary increments, as discussed briefly in the appendix.

case-specific calibration. Finally, in the appendix, a brief discussion about possible extensions of the model toward inclusion of aging and/or other forms of degradation is given.

2. RELIABILITY FORMULATION UNDER DAMAGE ACCUMULATION HYPOTHESIS

The objective of this study is to enable the computation of the failure probability of structures that degrade due to accumulating seismic damage. The effect of degradation is measured in terms of residual seismic capacity, $\mu(t)$. From the analytical point of view, the degradation process is that in Equation (1), where μ_0 is the initial capacity, $D(t)$ is the total degradation at the time t ; the initial time is assumed to be zero, that is, $t_0 = 0$.

$$\mu(t) = \mu_0 - D(t) \quad (1)$$

$D(t)$, in the case aging is neglected,² can be defined as the cumulated loss of resistance due to all earthquake events, $N_E(t)$, occurring until time t , in Equation (2). The damage increment in a single seismic shock ($\Delta\mu_i$), as well as $N_E(t)$, are random variables (RVs).

$$D(t) = \sum_{i=1}^{N_E(t)} \Delta\mu_i \quad (2)$$

Given the formulation in Equation (1), the probability the structure fails within t , $P_f(t)$, or the complement to one of the structural reliability $R(t)$, is the probability that the structure passes a threshold related to a certain limit state, μ_{LS} , at any time before t , Equation (3). In other words, it is the probability that in $(0, t)$ the capacity reduces travelling the distance, $\bar{\mu}$, between the initial value and the threshold. Note that, by definition, Equation (3) also provides the cumulative probability function of structural lifetime, $F_T(t)$.

$$P_f(t) = 1 - R(t) = F_T(t) = P[\mu(t) \leq \mu_{LS}] = P[D(t) \geq \mu_0 - \mu_{LS}] = P[D(t) \geq \bar{\mu}] \quad (3)$$

Reliability problems with formulations similar to that in Equation (3) were addressed in [4,11-12]. With respect to those studies, which consider damage increments in subsequent shocks as independent and identically-distributed RVs, the one herein relaxes an assumption that may be considered strong in the structural engineering context. In particular, it is explicitly considered that the damage increment RV in each shock is dependent on state of the structure at the time of the shock. This may happen because of two, possibly concurring, reasons:

- (1) the hysteretic behavior is evolutionary, that is the structure responds differently to future earthquakes as a function of the cumulative effect of shocks it has already experienced;
- (2) the damage measure considered is such that it is necessary to keep track of previous damages to measure the damage increment in a shock.

These issues are discussed in the following two sections. In the first one, accumulating damage, $D(t)$, is discussed from the structural engineering point of view first; in the second one, from the probabilistic modeling side.

² See, for example, [4] for an analogous formulation, which accounts for aging.

3. STATE-DEPENDENT SEISMIC DAMAGE INCREMENT

To model degradation in structures with possible energy dissipation during seismic shaking (e.g., hysteretic behavior) an index measuring accumulating damage, and its effect on structural performance, is needed. This has been, and currently is, a relevant topic in the earthquake engineering research. According to the review in [13], damage indices may be grossly categorized in two main classes labeled as *displacement-related* and *energy-related*. In the former case, the principle is that the structure reaches the limit state of interest because it exceeds a maximum displacement threshold, that is, maximum strain. The latter case refers to structures in which damage is related to the amount of energy dissipated by hysteretic loops. Indeed, the most representative strain-based damage index is the maximum displacement demand imposed by the seismic shock, while hysteretic energy (i.e., the summation of the areas of plastic cycles during seismic shaking) is the most direct energy-based index. Hybrid damage indices, accounting for both damage phenomena in a single metric, also exist; the best known is that by Park and Ang [14].

The main issue in modeling the stochastic evolution of accumulating seismic damage is that the probability of observing a certain increment of deterioration (i.e., the vulnerability) in a generic earthquake shock may be dependent on the history prior to its occurrence. If the damage increment in one earthquake depends on the seismic history only via the state of the structure at the time the seismic event occurs, then the stochastic process describing/predicting the evolution of structural conditions in time may be categorized as a Markovian one.

In structures, there are two concurrent seismic vulnerability issues which call for Markov-type reliability models: (1) the hysteretic behavior of the structure does not remain the same in subsequent earthquakes; and (2) the way damage is measured introduces a dependency on history, even if the structural behavior remains the same. In the following subsections these two issues are further discussed for two different structural systems, subsequently, it will be shown how state-dependent vulnerability, jointly with the classical probabilistic seismic hazard analysis, leads to a global Markovian failure risk process.

3.1. Energy-based damage indices

To better illustrate the dependency on the state due to evolutionary seismic response, it may be useful to refer to the simplest hysteretic behavior of a single degree of freedom (SDOF) system, that is, the elastic-perfectly-plastic (EPP) loop depicted in Figure 2 (left). The horizontal force versus horizontal displacement relationship defines the hysteretic (seismic) behavior of this simple structure.

If the response of such a system does keep the same strength and unloading/reloading stiffness through different cycles, and then in different seismic events (i.e., the cycle is stable), then the response is defined as non-evolutionary. It is easy to recognize, then, that if the damage is measured in terms of dissipated hysteretic energy (i.e., sum of hysteretic cycle areas) the damage increment in one earthquake only depends on the response of the system in that earthquake, not on what happened previously. This is because the areas of hysteretic loops only depend on the amplitude and number of cycles in the specific earthquake and not on the history (see also Figure 3).

To better clarify this concept, it may be useful to now consider an evolutionary SDOF system, for example the pinching (PIN) model in Figure 2 (right). This is a system adapted from [15], which has a multi-linear backbone curve and a behavior such that the reloading stiffness and target strength depend also on the residual displacement and the maximum/minimum transient displacements observed, that is the displacements from which the reloading starts in a new earthquake and the target to which the reloading path is oriented. Therefore, in this case, even if the hysteretic energy is chosen to measure the variation of damage in one event, to be able to compute it, one has to know displacement history from the previous earthquakes, in addition to the response in the current seismic shock. In this sense the PIN system, conversely to the EEP-SDOF, has a state-dependent response in the case of energy-based damage indices.

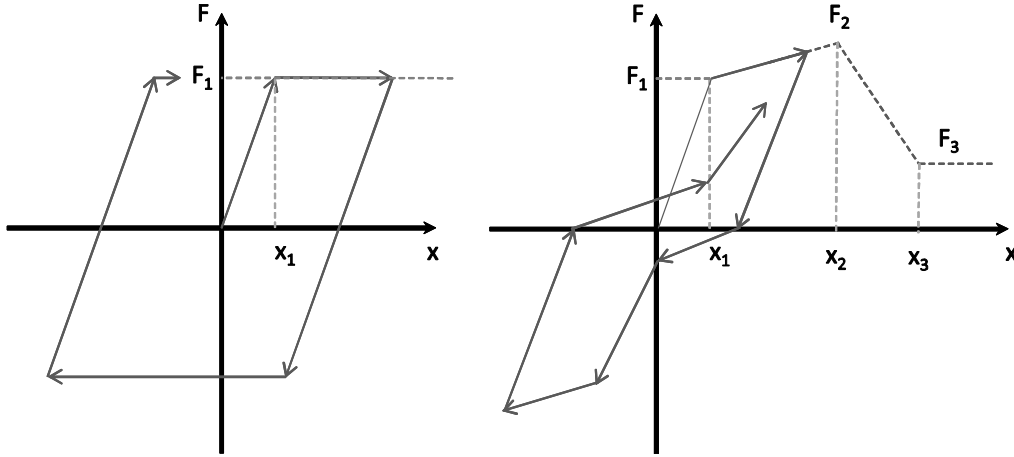


Figure 2. Elastic-perfectly-plastic non-evolutionary behavior (left), and a ‘pinching’ evolutionary model, adapted from [15] (right).

In general, in the case of evolutionary systems, seismic damage may change dynamic properties of the structure and then the response in further eventual shocks. This is also the case of systems defined in [13] as *degrading*. In these systems the hysteretic features degrade (change) as a function, for example, of the number of cycles the system has sustained. In this case the response is still state-dependent, but the state may be defined by different/more parameters as, for example, the number of plastic excursions.

3.2. Displacement-based damage measures

It has been clarified why the type of hysteretic behavior can induce state-dependent seismic response and then state-dependent vulnerability. However, even if the hysteretic loop of the structure may be considered non-evolutionary, that is, its shape does not change through shocks, as in the EPP-SDOF, the damage-criterion may still let the damage increment to be history-dependent. Indeed, considering a maximum-displacement-based (i.e., maximum strain) criterion, damage accumulation in an earthquake occurs only once the maximum displacement reached in it is larger than the maximum in those previous [13], which makes the probability of observing a certain damage increment dependent on the distance between the residual displacement from which shaking starts (depending on seismic history) and the maximum transient until that shock. This may be argued by Figure 3, where the effect of a generic seismic shock is depicted referring to the envelope response of an EPP-SDOF (the same reasoning applies to cyclic response).

In the figure it is illustrated that, while the area of hysteretic loops do not depend on the previous behavior of the structure, the maximum absolute displacement (and then the damage increment) does. Indeed, the amount of damage in a shock, which is the strain increment in this case, depends on the maximum recorded strain in previous shocks and also on the residual displacement. This is because the amount of plastic displacement required to increase damage is the difference between the maximum transient ever recorded and the residual displacement at the time of the shock. Therefore in this case the vulnerability is state-dependent also if the response is non evolutionary.

In a series of previous studies, stochastic modeling of damage accumulation in structures was addressed: [4], [11-12]. In these works, an energy-based criterion was considered with reference to EPP-SDOF systems, therefore a series of reliability models based on independent increments was developed. Herein, conversely, a Markovian model is developed to accommodate state-dependent vulnerability because either of the (1) hysteretic behavior and/or (2) the considered damage measure. In particular, to isolate the effect of (2), the developed model is calibrated for an EPP-SDOF system when damage is based on the maximum displacement demand (or an equivalent of it; e.g., the drift ratio), while, to consider also the effect of (1), a PIN-SDOF systems is investigated, with the same damage measure.

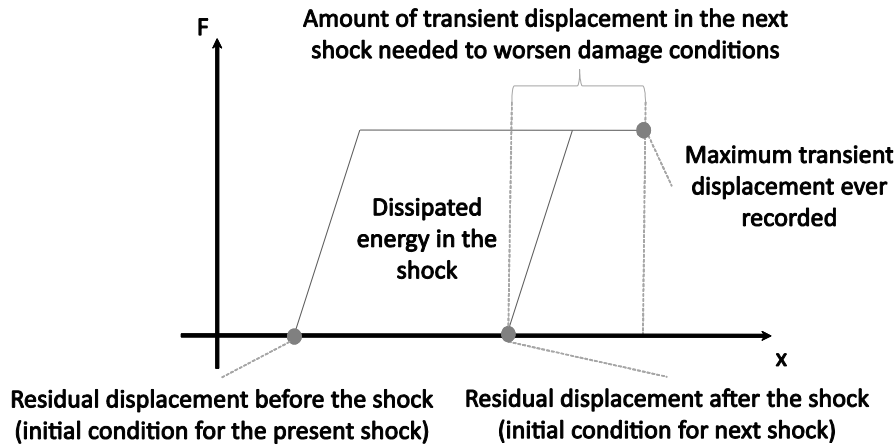


Figure 3. Envelope (simplistic) scheme of response in terms of maximum displacement and dissipated hysteretic energy. It is to note that if a maximum strain threshold has to be reached to collapse, the damage increment in one shock depends on the maximum ever recorded (which has to be surpassed for the damage to increase) and the residual displacement, which determines the starting point.

4. RELIABILITY MODEL FOR MARKOV-TYPE SEISMIC DAMAGE ACCUMULATION PROCESS

In this study damage accumulation is of concern; it is assumed that the attainment of a certain damage level (e.g., failure) may also be produced by multiple partially-damaging seismic shocks, and not only in a single catastrophic event as implicitly assumed in the *fragility* function employed in PBEE. An approach to address this issue was proposed in some studies; see for example [4]. As discussed, a remarkable limitation of this kind of models stays in the probabilistic assumption that damage increments are independent (and identically distributed), which means that the structure is characterized by the same structural response in different events and that the damage increment in one event is independent of the seismic history of the structure.

An alternative approach consists in modeling state-dependent vulnerability; i.e., modeling the probability of transition, in one event, between progressively worse damage states, given the state of the structure prior to the event. Indeed, if one is able to adjust the structural vulnerability as a function of the state of the structure, it is then possible to probabilistically predict the behavior of the already-damaged structure. This, although leading to a more elaborated formulation/calibration of the resulting models, enables to describe forms of stochastic dependence among damage increments, which approaches such as that in [4] neglect. In fact, if a probabilistic description of the occurrence of shock exists (as it happens in the case of classical probabilistic seismic hazard analysis; to follow), in conjunction with state-dependent vulnerability, one is able to probabilistically predict, in closed-form, the path of the structure from as-new condition to collapse, via multiple state-changing earthquakes.

A reliability solution of this type is derived herein based on modeling the damage process via a Markov chain (i.e., a discrete-time and discrete-state Markovian process). The time t is discretized in intervals of fixed width equal to Δ , which may be considered to be the time unit (e.g., one year), then $\Delta = 1$ (the index used to indicate a certain time in the discretized domain is k). The domain of the considered damage index (structural performance measure) is partitioned to have a finite number (n) of damage states (DS). The various $DS_i, i = \{1, 2, \dots, n\}$, are factually limit states, identifying intervals of the damage metric considered between as-built conditions and failure, the structure has to walk (not necessarily one-by-one) to reach collapse (Figure 4).

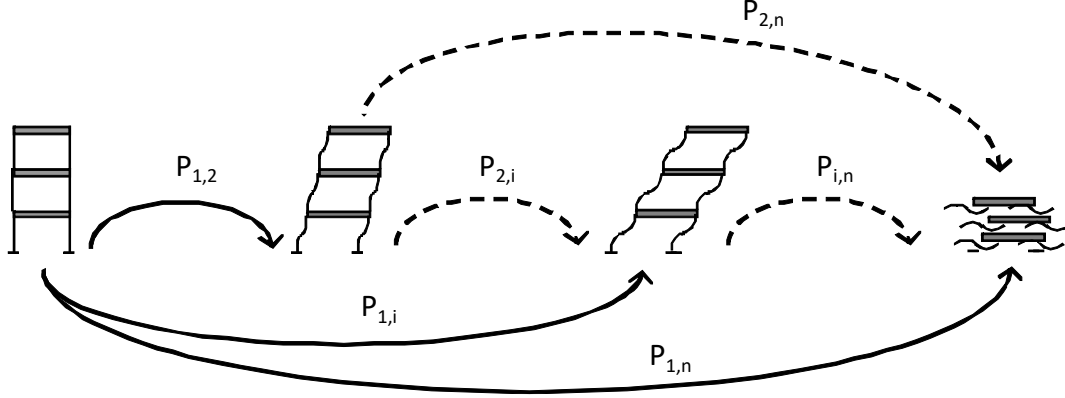


Figure 4. Sketch of discretization of degradation states of a damage-accumulating structure.

In this context, the transition probabilities between the i -th and j -th generic damage states, given the occurrence of an earthquake, are indicated as $P_{i,j}$. These are the probabilities that after one event the structure is in the j -th DS given that it was in the i -th DS previous to the earthquake. Arranging $P_{i,j}$ in the form of a matrix, which contains probabilities of observing transitions between any possible couple of damage states, computed under the hypotheses that a seismic event is occurred, a Markovian transition matrix in the case of event occurrence is obtained, Equation (4).

$$[P] = \begin{bmatrix} 1 - \sum_{j=2}^n P_{1,j} & P_{1,2} & \cdots & \cdots & P_{1,n} \\ 0 & 1 - \sum_{j=3}^n P_{2,j} & \cdots & \cdots & P_{2,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & 1 - P_{(n-1),n} & P_{(n-1),n} \\ 0 & \cdots & \cdots & 0 & 1 \end{bmatrix} \quad (4)$$

In the matrix, the rows and columns are labeled with the damage states of the structure; therefore, the first state is the as-new (or initial) conditions and collapse (or *failure*) is represented by the n -th state. The latter, in the Markov chain context, is referred to as an *absorbing state*, that is, once the structure is in it cannot escape from it. The lower triangle of the matrix is comprised of zeros because of the monotonic nature of deterioration. The matrix may be used entering the row with the pre-event condition of the structure to get the probability to find it in any of the other states given the occurrence of an earthquake.

Even if in the applications also an alternative approach will be used to calibrate the model (see section 5), it is worth noting that, consistent with the PBEE framework, the individual elements of the matrix can be computed combining: (i) the probability density function (PDF) of the intensity in a seismic shock (IM), conditional to the earthquake event occurrence (E), available from hazard analysis [9], and (ii) the transition probability of the structure conditional to IM, as per the simple application of the total probability theorem in Equation (5).

$$P_{i,j} = P[j\text{-th state} | i\text{-th state} \cap E] = \int_{im} P[j\text{-th state} | i\text{-th state} \cap IM = z] \cdot f_{IM|E}(z) \cdot dz \quad (5)$$

Note that, as per classical probabilistic seismic hazard analysis, or PSHA, the random variables representing ground motion intensities of different earthquakes are independent and identically distributed; i.e., $f_{IM|E}(z)$ is always the same in different shocks and does not depend on time. This leads to time-invariant $P_{i,j}$, if the other term in the integral also do not change with time; i.e., the

structure is not affected by aging (see the appendix for further discussions). Note also that the $P[j\text{-th state} | i\text{-th state} \cap IM = z]$ term is factually equivalent to (i.e., it is derived by) a *state-dependent fragility curve* of the damaged structure (e.g., [16]), as illustrated in section 5.

4.1. Occurrence of seismic shocks

PSHA typically refers to the HPP as the process modeling the occurrence of earthquakes at a specific seismic source. HPP is such that the counting process for earthquakes is completely defined by one parameter; i.e., the rate of occurrence of earthquakes, ν_E . In fact, it may be worthwhile to recall that, in PSHA, also the process of occurrence of events causing exceedance of a specific ground motion intensity threshold at a site of interest is represented by a HPP, whose rate, λ_{im} , is computed as in Equation (6), where it is assumed, for simplicity, that one seismic source affects the site of interest. In the equation, usually referred to as the *hazard integral*, $f_{M,R|E}$ is the distribution of magnitude and source-to-site distance of earthquakes from the source of interest, and $P[IM > im | M = x, R = y]$ is the probability of exceeding im in an earthquake of known magnitude (M) and source-to-site distance (R), or a *ground motion prediction equation* (GMPE).

$$\lambda_{im} = \nu_E \cdot P[IM > im | E] = \nu_E \cdot \int \int_{r \ m} P[IM > im | M = x, R = y] \cdot f_{M,R|E}(x, y) \cdot dx \cdot dy \quad (6)$$

On the premises of Equation (6), and if the unit-time rate of occurrence of earthquake shocks is small enough, such that the probability of observing more than one seismic event in the unitary time interval is negligible, it is possible to compute, for any pair of damage states ($i \neq j$), the probability of the structure passing from one to another in an unit-time interval $[k, k+1[$. It is simply the product of the rate of earthquake events times $P_{i,j}$, Equation (7). In other words, the unit-time transition probability between two states is the rate of earthquakes filtered by the probability that structure moves between the two states given the occurrence of one event.

$$P[j\text{-th state at } (k+1) | i\text{-th state at } k] = \nu_E \cdot P_{i,j} \quad (7)$$

4.2. Formulating the damage accumulation process

On the basis of Equation (7), the matrix reporting the probabilities of the structure moving between any two states in a unit-time interval, $[k, k+1[$, is given by Equation (8). In the equation: $[I]$ is the identity matrix representing the certitude that the structure remains in the same state if no earthquakes occur in the unit-time interval; and $(1 - \nu_E)$ is the probability of not observing an earthquake in the unit-time interval.

$$[P(k, k+1)] = \nu_E \cdot [P] + (1 - \nu_E) \cdot [I] = \begin{bmatrix} 1 - \sum_{j=2}^n \nu_E \cdot P_{1,j} & \nu_E \cdot P_{1,2} & \cdots & \cdots & \nu_E \cdot P_{1,n} \\ 0 & 1 - \sum_{j=3}^n \nu_E \cdot P_{2,j} & \cdots & \cdots & \nu_E \cdot P_{2,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & 1 - \nu_E \cdot P_{(n-1),n} & \nu_E \cdot P_{(n-1),n} \\ 0 & \cdots & \cdots & 0 & 1 \end{bmatrix} = [P_E] \quad (8)$$

In these conditions the stochastic damage accumulation process results to be an homogeneous Markov chain, which is completely characterized by the transition matrix, $[P_E]$, in Equation (8). This means that the transition matrix for m time units, $[P(k, k+m)]$, is given by the m -th power of the unit time transition matrix that characterizes the process, as in Equation (9).

$$\begin{aligned}
[P(k, k+m)] &= \\
&= \begin{bmatrix} 1 - \sum_{j=2}^n v_E \cdot P_{1,j} & v_E \cdot P_{1,2} & \cdots & \cdots & v_E \cdot P_{1,n} \\ 0 & 1 - \sum_{j=3}^n v_E \cdot P_{2,j} & \cdots & \cdots & v_E \cdot P_{2,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & 1 - v_E \cdot P_{(n-1),n} & v_E \cdot P_{(n-1),n} \\ 0 & \cdots & \cdots & 0 & 1 \end{bmatrix}^m = [P_E]^m \quad (9)
\end{aligned}$$

In other words, if one wants to know the probability of finding the structure in any of the possible states (including collapse) at time $k+m$, given a vector (size $1 \times n$) collecting the (initial) probabilities of structure being in one of the states at k , $[P_1^0 \ P_2^0 \ \cdots \ P_n^0]$, one has to simply do the product in Equation (10).

$$[P_1^0 \ P_2^0 \ \cdots \ P_n^0] \cdot [P_E]^m = [P_1(k, k+m) \ P_2(k, k+m) \ \cdots \ P_n(k, k+m)] \quad (10)$$

It is to note that the Markov chain resulting from this model is homogeneous (i.e., the transition matrix for m time units does not change through time; i.e., does not depend on k) because the transition probabilities given the occurrence of one event are dependent on the state of the structure and not on its age, and because the rate of occurrence of earthquakes also does not depend on time, due to PSHA hypotheses. However, the model is able to accommodate both for the variation in time of the transition probabilities due to aging of the structure, and rate of occurrence of earthquakes which is time-variant (for example during aftershocks sequences); these issues, which are outside the scope of this study, are briefly discussed in the appendix.

Based on the model of Equation (10) it is possible to compute the mean of the time to collapse, $E[k_f]$, that is the average time to get to the last (absorbing) state. The derivation of the relationship to compute the mean number of time units to collapse the structure is given in Equation (11), where $P[DS(i) < DS_n]$ is the probability that at time i the structure has not failed yet, as DS_n is the last (absorbing) state.³

$$\begin{aligned}
E[k_f] &= \sum_{i=0}^{+\infty} i \cdot P[k_f = i] = \sum_{i=0}^{+\infty} i \cdot (P[k_f \geq i] - P[k_f \geq i+1]) = \\
&= \sum_{i=1}^{+\infty} P[k_f \geq i] = \sum_{i=1}^{+\infty} P[DS(i) < DS_n] = \sum_{i=1}^{+\infty} \{1 - P[DS(i) \geq DS_n]\} = \\
&= \sum_{i=1}^{+\infty} \left\{ 1 - [P_1^0 \ P_2^0 \ \cdots \ P_n^0] \cdot [P_E]^i \cdot [0 \ \cdots \ 0 \ 1]^T \right\} \quad (11)
\end{aligned}$$

³ Note that the mean time-to-failure is not especially sensitive to the number of states selected for the discretization of the damage domain, because the simulation-based calibration procedures described in section 5, allow the model to well approximate it, even in the case a very small number of state is considered.

Finally, although it is out of the scope of this study, which focuses on damage accumulation, it is worth to briefly discuss how to include the effect of repair in the developed Markov chain. The effect of repair may be readily introduced in the context of this model following the approach described in [17]. The latter makes use of the so-called *reset matrix*, which is a transition matrix analogous to that describing damage accumulation, yet defined to represent maintenance in a probabilistic manner. In fact, it is filled with the probabilities that repair moves the structure from any damage state to any other damage state.

5. MODEL CALIBRATION

5.1. Damage states

In this section the calibration of the reliability model resulting in Equation (9) is illustrated. In particular, two structural systems, represented for simplicity, yet without harm to generality, via SDOF systems are discussed: the EPP and the PIN systems of Figure 2. The damage criterion considered is a strain-based one, that is the damage states are identified by means of maximum transient displacement thresholds until collapse (drift ratio, in fact, and to this aim both SDOFs are assumed to be 1m in height). Five damage states are arbitrarily defined: *as-new* (AN) conditions, *immediate occupancy* (IO), *life safety* (LS), *collapse prevention* (CP), and *failure* (F) indicating collapse. The DS' thresholds are identified on the backbones of the two systems in Figure 5. The corresponding values of drift rotations identifying the damage states are common to the two systems and are listed in Table 1.

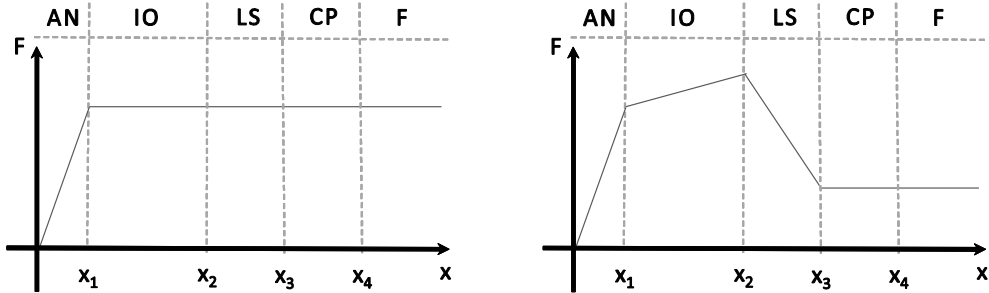


Figure 5. Damage states and limit state thresholds arbitrarily identified for the two systems (figure not to scale).

Table 1. Damage state thresholds in terms of transient drift angle for the two considered systems.

AN (x_1)	IO (x_2)	LS (x_3)	CP (x_4)	F
0.0076	0.0175	0.0497	0.1	—

A structural systems travels from a damage state (i) to a worse one ($j > i$) in one earthquake, if and only if, the maximum (over the history) transient drift has put the structure in state i and in the specific earthquake the maximum drift exceeds the historical maximum and is such to put the structure in state j . It is to note at this point that the transition probabilities in one earthquake depend not only on the transient drift thresholds identifying the DS', but also on the residual displacement the structure has at the time of the earthquake. It is easy to recognize, indeed, that the *jump* (in term of maximum transient displacement) between states the structure has to perform to worsen its structural conditions is dependent on the arrival point (transient drift threshold) but also on the starting point (residual displacement before the event); see section 4.

In this sense, the Markovian representation of the damage accumulation process would require the state of the structure identified by means of two parameters: the pair of maximum recorded transited displacement (which provides the limit to pass to worsen damage conditions) and the current residual displacement, which serves, in conjunction with the maximum displacement, to compute the increment of transient displacement needed to change the state; see Figure 3. A more rigorous

Markovian description of the damage accumulation process would lead to consider a transition matrix having much more states/rows than those considered in Table 1. Indeed, it should be necessary to consider states that account for any possible combination of transient and residual displacements.

This gives the chance to discuss an important issue in the context of this study, that is, the effect of the number of states on the reliability assessment. In fact, once the domain of the damage measure, continuous in principle, is discretized in order to introduce in a simple and effective manner, the dependence of future increments of degradation level on the state of the structure, then an approximation in the stochastic analysis is introduced. In particular, the discrete-time discrete-state Markov model tend to less faithfully reproduce the paths of the process being approximated, while capturing them on average. Such an approximation, arguably, tends to increase as a smaller number of states is considered (similar to when a continuous random variable is approximated by a discrete one). On the other hand, a large number of states is detrimental with respect to the effort for model calibration (see section 5). Thus, in setting the number of states, this trade-off has to be considered.

That being said, for what concerns the number of states selected herein, the choice made is to be consistent with the standard of damage conditions in PBEE (i.e., usually five). Moreover, as discussed in the following, the dependency of transition probabilities on the residual displacement will be only implicitly considered in the model calibration approaches employed in this study, consistently with the literature on the topic of state-dependent fragility curves. The main reason for this is that the complexity of the model that explicitly considers the dependence on the residual displacement makes its calibration hardly feasible in practice.

5.2. Options to fill the transition matrices in one event and in one time unit

The transition probabilities in one earthquake ($P_{i,j}$) needed to fill the matrix in Equation (4), lying at the core of the model, can be computed following two (alternative) approaches.

The first one is that of Equation (5), in which each $P_{i,j}$ terms computed via state-dependent fragilities (i.e., transition probability from a state to a worse one, given a specific value of earthquake intensity) times the distribution of intensity in one earthquake. Equation (12) shows how to retrieve the vulnerability term of Equation (5) from state-dependent fragility curves (at the right hand side).

$$P[j\text{-th state} | i\text{-th state} \cap IM = z] = P[j\text{-th state or worse} | i\text{-th state} \cap IM = z] + \\ -P[(j+1)\text{-th state or worse} | i\text{-th state} \cap IM = z], \quad j \geq i \quad (12)$$

It may be interesting to note that, using state-dependent fragility curves, Equation (7) may be rewritten as in Equation (13), a format common in PBEE, where $|d\lambda_{im}(z)|$ is the absolute value of the derivative of the hazard curve for the site of the construction [5].

$$v_E \cdot P_{i,j} = \int_{im} P[j\text{-th state} | i\text{-th state} \cap IM = z] \cdot |d\lambda_{im}(z)|, \quad j > i \quad (13)$$

To compute state-dependent fragility function the approach of [16] may be pursued, which is based on back-to-back incremental dynamic analysis of the damaged structure. This method is used for the EPP system in section 5.5. Therefore, while the interested reader is referred to [16] for details, a summary of the steps, as they are employed herein to compute the transition probabilities in one event, is given here:

1. a set of ground motion records is chosen to simulate seismic structural behavior;
2. each record is scaled in amplitude (by means of the chosen IM; assumed to be *sufficient* in the sense of [18]) until the structural response of the as new structure reaches a threshold corresponding to any of the damage states considered;

3. the distribution of the IM values from step 2 provides four empirical probability distribution functions, representing the transition probability from AN to any other DS in one event of given intensity (for convenience lognormal models, featuring the sample means and variances of the logs, is used to replace the empirical distributions);
4. because each of the analyses of step 2 also provides a damaged structural model (including the residual displacement from the last shock), dynamic analyses of the damaged models are carried out sampling for each model a record from the set of step 1 – these analyses entail scaling the record until it is reached each of the damage states worse than that of the analyzed damaged models;
5. step 3 is repeated for the IMs from step 4 grouped by means of the departure damage states; therefore, this step provides the remaining set of state-dependent fragility functions, in analogy to step 3, which provides damage fragility for the as-new model;
6. the probability of transition from a DS to another (worse) DS are obtained by difference of the fragility curves just retrieved as per Equation (12);
7. each of the fragility functions obtained in steps 1-5 is integrated with the hazard curve, as in Equation (13), to get the $i \neq j$ terms of the transition matrix in Equation (8);
8. finally, the terms for $i = j$ are obtained as a complement to one of the sum of the others for the same i ; see Equation (8).⁴

An original alternative approach is also followed in this paper. It is based on Monte Carlo simulation, of structural response to sequences of seismic events, to directly compute the probabilities appearing in Equation (4) (i.e., the transition probabilities given the occurrence of one event), which are the core terms to get the unit-time transition matrix in Equation (8). The simulation procedure (the flowchart of which is given in Figure 6) deploys in the following steps:

1. according to the distribution of intensity in one earthquake (obtained from the hazard curve for the site of the construction divided by v_E), a realization of IM (assumed to be *sufficient* in the sense of [18]) is sampled;
2. one record is randomly selected from a set of records chosen to analyze the structure;
3. the selected record is scaled in amplitude in order to feature the IM value from step 1;
4. time-history analysis of the structure to the scaled record is carried out;
5. the resulting damage state of the structure is assessed and saved together with the state previous to the analysis;
6. steps 1-5 are repeated with the structural model in initial conditions which are those final of the last time-history (including residual displacement, which becomes initial condition for the new analysis), until the structure gets the final (F) state;
7. steps 1-6 are repeated (i.e., a new sequence is simulated), resetting at the beginning the structural state to AN conditions structural state (i.e., the first nonlinear dynamic analysis of the sequence is performed on as-new model), until a *sufficiently* large number of transitions between any pair of damage states is observed.

⁴ It is to note that, the method in [16] implicitly relies on the assumption that the residual displacement for a structure in a state mainly depends on the threshold to enter the state and that (in particular) the earthquakes not causing transitions also provide negligible change in the distribution of the residual displacement of the structure.

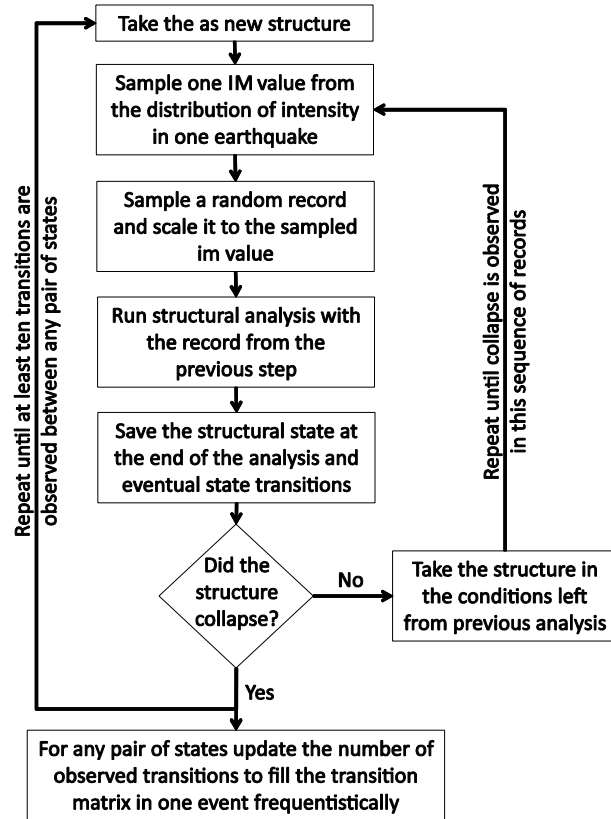


Figure 6. Flow chart of the simulation procedure to get the transition probabilities given the occurrence of one event; i.e., Equation (8).

Note that both approaches to calibrate the model, simulating the structural behavior considering the residual displacement observed in previous analyses, implicitly account for the dependency of the transition probabilities from the residual displacement. However, the one from [16] is significantly more efficient (i.e., less demanding in terms of structural analyses) than the simulation-based just illustrated. On the other hand, as a trade-off, it may be demonstrated that it requires to assume that the distribution of the residual displacement given the structure is in a state, is independent of the path (history) that took the structure to that state. This latter hypothesis has to be verified case-by-case and therefore a full comparison of the two methods, which should lead in principle to the same results, cannot be fully addressed in the context of this paper.

Also note that both approaches require the distribution of intensity in one earthquake, that is, the probability distribution of the ground motion intensity when one earthquake occurs at the site. It may be easily obtained from the annual hazard curves dividing by the rate of earthquakes on the seismic source. Thus, in the following subsection, this issue is addressed first; subsequently, the transition matrices in one earthquake are calibrated for the PIN and EPP systems, following Figure 6 for the former and the state-dependent fragility approach for the latter, to finally get unit-time transition matrix and perform the reliability assessment for both SDOFs.

5.3. Seismic hazard and IM distribution

The earthquake damage accumulation process requires the distribution of earthquake intensity given the occurrence of a seismic shock. This means to carry PSHA for the site of the construction. The chosen ground motion intensity measure is the spectral acceleration at the elastic period of the two systems, $Sa(T = 0.5s)$, which is the same for both (to follow). Both systems are supposed located in Sulmona (close to L'Aquila, in central Italy). PSHA for this site (Figure 7, left) was carried out via

the FORTRAN code also used in [19]. In the software, seismogenic source zones of Italy are those of [20], while seismic parameters of each zone are those from [21,22]. The adopted GMPE is that of [23]. It was assumed that the coordinates of the epicenter are uniformly distributed over each source zone. Because of distance applicability limits of the GMPE, contributions distant more than 200 km from the site were neglected in hazard computations.

Figure 7 (left) shows the source zones considered, while Figure 7 (right) provides the resulting distribution of intensity at the site given the occurrence of an earthquake. Further details about PSHA for the site and a summary of the source zone parameters may be found in [19]. However, a relevant information for this study is that the rate of occurrence of earthquakes within the magnitude bounds of the considered seismic sources, and then at the site, is $\nu_E = 1.95$ [events / year]. This annual rate times the curve in Figure 7 (right) provides the annual rate of exceedance of IM at the site; i.e., the hazard curve; see Equation (6).

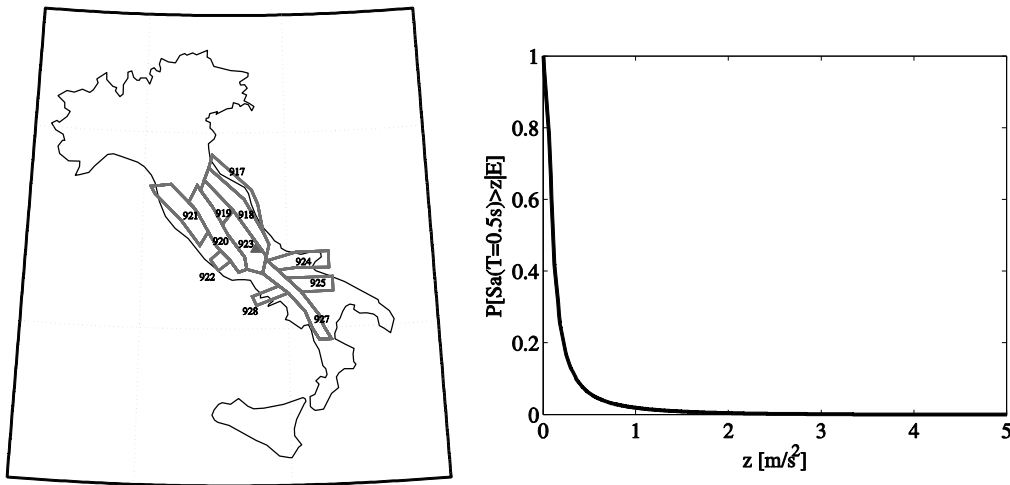


Figure 7. Considered site in Italy (triangle), and seismic source zones considered according to the model of [19] (left); distribution of ground motion intensity given the occurrence of an earthquake at the site (right).

5.4. Evolutionary (pinching) system

The first example of model calibration refers to a PIN-SDOF system of the type in Figure 2 (right). Weight is 100 kN and the yielding force (F_1) is equal to 12.25 kN, which corresponds to a strength reduction factor equal to 4 when the mass acceleration is equal to 0.49g; drift at the yielding is equal to 0.76%. $\{F_2, F_3, x_2, x_3\}$ the parameters of Figure 2 are equal to $\{13.8 \text{ kNm}, 1.38 \text{ kNm}, 0.0175 \text{ rad}, 0.1 \text{ rad}\}$, respectively; the unloading/reloading rules are from [15].

The described Monte Carlo simulation approach was followed to get the $P_{i,j}$ transition probabilities in one event. Indeed, an illustrative example (realization) of a complete simulated sequence of events, leading the structure to travel from AN conditions to F damage state, is shown in Figure 8. The sequence features about nine-hundreds sampled intensities and ground motions.⁵ Transition to the IO state occurs during the fourth record while, due to the 749th, 840th and 868th extracted intensities and consequent ground shaking, the SDOF reaches LS, CP and F, damage states, respectively. Total hysteresis after each state transition, is also reported in the picture.

In the case of the PIN systems, a total of sixty seismic sequences have been simulated. Given that the structure is in damage state i , the probability the structure moves to a damage state j , ($i+1 \leq j \leq n$), in one event, $P_{i,j}$, is estimated via the ratio of the number of transitions between the

⁵ In fact, most of them is of small intensity, as expected from the hazard curve, creating no transition between damage states. For example, the first three of them are under the yielding threshold of the structure (and are not reported in the figure).

states and the number of sampled IM values when the structure was in state i . Then the $P_{i,j}$ probabilities are used in conjunction with the annual occurrence rate of events at the site to get the annual transition matrix of Equation (8), as per Table 2.

Note that, in this case the annual rate of earthquakes is much larger than one, being 1.95 events per year (section 5.3). Therefore, in principle, Equation (8) should not be applied as the rate cannot be confused with the probability of occurrence of one earthquake. The problem can be easily solved obtaining, for example, the monthly earthquake rate dividing 1.95 by 12 and then considering the unit-time to be the month, while keeping the same $[P]$ matrix. Indeed the latter, being transition matrix in one generic earthquake, is independent of the event rate. However, as a peculiar effect of the combination of hazard and vulnerability in this specific application (not to generalize), it happens that $[P]$ matrix is dominated by the principal diagonal, and the approximation in Equation (8) works, in this case, also if the rate is kept yearly, without the need to go to a smaller time unit.

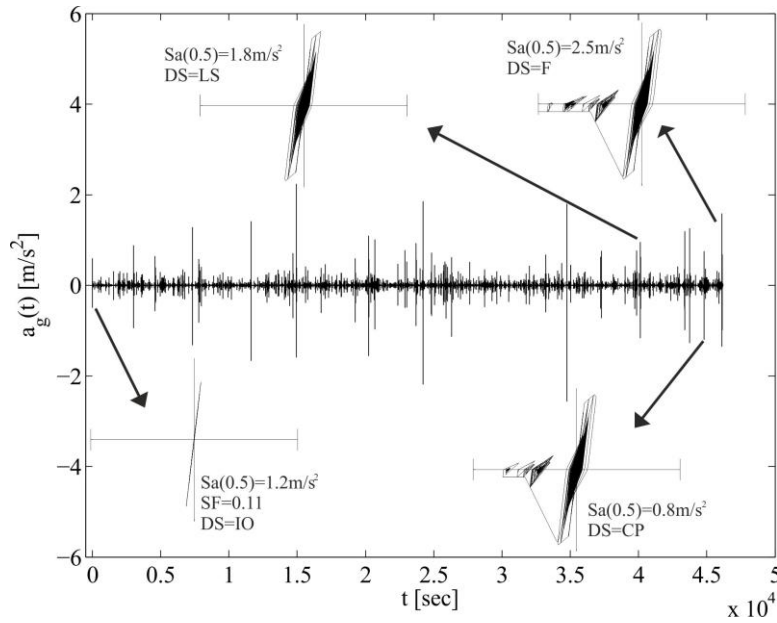


Figure 8. Realization of a seismic sequence from as-new conditions to collapse used to assess transition probabilities in the case of occurrence of one event.

At this point, the reliability assessment can be performed. Indeed, Figure 9 shows curves, function of time, which represent the probability the structure gets any damage state given that it starts from any other damage state, as per Equation (9). It may be seen from the pictures that the probability of remaining in a damage state generally gets lower as time increases, while the probability of getting to failure increases with time units, and is non-monotonic for intermediate damage states, as expected. Finally, the mean times to failure according to Equation (11) may be computed for the PIN structure in AN conditions at time zero, it results $E[k_f] = 181 [years]$.

Table 2. Transition probabilities from a state to another in a unit-time interval (i.e., $[P_E]$) for the PIN system.

	AN	IO	LS	CP	F
AN	9.66E-01	2.65E-02	3.88E-03	4.93E-04	3.40E-03
IO	0	9.89E-01	5.27E-03	6.32E-04	4.64E-03
LS	0	0	9.93E-01	9.11E-04	5.69E-03
CP	0	0	0	9.67E-01	3.31E-02
F	0	0	0	0	1

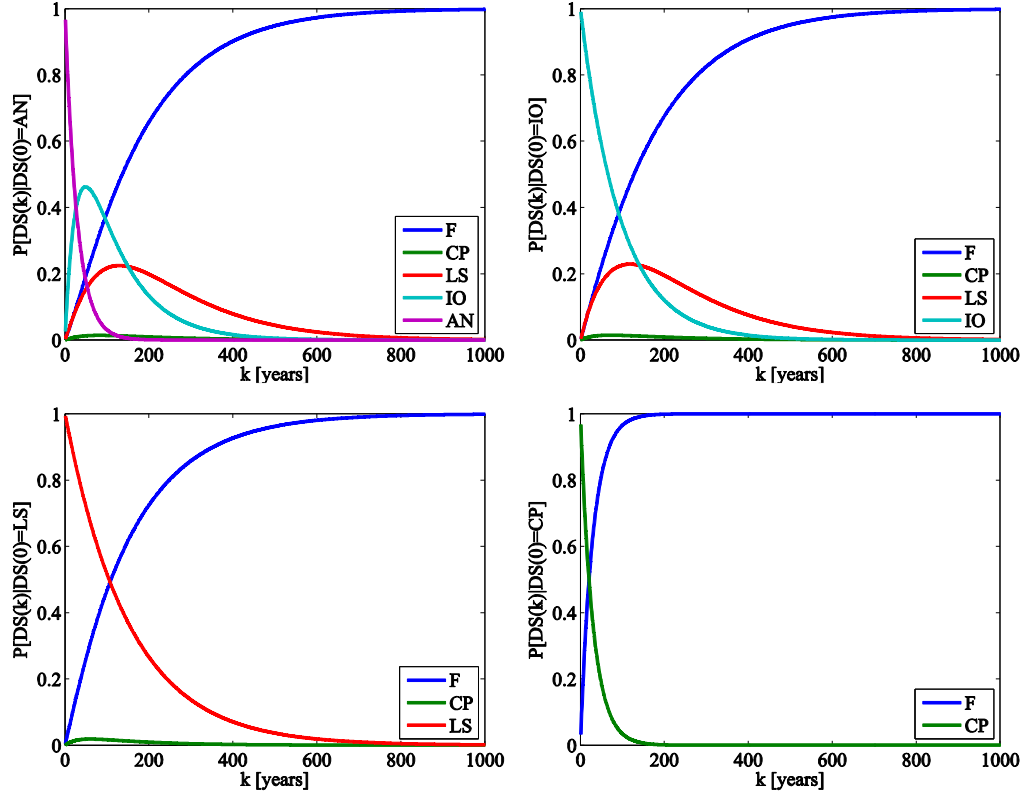


Figure 9. Damage state probabilities, from any initial state to any other damage state, as a function of time for the PIN system.

5.5. Elastic-perfectly-plastic system

In the case of EPP (having the same period, weight and yielding force of the PIN system), the approach of [8,16] has been used to fill the unit-time transition matrix of Equation (8). In fact, a different approach, with respect to the PIN system was chosen to report in this study both the options readily available to calibrate the reliability model.

The records employed in the analysis summarized in the steps listed in section 5.2, are the same as those of the simulations for the PIN system. The resulting state-dependent (lognormal) fragility curves for the EPP system are given in Figure 10.

After taking the differences of these, as per Equation (12), and the integration with the site-specific hazard, as per Equation (13), the unit-time transition matrix of Equation (8) is obtained; see Table 3. Consequently, the time-variant probabilities of getting in any DS state for the structure starting in any other state, from Equation (9), are given in Figure 11. It may be observed that the EPP is, as expected, more reliable than the PIN system. Indeed, the mean time to failure from as-new conditions may be computed also for the EPP structure; it results $E[k_f] = 718[\text{years}]$. The large average time to collapse of this system, in comparison with the PIN systems, confirms the comparatively lower vulnerability of this hysteretic loop.

Table 3. Transition probabilities from a state to another in a unit-time interval (i.e., $[P_E]$) for the EPP system.

	AN	IO	LS	CP	F
AN	9.66E-01	2.55E-02	6.48E-03	1.26E-03	7.18E-04
IO	0	9.85E-01	1.25E-02	1.47E-03	7.48E-04
LS	0	0	9.96E-01	2.84E-03	1.01E-03
CP	0	0	0	9.98E-01	1.71E-03
F	0	0	0	0	1

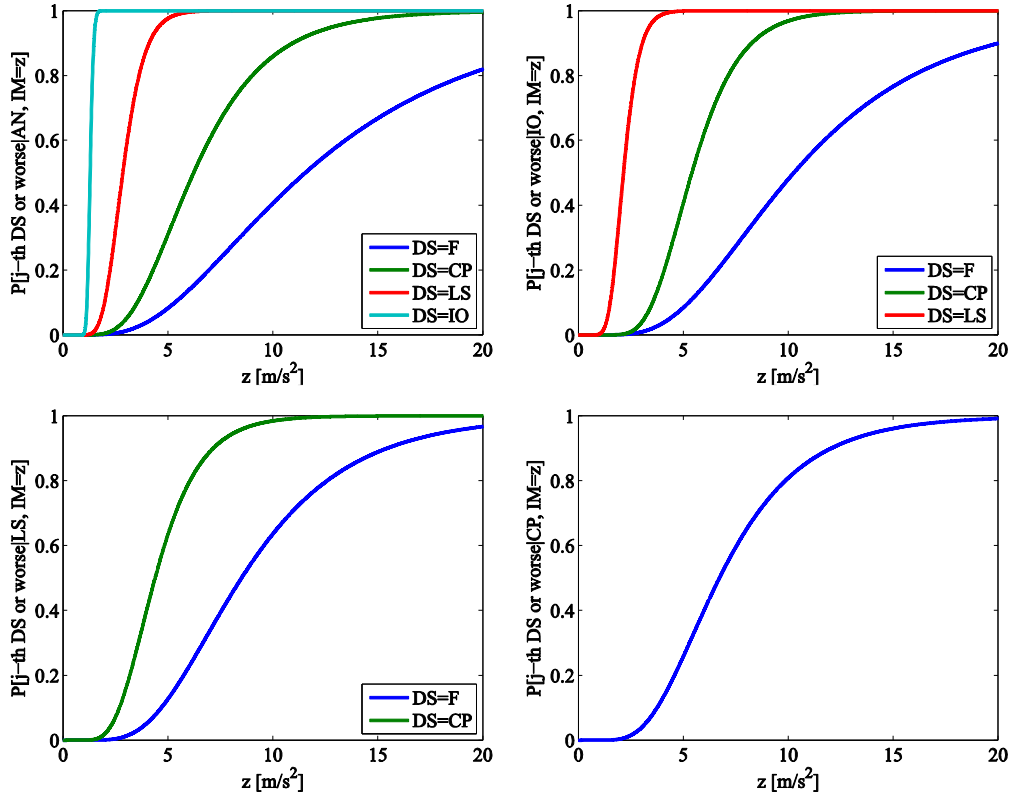


Figure 10. State-dependent lognormal fragility curves for the EPP system derived via the method in [16].

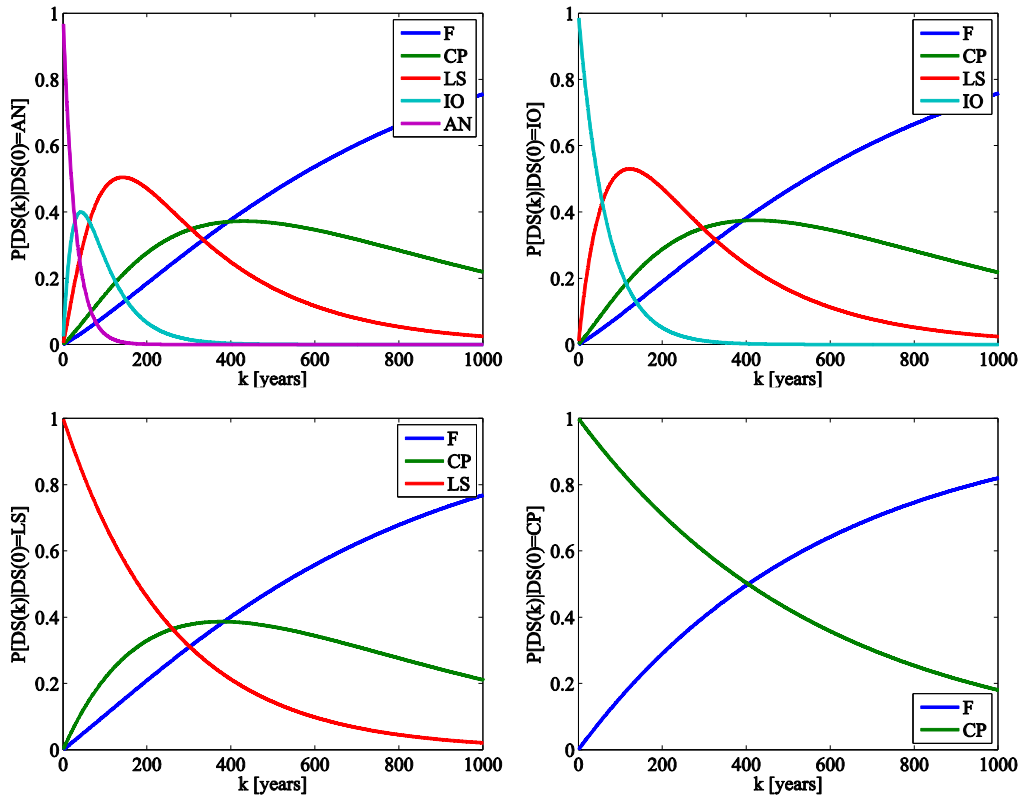


Figure 11. Damage state probabilities, from any initial state to any other damage state, as a function of time for the EPP system.

6. CONCLUSIONS

In the study, a reliability model for structures cumulating seismic damage was presented. The model is based on a Markovian representation of the degradation (i.e., seismic damage accumulation) process. The model is (pure) state-dependent, that is the evolution of the process after a certain time probabilistically depends only on the state of the structure at that time. Thus it relies on a stationary transition matrix, which completely characterizes the stochastic process, that is, a homogenous Markov chain. The transition matrix collects the probabilities that the structures passes from any damage state (which collectively define the ensemble of the possible conditions of the structure) to any another, in a unitary time interval. The matrix is obtained multiplying the rate of occurrence of events on the seismic source of interest by the probabilities of transition between different states during one earthquake.

Transition probabilities given the occurrence of one event may be obtained by simulation of seismic sequences, or integrating state-dependent fragility curves with the distribution of ground motion intensity in one earthquake. Both approaches to case-specific model calibration were followed, for illustrative purposes, in the study.

From the structural engineering point of view, it was shown that either the hysteretic behavior and/or the type of index used to measure damage, can lead to a state-dependent representation of the degradation phenomenon, which makes the Markov chain a viable and closed-form solution to the damage accumulation reliability problem. Indeed, it allows the reliability assessment in the life-cycle with a very low computational demand (simply taking powers of the annual transition matrix over the time interval of interest). Finally, it enables to remove the conventional hypotheses of a quite large deal of literature, which assumes independent, and often identically distributed, degradation increments.

To discuss possible strategies to calibration of model's parameters, two applications were set up. They both rely, for simplicity, to single-degree-of-freedom systems ideally located in central Italy, a region of relatively high seismic hazard in the country. For both systems the damage measure is strain-based one, which implies that damage can increase in a shock only if the maximum displacement recorded in the seismic history is exceeded in that shock. The first system is a pinching systems with evolutionary hysteretic behavior. For this structure, the stochastic dependence of the damage increment on the state primarily arises from the hysteretic loop. The second is an elastic perfectly plastic single degree of freedom system. This system has a seismic response, which remains unaltered through seismic shocks; however, the use of a strain-based damage measure introduces a form of stochastic dependence of the damage increment on the seismic history of the structure, which lets the damage accumulation process be a state-dependent one as well.

For both systems it was discussed how the state is defined by a number of parameters: the maximum/minimum recorded displacements and the residual displacement. For the two systems, transition probabilities were computed suppressing the stochastic dependence of the state on the residual displacement (so as to reduce the number of states and, consequently, the size of the transition matrix) and combined with hazard for the considered site, resulting in unit-time transition probabilities. Results of the reliability assessment for the illustrative applications allow appreciating the generality of the developed approach for life-cycle analysis of degrading earthquake-resistant structures.

Finally, a discussion is given in the appendix about how the model may virtually account for: (i) non-stationary earthquake occurrence rates (e.g., during aftershock sequences); (ii) non-stationary fragility curves because of aging, via a time-variant transition matrix given (i.e., an age- and state-dependent Markovian model); (iii) the combination of seismic damage accumulation and/or aging with other degradation phenomena such as fatigue or other non-seismic shocks.

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APPENDIX: MODEL EXTENSIONS

This section briefly discusses some issues the Markovian model developed may take into account with formally simple extensions. In particular, in the following subsections three cases are considered: (i) the rate of occurrence of earthquakes is time-variant, for example, as in the case of aftershock sequences, where the occurrence of aftershocks is represented by means of a non-homogeneous Poisson process; (ii) the seismic behavior of the structure changes in time because of, for example, aging of structural characteristics; (iii) other shocks different from earthquakes, as traffic load or fatigue, may let damage to accumulate and the structure to degrade in combination with seismic shocks.

A.1. Time-variant earthquake occurrence rate

There are cases of engineering interest in which the rate of occurrence of earthquakes is not time-invariant such as in the classical probabilistic seismic hazard analysis. This is the case, for example, of aftershock sequences. According to [24], during aftershock sequences the rate of exceedance of IM at the site of the structure is given by Equation (14), which is equivalent to Equation (6), except that: (i) the rate of occurrence of aftershocks, $\nu_{A|m_E}(t)$, conditional on the magnitude of the mainshock, changes (decreases) as the time since the mainshock increases, and that (ii) the distribution and magnitude and source to site distance of aftershocks, $f_{M_A, R_A|m_E, r_E}$, is conditional to the magnitude (M_E) and distance (R_E) of the mainshock (location, in fact).

$$\lambda_{im} = \nu_{A|m_E}(t) \cdot \int \int_{r_m} P[IM > im | M_A = x, R_A = y] \cdot f_{M_A, R_A|m_E, r_E}(x, y) \cdot dx \cdot dy \quad (14)$$

If one wants to model the damage accumulation during aftershock sequences, one may apply a Markovian model such as in Equation (8), where the unit-time transition matrix is that in Equation (15), where k is the number of time units from the mainshock. In the equation the $P_{i,j}$ terms represent the transition probabilities between states i and j given the occurrence of an aftershock. Therefore, they may be obtained, for example, integrating the state-dependent fragility, as in Equation (13), just considering that the IM is that of aftershocks.

$$\begin{aligned} & [P_E(k, k+1)] = \\ & = \begin{bmatrix} 1 - \sum_{j=2}^n \nu_{A|m_E}(k) \cdot P_{1,j} & \nu_{A|m_E}(k) \cdot P_{1,2} & \cdots & \cdots & \nu_{A|m_E}(k) \cdot P_{1,n} \\ 0 & 1 - \sum_{j=3}^n \nu_{A|m_E}(k) \cdot P_{2,j} & \cdots & \cdots & \nu_{A|m_E}(k) \cdot P_{2,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & 1 - \nu_{A|m_E}(k) \cdot P_{(n-1),n} & \nu_{A|m_E}(k) \cdot P_{(n-1),n} \\ 0 & \cdots & \cdots & 0 & 1 \end{bmatrix} \quad (15) \end{aligned}$$

Clearly, the transition matrix changes with time leading to a non-homogeneous Markov chain, in this case the probabilistic prediction of the evolution of damage is given by Equation (16) where the

product of the transition matrices for various instants replace the powers of the stationary transition matrix in Equation (9).

$$[P(k, k+m)] = \prod_{i=1}^m [P_E(k+i-1, k+i)] \quad (16)$$

A.2. Including the effect of aging on seismic vulnerability

It may be the case that the transition probability between states is time-variant because the structural characteristics change with time. This is often referred to as aging, and it is usually due to degradation of material characteristics, corrosion and similar phenomena. This issue renders the unit-time transition matrix non-stationary, as in the previously examined case; however, the transition probabilities carry this variability rather than the earthquake occurrence rate, Equation (17). To calibrate this model, one must be able to probabilistically characterize the transition probabilities as a function of time. Once the transition matrix is defined for this model, which results being an age- and state-dependent one, Equation (16) applies again for probabilistic predictions over an arbitrary number of time intervals.

$$[P_E(k, k+1)] = \begin{bmatrix} 1 - \sum_{j=2}^n v_E \cdot P_{1,j}(k, k+1) & v_E \cdot P_{1,2}(k, k+1) & \cdots & v_E \cdot P_{1,n}(k, k+1) \\ 0 & 1 - \sum_{j=3}^n v_E \cdot P_{2,j}(k, k+1) & \cdots & v_E \cdot P_{2,n}(k, k+1) \\ \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & 1 \end{bmatrix} \quad (17)$$

A.3. Including other sources of age -and state-dependent deterioration

There are cases in which there are other sources of damage accumulation different from earthquakes, yet producing a similar effect. This may be the case of traffic loads, fatigue and other shocks, the occurrence of which, differently from earthquakes, cannot be directly observed. This degradation phenomenon, therefore, may be seen progressive with respect to that of seismic origin, which is sudden. The Markov chain is virtually able to include the effect of a secondary damage process. This requires the definition of a unit-time transition matrix, $[P'(k, k+1)]$, for the progressive degradation.

The matrix should feature the same set of states used to define damage in the seismic case. The matrix in Equation (18) reports the probabilities that the structure moves between states in the $(k, k+1)$ time interval due to progressive deterioration; note that, in general, this matrix is time-variant.

$$[P'(k, k+1)] = \begin{bmatrix} 1 - \sum_{j=2}^n P'_{1,j}(k, k+1) & P'_{1,2}(k, k+1) & \cdots & \cdots & P'_{1,n}(k, k+1) \\ 0 & 1 - \sum_{j=3}^n P'_{2,j}(k, k+1) & \cdots & \cdots & P'_{2,n}(k, k+1) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & 1 - P'_{(n-1),n}(k, k+1) & P'_{(n-1),n}(k, k+1) \\ 0 & \cdots & \cdots & 0 & 1 \end{bmatrix} \quad (18)$$

Consider now that the structure is subjected to both seismic and progressive degradation phenomena [25]. In a unitary time interval, two cases are possible: (i) no earthquakes occur, then degradation is due to the secondary cause only; (ii) an earthquake occurs, then the structure can travel to a worse DS because of both phenomena. Applying the total probability theorem with respect to the occurrence of earthquakes, the global transition matrix across the unitary interval, $[P(k, k+1)]$, is given by Equation (19). Consequently, the transition probabilities in m intervals are given by Equation (20).

$$[P(k, k+1)] = v_E \cdot [P_E(k, k+1)] \cdot [P'(k, k+1)] + (1 - v_E) \cdot [P'(k, k+1)] \quad (19)$$

$$[P(k, k+m)] = \prod_{i=1}^m \left\{ v_E \cdot [P_E(k+i-1, k+i)] \cdot [P'(k+i-1, k+i)] + (1 - v_E) \cdot [P'(k+i-1, k+i)] \right\} \quad (20)$$

Equation (19) represents an age- and state-dependent reliability model. Obviously, if both $[P_E]$ and $[P']$ are stationary, then the unit-time transition matrix is time-invariant, then the more simple Equation (21) applies.

$$[P(k, k+m)] = \left\{ v_E \cdot [P_E] \cdot [P'] + (1 - v_E) \cdot [P'] \right\}^m \quad (21)$$