On Bandwidth Maximization of Fixed-Geometry Arrays through Convex Programming

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Abstract

We propose a new approach to solve the problem of optimal power synthesis of array antennas, so that maximum possible bandwidth can be granted to fixed sidelobe-level performances. The proposed approach can be applied to any kind of fixed-geometry array that radiates pencil beams and to linear equispaced arrays that generate shaped patterns. The designing problem is cast as a sequence of Convex Programming optimizations. Numerous numerical experiments, including full-wave synthesis of realistic antennas, were carried out and their results discussed here to assess the array antennas' capability of achieving ultra-wideband performances.

Keywords: Array optimization, power synthesis, UWB antennas.

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I. Introduction

In designing array antennas, two of the most important performance indicators are the bandwidth and the sidelobe level (SLL) [1].

The bandwidth is defined as the continuous range of frequencies over which the antenna can correctly operate, and it is usually described in terms of fractional bandwidth (FBW) [2]-[4]. It varies between 0 and 2 and, for a system which operates between the minimum frequency $f_{\min}$ and the maximum frequency $f_{\max}$, it is given by:

$$FBW = \frac{f_{\max} - f_{\min}}{f_c} = 2\frac{f_{\max} - f_{\min}}{f_{\max} + f_{\min}} \tag{1}$$

wherein $f_c=(f_{\min}+f_{\max})/2$ denotes the center frequency.

An antenna is considered ‘wideband’, as long as 0.2<FBW<0.5 and ‘ultra-wideband’ (UWB) as long as FBW≥0.5 [4]. Wideband systems offer several advantages over narrowband devices, including high precision in ranging, shortness in broadcast time, low electromagnetic radiation (safe for human body even at a short distance), and low energy consumption in processing [2].

UWB antennas applications include medical imaging, tracking and real-time locating systems, personal area networks, see-through-wall imaging, and time-of-arrival-based localization approaches [3]. Procedures for effective array pattern synthesis can be found in [4]-[9].

The SLL is defined as the sidelobes’ maximum value (relative to the maximum value of the pattern) [1]. For example, in the case of 1-D sources, it is expressed as:

$$SLL = \max_{\theta\in\Omega}\left|\frac{F(\theta)}{F(\theta_0)}\right|^2 \tag{2}$$

wherein F denotes the radiated field, θ is the angle between the antenna axis and the observation direction, θ_0 is the target direction, and Ω represents the sidelobes region (which is fixed by taking into account the expected beamwidth of the system as well as the radiation requirements pertaining to the application scenario at hand [10]).

Systems granting a low SLL are essential in a number of applications, including radar and satellite communications, wireless power transmission, hyperthermia treatments, and synthesis of high-beam-efficiency antennas [10]-[14]. Effective synthesis techniques have been given in cases of both single [12]-[17] and reconfigurable [18]-[22] beam arrays.

Unfortunately, almost all the techniques that pursue SLL minimization have an important limitation, i.e., using either a single-frequency or a narrow-band assumption [4]. Obviously, while simplifying the consequent synthesis algorithm, this limitation strongly reduces the applicability range of

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the achieved solutions. Therefore, it makes sense to introduce a new synthesis approach that allows for the simultaneous optimization of both SLL and FBW performances. It was with this spirit that the authors of [4] proposed a Particle Swarm Global Optimization algorithm aimed at jointly determining the array-element locations and excitations such to minimize the SLL for a given fixed FBW performance.

By taking into account the results shown in [4], and exploiting the fundamental approaches available for the single-frequency optimal synthesis, a new and effective approach is proposed here for designing array antennas in such a way that, once the minimum guaranteed SLL performance has been fixed, maximization of FBW follows. Also, a new method is introduced to identify the *maximum possible FBW* over which an array of given size and geometry can achieve a fixed SLL performance.

The proposed tools facilitate the rapid synthesis of pencil beams through any kind of fixed-geometry arrays, i.e., arrays having whatever layout and element patterns (also, in the presence of mutual coupling, as well as sparseness and mounting-platform effects). The general methodology introduced here can be used for the mask-constrained power synthesis of shaped patterns also, provided the field can be expressed in terms of the array factor. In all the cases, the overall synthesis problem is cast as a sequence of fast Convex Programming (CP) optimizations, with decisive advantages in terms of computational burden and solutions' optimality.

The remainder of this paper is organized as follows: Section II briefly recalls some fundamental results available for the optimal synthesis of single-frequency pencil beams with minimum SLL; Section III describes the proposed approach; Section IV establishes, through several numerical experiments [including Computer Simulation Technology™ (CST) full-wave simulations] the capability of the proposed approach to design realistic UWB systems; Section V presents the conclusions drawn from this study.

II. Review of Fundamentals

In the case of fixed-geometry arrays (wherein the excitation weights are the only unknown parameters of the design problem), the approach in [17] was the first to address the optimal[1] synthesis of single-frequency pencil beams as a CP optimization. In particular, in [17] the array excitations are found in such a way that:

[1] Pencil beams' synthesis is referred to as 'optimal' as long as it provides, for fixed antenna resources and a given null-to-null beamwidth, the best possible SLL performance. Once the array geometry, Ω, and θ_0 are fixed, such a goal can be pursued by minimizing (2).

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$$\max \mathrm{Re}[F(\theta_0)] \tag{3.a}$$

subject to:

$$\begin{cases} \mathrm{Im}[F(\theta_0)] = 0 & (3.b) \\ \mathrm{Re}[F(\theta_0)] \geq 0 & (3.c) \\ |F(\theta)|^2 \leq A \quad \forall \theta \in \Omega & (3.d) \end{cases}$$

wherein A denotes the maximum permitted power-pattern value inside the sidelobes region. In fact, the constraints (3.b) and (3.c) ensures that maximizing the function (3.a) is equivalent to maximizing the square amplitude of the field in the target direction. Therefore, the maximization of the power pattern is pursued as the maximization of a linear functional of the unknowns, with relevant advantages in terms of computational times and optimality of the results.

As a straightforward consequence, if the goal is not to minimize the SLL but to simply guarantee that it does not exceed a given value, say A, the problem can be solved by finding an intersection amongst the following convex constraints:

$$\begin{cases} F(\theta_0) = 1 & (4.a) \\ |F(\theta)|^2 \leq A \quad \forall \theta \in \Omega \ . & (4.b) \end{cases}$$

An important advantage of formulation (4) is that it allows, for a fixed guaranteed SLL performance, the insertion of an objective function aimed at maximizing an additional antenna parameter, e.g., gain or directivity [15].

It is well known that formulations (3) and (4) can also be applied by using, instead of the constant A, an arbitrary upper-bound function, say $UB(\theta)$ [17]. Moreover, if one can address the designing problem by dealing just with the array factor, and as long as the array layout is centrosymmetric, the synthesis can even be cast as a Linear Programming (LP) problem. In fact, under these hypotheses, the optimal field satisfying (3) and (4) is *real* [15] and hence the quadratic constraints (3.d) and (4.b) can be linearized as follows:

$$-\sqrt{A} \leq F(\theta) \leq \sqrt{A} \quad \forall \theta \in \Omega \ . \tag{5}$$

The proposed design procedure, presented below, takes advantage of the results recalled above.

III. The Proposed Approach

The goal is to identify the array elements’ complex excitations in such a way as to maximize FBW and simultaneously guarantee fixed SLL performance over the whole operating frequency range.

Notably, the general method that is presented here includes the cases where:

(i) UB is not a constant function,

(ii) the fields radiated by the array’s single elements are frequency-dependent,

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(iii) mutual coupling and mounting-platform effects are present,

(iv) any combination of the above.

To introduce the synthesis approach, a one-dimensional fixed-geometry array is considered, which is disposed along the z-axis and composed of N elements, having arbitrary locations $z_1,\dots,z_N$ and complex excitation weights $w_1,\dots,w_N$.

By referring to scalar fields and denoting the operating frequency and the speed of light in vacuum with, respectively, f and c, the far field radiated by such an antenna can be expressed as:

$$F(r,f,\theta)=\frac{e^{-j\frac{2\pi f}{c}r}}{r}\sum_{n=1}^{N}w_n g_n(f,\theta)e^{j\frac{2\pi f}{c}z_n\cos\theta} \tag{6}$$

wherein r and g_n respectively denote the distance from the origin and the n-th Active Element Pattern (AEP) [23], i.e., the array's radiation pattern when only the n-th element is excited (with w_n=1) [2].

Let us now suppose fixing $f_{\max}$ (see Appendix) and introduce a discrete set of frequencies $f_1,\dots,f_K$ such that $f_1=f_{\max}$ and $f_k=f_{\max}-(k-1)\Delta f$, $k=2,\dots,K$, Δf denoting the difference between two consecutive frequencies. Let us also temporarily neglect the field's dependence on r and suppose that the signal to be transmitted does not exhibit a fast oscillating behavior over the frequency domain $[f_1, f_K]$.

Under such hypotheses, the problem of maximizing the array's FBW for a given SLL performance can be solved as shown below:

$$\max_{w_1,\dots,w_N} K \tag{7}$$

$$subject\ \ to:$$

$$\begin{cases} F(f_k,\theta_0)=1 \qquad k=1,\dots,K & \text{(8.a)} \\ |F(f_k,\theta)|^2 \le UB(\theta) \quad k=1,\dots,K \quad \forall\theta\in\Omega\ . & \text{(8.b)} \end{cases}$$

In fact, finding an intersection amongst *convex* constraints (8) is equivalent to satisfying the constraints (4) over the whole frequency range $[f_1, f_K]$, while the objective function in (7) allows maximizing the size of this range. In particular, as $f_{\min}=f_K$, by virtue of (1) the synthesized array will provide the following bandwidth performance:

$$FBW=\frac{1}{\frac{f_1}{(K-1)\Delta f}-\frac{1}{2}} \tag{9}$$

wherein it is guaranteed that $f_1/[(K-1)\Delta f]\geq 1$ (since $f_K\geq 0$). Moreover, the antenna can transmit the input signal in the target direction with no distortion. In fact, constraint (8.a) and the exponential functions in

[2] Note that, in (6), all AEPs are supposed to be 'phase-adjusted', i.e., computed by placing the coordinate origin at each element center (and then identifying the corresponding location-related phase term) [23],[28].

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(6) entail, respectively, a *uniform magnitude* and a *linearly-varying phase* of the frequency response in the whole operating range [f_1, f_K], and hence the input signal will be delayed by precisely the same amount of time at all frequencies.

Interestingly, problem (7)-(8) can even be cast in a simpler fashion, subject to two more assumptions:

1. if the target direction is set as the antenna boresight, i.e., $\theta_0=\pi/2$, then the location-related phase term in (6) disappears;
2. if the elements' locations and AEPs are centrosymmetric, i.e., $z_n=-z_{N+1-n}$ and $g_n=g^*_{N+1-n}$ (* denoting complex conjugation) for either n=1,..,N/2 (as long as N is even) or n=1,..,(N-1)/2 (as long as N is odd, with z_0=0), then the optimal field is a *real* function [15], and hence the constraints (8.b) can be rewritten as *linear* inequalities.

Under these two hypotheses, problem (7)-(8) can be rewritten as:

$$\max_{w_1,\ldots,w_N} K \tag{10}$$

subject to:

$$\begin{cases} \sum_{n=1}^{N} w_n g_n\left(f_k,\theta_0\right)=1 \quad k=1,\ldots,K & \text{(11.a)} \\ -\sqrt{UB(\theta)} \le F\left(f_k,\theta\right) \le \sqrt{UB(\theta)} \quad k=1,\ldots,K \quad \forall\theta\in\Omega & \text{(11.b)} \end{cases}$$

Notably, condition (11.a) [as well as (8.a)] implies that each element is by itself a wideband (which is a reasonable assumption for the kind of arrays being designed)[3].

Going back to formulation (7)-(8), a solution can be found by exploiting the fact that, by virtue of band-limitedness of F [24], constraints (8.a) and (8.b) can be substituted with sufficiently-fine discretization and hence can be seen as a system of *linear* equalities and *quadratic* inequalities in the unknowns. Therefore, finding an intersection between these constraints is a CP problem, implying that it can be solved in a fast and effective manner. Hence, the optimal solution of the problem (7)-(8) can be quickly obtained by applying the following iterative strategy:

For a given set of array element locations and a fixed value of $f_{\max}$, *UB*, *Δf*, *and Ω, solve the convex problem (8) repeatedly for increasing values of K, until constraints become so strict that no solution exists anymore.*

[3] The approach would also work in case of compensation effects from different elements of the array (so that, if one element exhibits some variation from *f*, the latter is compensated by an opposite variation of another element).

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The proposed synthesis procedure can be considerably simplified, provided that none of the conditions (i)-(iii) holds true and the radiation pattern simply exhibits a kind of 'shrinking' (in angle) with increasing frequency. In fact, in such a case, the radiation behavior at f_{min} and at f_{max} completely determines the pattern in the whole frequency range $[f_{min}, f_{max}]$ and hence it is not necessary to enforce constraints (8) at all the intermediate frequencies. In particular, as the pattern shrinks with increasing frequency, the constraints at the lowest frequency will ensure the correct main-beam shape at all frequencies, while the ones at the highest frequency will provide the correct sidelobes behavior at all frequencies.

While noting that a-priori choice of some input parameters (such as f_{max}) can be made according to the criteria reported in Appendix, a number of interesting and useful features of the overall approach are reported in the following paragraphs.

As a first comment, it is to be noted that, if there exists more than one intersection with (8), an objective function that leads to the identification of the 'most convenient' one can be added. For instance (see numerical examples under Section IV), for a given f_{max}, the problem can be solved by adding the minimization of the following *convex* objective function:

$$\Theta(w_1,...,w_N)=\sum_{k=1}^{K}\int_{0}^{\pi}\left|F(f_k,\theta)\right|^2\sin\theta d\theta \tag{12}$$

which allows maximizing the average antenna directivity over the whole frequency range $[f_1, f_K]$.

Second, a number of straightforward extensions are possible. In fact, by applying the theory in [15], the approach can be easily extended to any kind of planar or conformal fixed-geometry array radiating pencil beams granting the maximum possible FBW. Moreover, in case the adopted array layout is a 1-D sufficiently-dense [11] equispaced one, the achieved solutions can be further processed by exploiting the theory in [25]-[28] to design circular sources as well as reconfigurable and sparse arrays.

Third, if the synthesis can be tackled in terms of array factor, and the array layout is linear and equispaced, the proposed approach can be used also for the optimal synthesis of wideband shaped beams. In fact, under such hypotheses, the square-amplitude far-field distribution can be rewritten as:

$$P(f,\theta)=\left|\sum_{n=1}^{N}w_n e^{j\frac{2\pi f}{c}nd\cos\theta}\right|^2=\sum_{p=-N+1}^{N-1}D_p e^{j\frac{2\pi f}{c}pd\cos\theta} \tag{13}$$

wherein $P(f, \theta)\geq 0$ $\forall(f, \theta)$, $D_{-p}=D_p^*$, and d denotes the inter-element spacing. Then, the problem of synthesizing a shaped beam granting the maximum possible FBW while being confined between the lower-bound function L and the upper-bound function U can be solved as follows:

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$$\max_{D_{-N+1},\ldots,D_{N-1}} K \tag{14}$$

subject to:

$$\begin{cases} 0 \le L(\theta) \le P(f_k,\theta) \le U(\theta) \quad k = 1,\ldots,K \quad \forall\theta & (15.a) \\ D_{-p} = D_p{}^* \quad p = 1,\ldots,N-1 & (15.b) \end{cases}$$

wherein possible distortion effects can be controlled by enforcing a small ripple in the shaped-beam region.

It is also worth noting that, subject to adopting a sufficiently fine [24] discretization, the constraints (15) can be seen as a system of ordinary *linear* inequalities in the unknowns. Therefore, finding an intersection among them is an LP problem, and the FBW maximization can be pursued through the same iterative approach presented above with reference to pencil beams. Then, the actual array excitations will be identified by executing the spectral-factorization procedure described in [25], i.e.:

$$P(f,\theta) = \left(\sum_{n=1}^{N} w_n{}^{j\frac{2\pi f}{c}nd\cos\theta}\right)\left(\sum_{n=1}^{N} w_n{}^{j\frac{2\pi f}{c}nd\cos\theta}\right)^* \tag{16}$$

Interestingly, the solution to the problem (14)-(16) can provide multiple equivalent excitation sets (in addition to phase ambiguities [30]) and guarantee that, at a given observation direction in the shaped zone, the signal is adequately flat in its amplitude over the entire considered frequency band. On the other hand, it is worth noting that, unless the factors in (16) are real, the overall phase behavior with frequency is not linear (so that phase-compensation techniques have to be used, or frequency-division multiplexing schemes have to be considered).

IV. Assessment of Performances

Many numerical experiments have been performed to ascertain the effectiveness of the proposed approach.

All simulations (except for the ones involving a non-broadside target direction) have been carried out by setting f_{max}=2.5 GHz, $\Omega=\{\theta|\ 0°\le\theta\le75° \cup 105°\le\theta\le180°\}$, $\theta_0\le90°$, and utilizing a centrosymmetric linear array layout composed of N=20 elements with a constant inter-element spacing[4] equal to d=15 cm.

[4] The layout's symmetry and constant spacing led in all the performed synthesis experiments (but for the ones pertaining to a non-broadside target direction) to purely real excitations. This finding agrees with the theory developed in [15] and updated in [29], and it does not affect the actual maximum radiation performances of the synthesized systems.

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The θ domain has been discretized into 2000 samples and all numerical simulations have been run in such a way to achieve UWB performances.

For each frequency, 14 experiments have been carried out by enforcing, both in the absence as well as in the presence of the objective function (12), the constraints SLL≤{-20; -25; -30; -35; -40; -45; -50} dB, with a total of more than 80 involved radiation patterns.

The complete synthesis of each array, using a calculator having a 2.5 GHz CPU and a 10 GB RAM, required less than 1 minute, on the average. Each single CP optimization lasted no more than a couple of seconds. Such a short time enabled the easy identification of the Δf value such to maximize the FBW for a fixed value of K. All this has been done by following the rules reported in Appendix and in such a way as to achieve the maximum FBW for K=6.

Four different synthesis scenarios (depending on the adopted power-pattern mask, objective function, and radiating elements) were considered and the scenario-wise results are discussed below.

IV.1 Assessment by exploiting a constant SLL mask and realistic radiating elements

In the first set of simulations, the upper-bound mask in (8.b) was chosen as a constant function and the practical reliability of the results validated by considering an array of realistic radiating elements. Since the fields generated by these elements actually decay in the sidelobes region, exhibit wideband performances in the broadside direction, and guarantee (also in view of the adopted inter-element spacing) the absence of relevant mutual coupling effects, we solved the synthesis problem by applying the 'simplified' procedure described in Section III. Therefore, we enforced the constraints only at the two frequencies $f_{\min}$ and $f_{\max}$. The achieved results are discussed in detail below.

First, the proposed procedure was executed by using isotropic elements and without adding any objective function. In all the cases, the design constraints were satisfied. For example, Table I reports the radiation performances (in terms of SLL, directivity, and half-power beamwidth) achieved when the constraint SLL≤-30dB was enforced. The achieved FBW is shown in Fig. 1 as a function of the enforced SLL values. As can be seen from this figure, a price must be paid in terms of SLL performance (and vice versa) to increase FBW. In particular, decreasing the SLL from -20 to -50 dB entailed the decreasing of FBW from 1.22 to 0.68. Notably, even the lowest achieved FBW value, i.e., 0.68, guarantees ultra-wideband performances.

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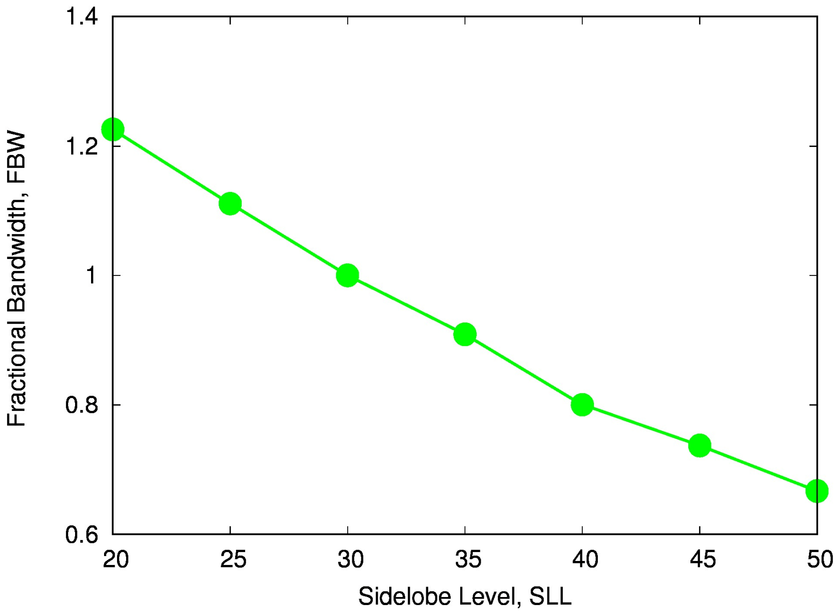
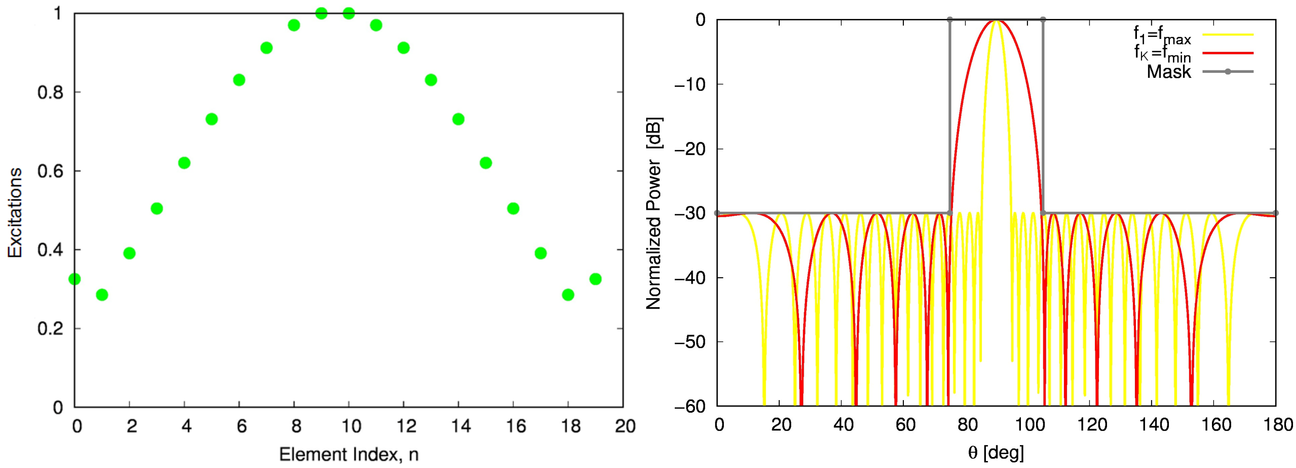


Figure 1. Graph showing that the FBW achieved is a function of enforced SLL. The results were obtained by setting *UB* as a constant, with $\Omega=\{\theta|\ 0°\leq\theta\leq75° \cup 105°\leq\theta\leq180°\}$. For f_{max}=2.5 GHz, decreasing the maximum SLL from -20 to -50 dB entailed the decreasing of FBW from 1.22 (f_{min}=0.61 GHz) to 0.68 (f_{min}=1.23 GHz), guaranteeing, in any case, UWB performances.



(a) (b)

Figure 2. (a) Real and positive excitations and (b) UWB power pattern achieved by maximizing the directivity and enforcing SLL≤-30dB for $\Omega=\{\theta|\ 0°\leq\theta\leq75° \cup 105°\leq\theta\leq180°\}$.

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	Directivity [dB]		HPBW [deg]		SLL[dB]	
	Not pursuing Max Directivity	Pursuing Max Directivity	Not pursuing Max Directivity	Pursuing Max Directivity	Not pursuing Max Directivity	Pursuing Max Directivity
f_1=f_{max}	14.46	14.60	3.93	3.79	-30.2	-30.0
f_2	13.12	13.36	5.24	5.05	-30.2	-30.0
f_3	12.25	12.40	6.56	6.31	-30.2	-30.0
f_4	10.80	10.94	9.38	9.02	-30.2	-30.0
f_5	10.21	10.37	10.65	10.25	-30.2	-30.0
f_6=f_{min}	9.70	9.86	11.94	11.49	-30.2	-30.0

Table I. Performances pertaining to the synthesis scenario of Subsection IV.1 (for SLL≤-30 dB).

Acronym	Meaning	Value
S_l	Substrate length	70 mm
S_w	Substrate width	50 mm
S_t	Substrate thickness	1.6 mm
P_l	Patch length	38.4 mm
P_w	Patch width	30 mm
C_r	Cutting radius	19.2 mm
W_l	Feeding line length	22 mm
W_f	Feeding line width	3 mm
G_l	Ground plane length	22 mm
G_w	Ground plane width	50 mm
M_t	Metal thickness	0.035 mm
ε_r	Relative permittivity	4.3 F/m
μ_r	Relative permeability	1 H/m
$\tan\delta$	Dielectric loss tangent	0.025

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d	Inter-element spacing	$15x10^{-2}$ m

Table II. Geometrical and electrical parameters of the microstrip antenna shown in Fig. 3.

	Directivity [dB]		HPBW [deg]		SLL[dB]	
	Not pursuing Max Directivity	Pursuing Max Directivity	Not pursuing Max Directivity	Pursuing Max Directivity	Not pursuing Max Directivity	Pursuing Max Directivity
$f_1=f_{max}$	13.42	13.54	4.85	4.78	-50.1	-50.0
f_2	12.17	12.28	6.47	6.37	-50.1	-50.0
f_3	11.66	11.77	7.36	7.20	-50.1	-50.0
f_4	11.19	11.32	8.10	7.97	-50.1	-50.0
f_5	10.78	10.90	9.00	8.86	-50.1	-50.0
$f_6=f_{min}$	10.59	10.70	9.34	9.22	-50.1	-50.0

Table III. Performances pertaining to the synthesis scenario of Subsection IV.3 (for SLL≤-50dB).

As regards the applications that require the joint optimization of bandwidth and gain performances (see for instance [31]), all the above experiments were repeated in such a way as to maximize the antenna's average directivity over the whole frequency operating range. To this end, the objective function (12) was minimized subject to the usual constraints (8). The results achieved for the case in which SLL≤-30dB was enforced are summarized in Table I (in terms of radiation parameters) and in Fig. 2 (in terms of array excitations and power pattern). As can be seen, solving this problem led to, besides a directivity increase, a slight increase in sidelobes' amplitude and a slight decrease in HPBW. The 'SLL vs. FBW' trade-off curve remained unchanged, compared to that of Fig. 1, and hence it is not presented here.

The reliability of the proposed synthesis strategy was validated using an array of realistic radiating elements and feeding them with the excitations shown in Fig. 2. For this purpose, the microstrip patch

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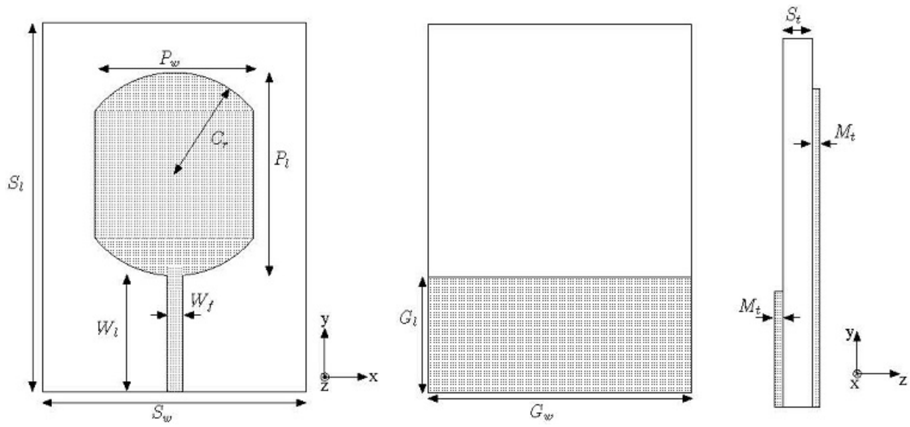
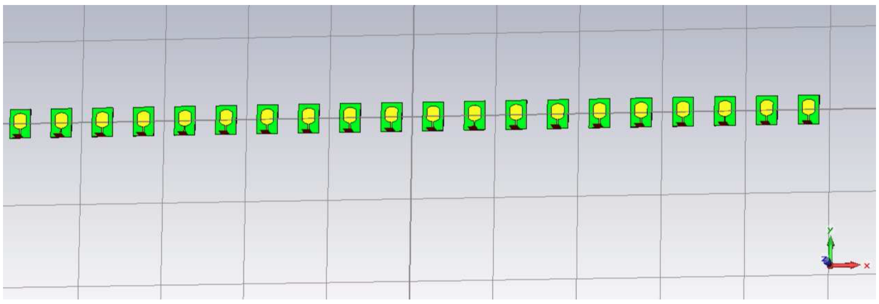
array shown in Fig. 3 was designed and characterized by the parameters listed in Table II. The K=6 radiated power patterns were computed through a CST™ full-wave simulation and the same are shown in Fig. 4. As can be noticed, the behavior of the power patterns turns out to be very similar to that of the ideal one shown in Fig. 2. In particular, for SLL≤-30dB, the antenna guarantees the same UWB performance as that of the one shown in Fig. 2, i.e., FBW=1.03 and f_{min}=0.8 GHz.



(a)



(b)

Figure 3. Microstrip array simulated using the CST full-wave software: (a) single element; (b) overall view.

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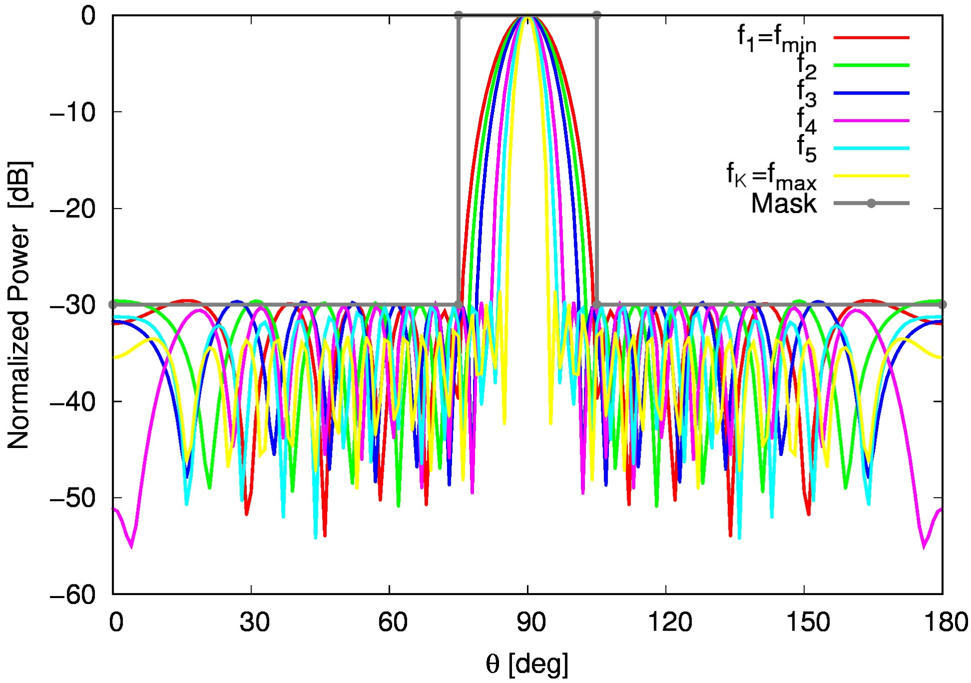

Figure 4. UWB power pattern, computed through CST™ full-wave simulations, generated by the microstrip array depicted in Fig. 3 when fed by the excitations shown in Fig. 2. The antenna guarantees SLL≤-30 dB as well as UWB performances (i.e., f_{min}=0.8 GHz, f_{max}=2.5 GHz, FBW=1.03).

IV.2 Full-wave synthesis of a microstrip patch array

In the second set of numerical experiments, the full-wave synthesis of a UWB realistic microstrip patch array was carried out. In particular, starting from the antenna structure and specifications described under Section IV.1 (i.e., SLL≤-30 dB, K=6, f_{min}=0.8 GHz, f_{max}=2.5 GHz, and the array described by Fig. 3 and Tab. II), the overall radiating system was made to be closer to a practical implementation by extending the substrate layer below different elements, as well as between each element and the neighbouring ones. The new array structure is shown in Fig. 5. Then, the array excitations were determined by taking into account the actual element patterns (including mutual coupling and mounting-platform effects) right from the scratch (i.e., during the numerical optimization of the unknowns). In particular, for each frequency, 20 different AEPs were computed the by performing 20 distinct CST full-wave simulations (for a total of 120 experiments), and then the optimization problem (7)-(8) was solved by following the guidelines given under Section III. For example, the AEP of the 10-th array element, computed through a full-wave simulation at 1.1 GHz, is shown in Fig. 6.

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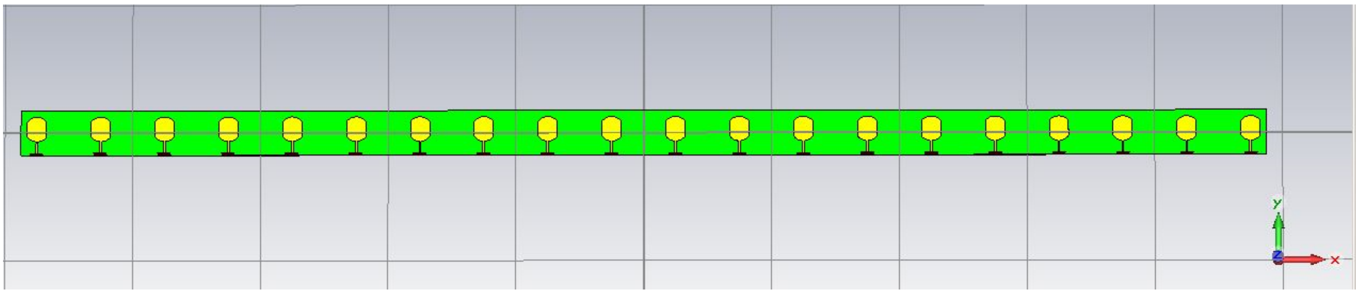
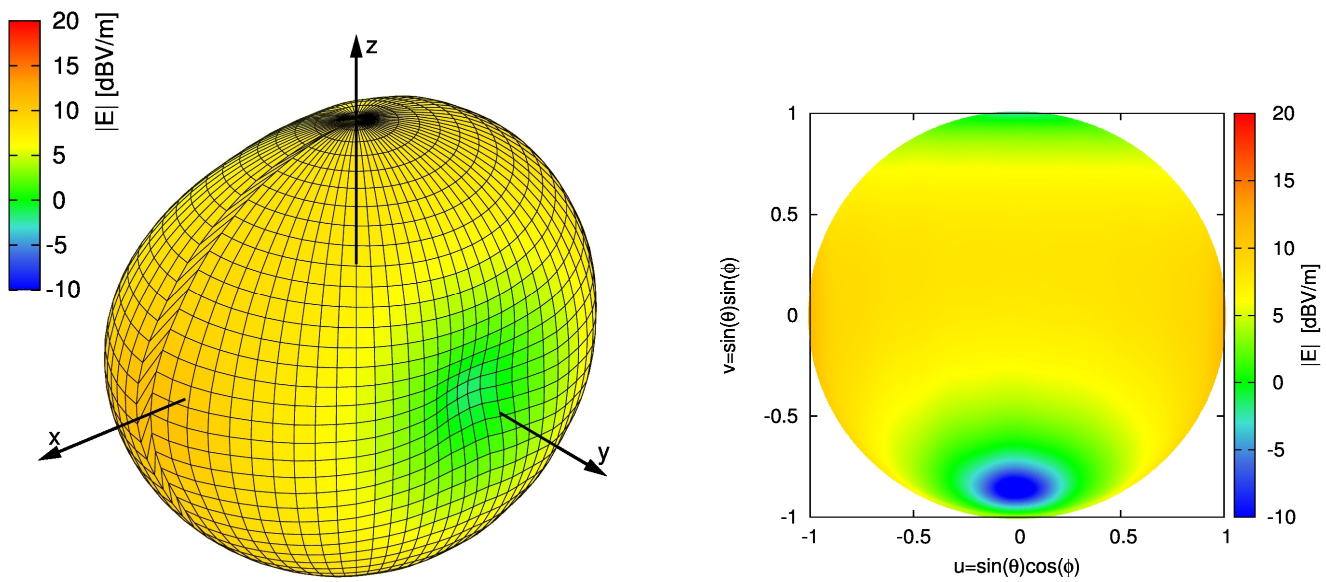


Figure 5. Radiating system exploited for the full-wave synthesis experiment, performed as described in Section IV.2. The substrate layer (described in Table II) was extended below different elements and between each element and the neighboring ones.

The final outcome of the optimization is depicted in terms of excitations and power patterns in figures 7 and 8, respectively. It can be seen that all the radiation constraints were fulfilled and UWB performances (i.e., FBW=1.03) achieved. Differently from the previous test cases, the excitations plotted in Fig. 7 exhibit a slightly asymmetrical behavior. This is due to the fact that the synthesis of the UWB microstrip patch array has been performed by considering the actual AEPs, that are also non-symmetric. Therefore, since the symmetry has not been forced, but only the satisfaction of the upper power bound and the maximization of the FBW, also asymmetrical power patterns (as the ones shown in Fig. 8 and generated by the excitations of Fig. 7) are suitable solutions of the problem at hand.



(a) (b)

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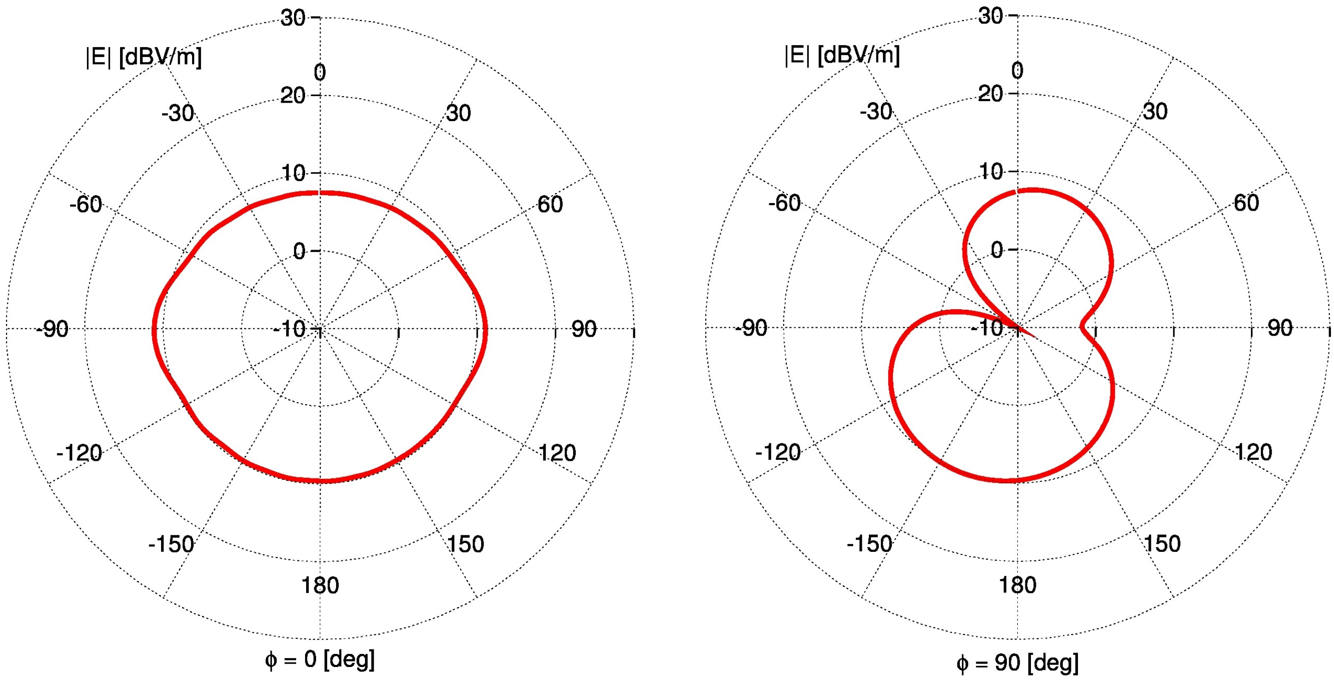

(c)

Figure 6. AEP of the 10-th element of the realistic array shown in Fig. 5, computed through a CST full-wave simulation at 1.1 GHz (θ and ϕ denoting the usual elevation and azimuth angles with respect to the aperture antenna plane): (a) 3-D spatial view; (b) spectral behavior; (c) main cuts through polar coordinates.

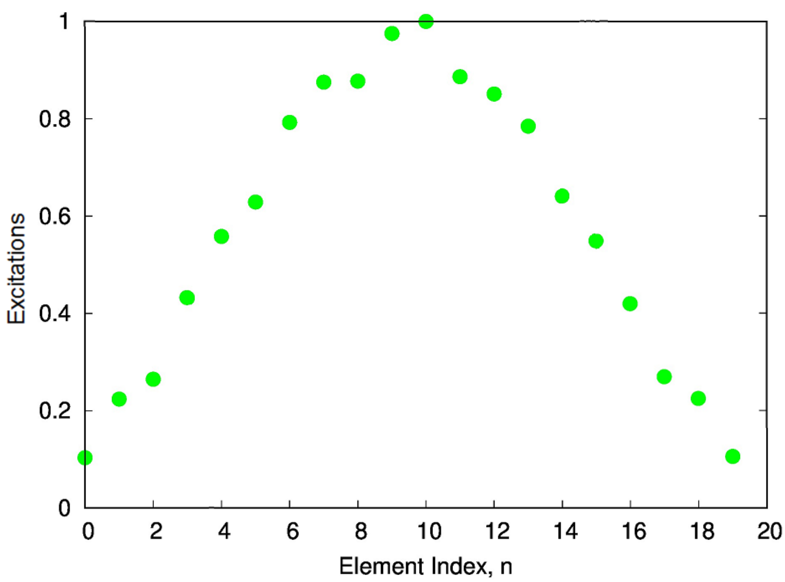

Figure 7. Real and positive excitations corresponding to the full-wave synthesis of the UWB patterns as per Fig. 8.

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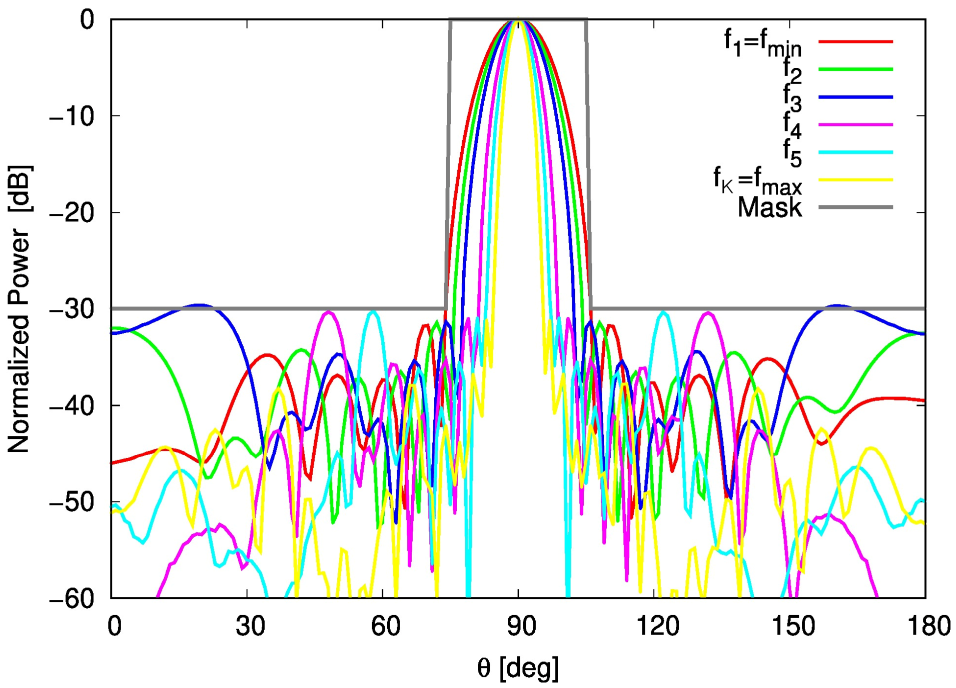

Figure 8. Superposition of the K=6 power patterns generated by the microstrip patch array shown in Fig. 5 under the excitations depicted in Fig. 7. The antenna guarantees SLL≤-30 dB as well as UWB performance (i.e., f_{min}=0.8 GHz, f_{max}=2.5 GHz, FBW=1.03).

IV.3 Assessment in the presence of notches on the power pattern

The third set of numerical experiments was performed by exploiting isotropic elements and adding, to the radiation constraints mentioned in Subsection IV.1, two 10 dB-deep notches (relative to the maximum permitted amplitude of the sidelobes) in the region $\Psi=\{\theta|\ 30°\leq\theta\leq 50° \cup 130°\leq\theta\leq 150°\}$. This kind of mask would be of interest in many practical applications, including monopulse radar and telecommunications [10].

As in the previous Subsection, for each frequency and sidelobes mask, 14 numerical experiments were performed, first by solving the problem (8) without adding any objective function and then by adding the directivity maximization.

Owing to the sidelobes' mask non-uniformity, this time, the problem was solved by enforcing the radiation constraints not only at f_{min} and f_{max} but also at all intermediate frequencies (for a total of more than 80 involved radiation patterns). The FBW achieved for these particular technical requirements is shown in Fig. 9 as a function of the enforced SLL value. As can be seen, decreasing SLL from -20 to -50 dB induced FBW to decrease from 1.18 to 0.63. Therefore, from the results reported under Subsection IV.1 it follows that the notches' requirement induced decrease in the maximum and minimum FBW values

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from 1.22 to 1.18 and from 0.68 to 0.63, respectively. Despite this, ultra-wideband performance is still guaranteed even for SLL≤-50 dB.

For example, the results achieved when the constraint SLL≤-50dB was enforced and maximum directivity was pursued are summarized in Table III (in terms of SLL, directivity, and half-power beamwidth) and in Fig. 10 (in terms of array excitations and power pattern).

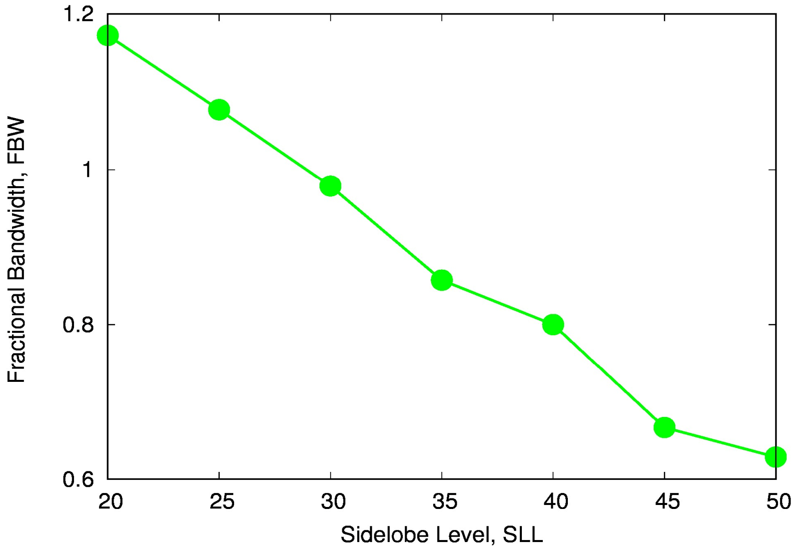
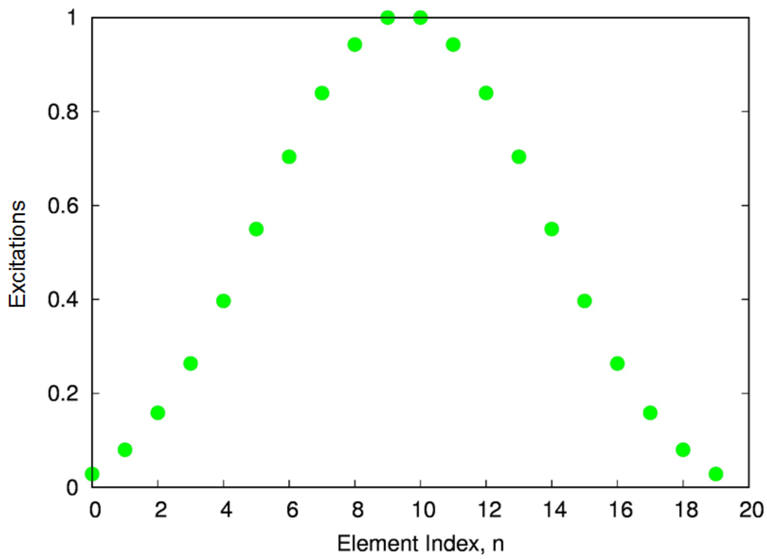


Figure 9. FBW of the particular array described in Section IV.3 as a function of the enforced SLL plus notches constraints. For f_{max}=2.5 GHz, decreasing the required maximum SLL from -20 to -50 dB entailed decreasing the FBW from 1.18 (f_{min}=0.64 GHz) to 0.63 (f_{min}=1.30 GHz), guaranteeing in any case UWB performances.



(a)

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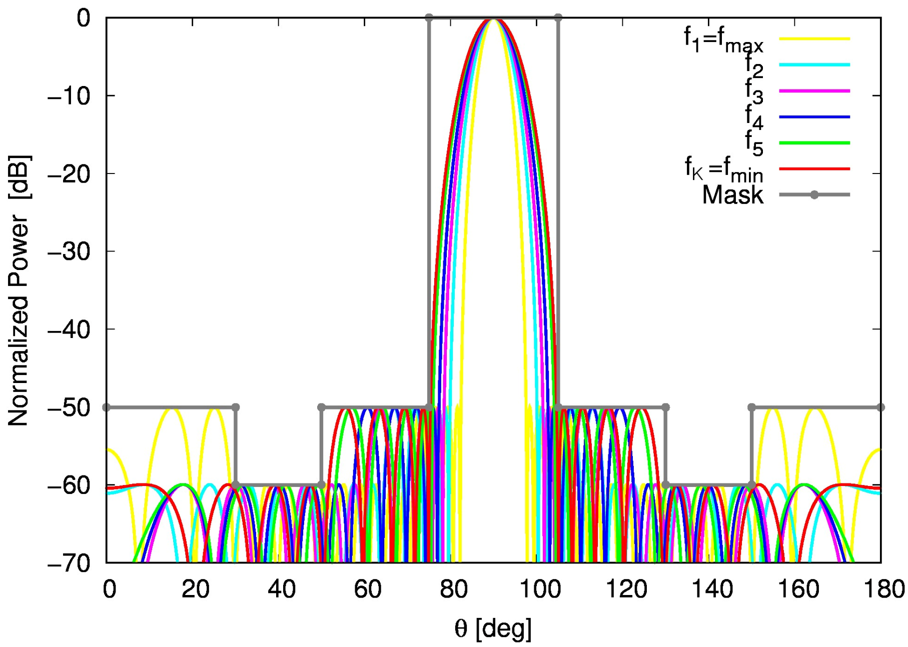

(b)

Figure 10. (a) Real and positive excitations and (b) UWB power patterns achieved by enforcing SLL≤-50 dB and notches in the region $\Psi=\{\theta|\ 30°\leq\theta\leq 50° \cup 130°\leq\theta\leq 150°\}$.

IV.4 Non-broadside radiation

The fourth and final set of numerical examples has been carried out in order to assess the effectiveness of the presented synthesis technique in providing high-FBW performances even in case of non-broadside radiation. To this end, we set θ_0=60°, f_{max}=10 GHz, Δf=365 MHz, and challenged the proposed approach to identify the optimal excitations guaranteeing SLL≤-25 dB for $\Omega=\{\theta|\ 0°\leq\theta\leq 40° \cup 80°\leq\theta\leq 180°\}$ and K=6, i.e., wideband performances (with FBW=0.201).

An array composed of N=25 isotropic elements with a constant inter-element spacing $d=0.5c/f_{min}$ (such to comply with all rules reported in the Appendix) has been adopted as radiating system.

The achieved complex excitation weights and the K=6 corresponding power patterns are shown in figures 11 and 12, respectively.

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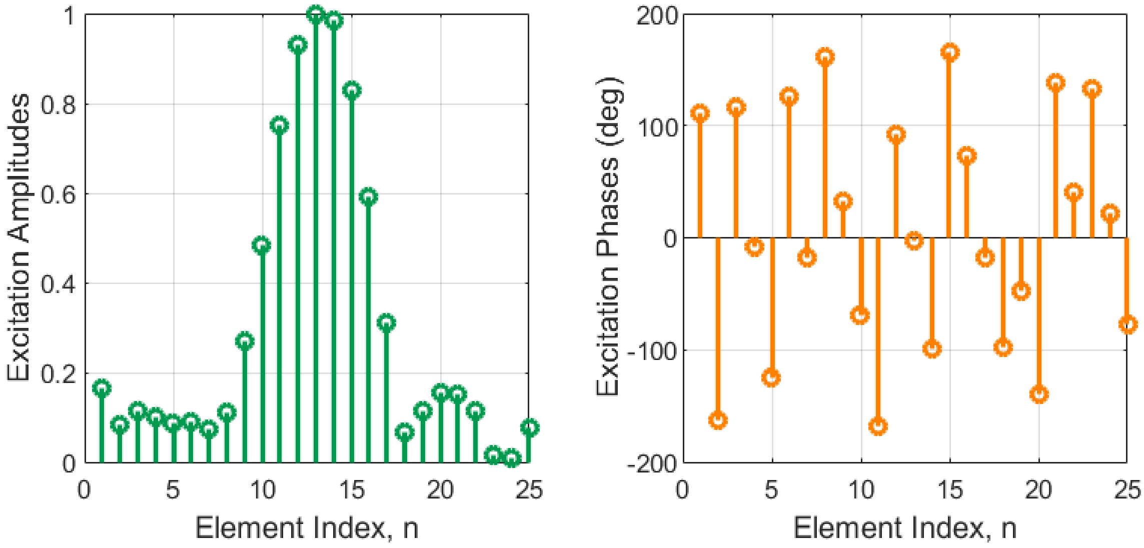

(a) (b)

Figure 11. Complex excitations [amplitude (a) and phase (b) distributions] providing a non-broadside radiation as per constraints listed in Section IV.4.

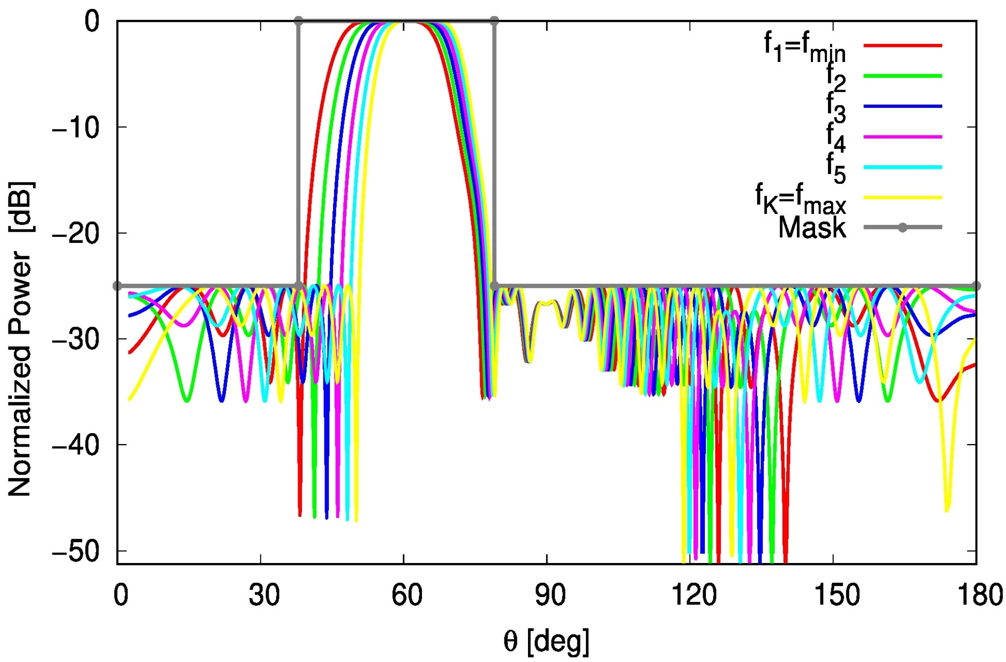

Figure 12. Wideband power pattern corresponding to the excitations depicted in Fig. 10. The resulting directivity values are between 7.9 dB (for f=f_{min}) and 8.8 dB (for f=f_{max}). FBW=0.201.

As can be seen, the proposed approach led to the fulfillment of all technical requirements. Moreover, since the approach is aimed at finding a unique excitation set common to all frequencies, a slight (unavoidable) misalignment of the beams has been experienced. On the other side, the adoption of a frequency-independent excitation set allows reducing as much as possible the beam forming network's

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cost and complexity as well as maximizing the easiness of the system reconfiguration amongst the different frequencies.

V. Conclusions

An innovative and effective approach to the optimal synthesis of fixed-geometry arrays, guaranteeing the maximum possible bandwidth performance for a fixed SLL power mask, has been proposed.

The capability of designing ultra-wideband arrays, granting extremely low SLL values and high-directivity performances, has been assessed through a large set of numerical experiments. The optimality of the achieved results, as also of the corresponding execution times, has been ensured by casting the problem as a sequence of convex optimizations.

The solutions obtained can be a key enabling technology for a number of recent applications, including radar and remote sensing as well as next-generation small satellites and 5G communications [32]-[36].

Appendix

The aim of this Appendix is to derive some criteria that enable the optimal choice of the input parameters required for the solution of problem (7)-(8). In fact, by referring to 1-D arrays and denoting the average and minimum spacings between adjacent elements as d_{av} and $d_{\min}$, a number of simple rules, related to mutual coupling effects as well as to FBW and SLL performances, can be framed as follows.

As a first rule, to counteract the presence of ‘pseudo-grating’ [35] lobes in the visible region, which may entail compromise in fulfillment of constraints (8.b), d_{av} should be lower than the *minimum* operating wavelength. This condition, in turn, is equivalent to require that:

$$f_{\max} < \frac{c}{d_{av}} \tag{17.a}$$

Moreover, to keep mutual coupling effects under control, $d_{\min}$ should be larger than the *maximum* operating wavelength multiplied by the factor 0.4 [4]. This entails that:

$$f_{\min} > 0.4\frac{c}{d_{\min}} \tag{17.b}$$

By exploiting the relationship between $f_{\min}$ and $f_{\max}$, i.e., $f_{\min}=f_{\max}-(K-1)\Delta f$, conditions (17.a) and (17.b) can be unified into the following rule:

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$$\left[\frac{0.4c}{d_{\min}}+(K-1)\Delta f\right]< f_{\max}<\frac{c}{d_{av}} \tag{18}$$

which, for an equispaced array with $d_{av}=d_{\min}=d$, can in turn be written as:

$$\left[0.4c+(K-1)d\Delta f\right]< f_{\max}d<c \tag{19}$$

As a second useful rule, it is worth noting that, by virtue of equation (1), achieving FBW≥τ requires that:

$$f_{\max}\geq f_{\min}\frac{2+\tau}{2-\tau} \tag{20}$$

which implies that the following conditions must be satisfied to realize wideband and ultra-wideband systems, respectively:

$$1.22 f_{\min}< f_{\max}<1.67 f_{\min} \quad \Leftrightarrow \quad \textit{wideband array} \tag{21.a}$$

$$f_{\max}\geq 1.67 f_{\min} \quad \Leftrightarrow \quad \textit{ultrawideband array} \tag{21.b}$$

Notably, by exploiting again the relationship between $f_{\min}$ and $f_{\max}$, the results of constraints (21.a) and (21.b) are equivalent to, respectively:

$$2.5<\frac{f_{\max}}{(K-1)\Delta f}<5.5 \quad \Leftrightarrow \quad \textit{wideband array} \tag{22.a}$$

$$\frac{f_{\max}}{(K-1)\Delta f}\leq 2.5 \quad \Leftrightarrow \quad \textit{ultrawideband array} \tag{22.b}$$

Setting the input parameters in such a way that they accommodate (19) and (22) the same time, and that $f_{\max}$ is not so high as to allow sidelobes enter the main-beam region $\theta\epsilon[0,180°]\backslash\Omega$, will enable the proposed synthesis strategy work in optimal conditions.

References

[1] C. A. Balanis, *Antenna Theory, Analysis, and Design*, §2 “Fundamental parameters of Antennas,” pp. 30-70, John Wiley & Sons, 2005.

[2] M. Capek, L. Jelinek, and P. Hazdra, “On the functional relation between quality factor and fractional bandwidth,” *IEEE Transactions on Antennas and Propagation*, vol. 63, n. 6, pp. 2787-2790, 2015.

[3] B. Allen, T. Brown, K. Schwieger, E. Zimmermann, W. Q. Malik, D. J. Edwards, L. Ouvry, and I. Oppermann, “Ultrawideband: applications, technology and future perspectives,” *Proceedings of the International Workshop on convergent technologies (IWCT)*, 6-10 June 2005, Oulu, Finland.

[4] P. J. Bevelacqua and C. A. Balanis, “Geometry and weight optimization for minimizing sidelobes in wideband planar arrays,” *IEEE Transactions on Antennas and Propagation*, vol. 57, n. 4, pp. 1285-1289, 2009.

[5] M. Ettorre, W. A. Alomar, and A. Grbic, “Radiative wireless power-transfer system using wideband, wide-angle slot arrays,” *IEEE Transactions on Antennas and Propagation*, vol. 65, n. 6, pp. 2975-2982, 2017.

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https://www.tandfonline.com/doi/pdf/10.1080/09205071.2020.1724832?needAccess=true
DOI: 10.1080/09205071.2020.1724832

[6] P. Rocca, R. J. Mailloux, and G.Toso, "GA-based optimization of irregular subarray layouts for wideband phased arrays design," *IEEE Antennas and Wireless Propagation Letters*, vol. 14, pp. 131-134, 2015.

[7] S. G. Zhou, Z. H. Peng, G. L. Huang, J. Y. Li, and C. Y. D. Sim, "Design of wideband and dual polarized cavity antenna planar array," *IEEE Transactions on Antennas and Propagation*, vol. 64, n. 10, pp. 4565-4569, 2016.

[8] J. Wu, Y. J. Cheng, and Y. Fan, "Millimeter-wave wideband high-efficiency circularly polarized planar array antenna," *IEEE Transactions on Antennas and Propagation*, vol. 64, n. 2, pp. 535-542, 2016.

[9] Q. Liu, Z. N. Chen, Y. Liu, and C. Li, "Compact ultrawideband circularly polarized weakly coupled patch array antenna," *IEEE Transactions on Antennas and Propagation*, vol. 65, n. 4, pp. 2129-2134, 2017.

[10] P. Rocca and A. F. Morabito, "Optimal synthesis of reconfigurable planar arrays with simplified architectures for monopulse radar applications," *IEEE Transactions on Antennas and Propagation*, vol. 63, n. 3, pp. 1048-1058, 2015.

[11] O. M. Bucci, T. Isernia, and A. F. Morabito, "Optimal synthesis of circularly symmetric shaped beams," *IEEE Transactions on Antennas and Propagation*, vol. 62, n. 4, pp. 1954-1964, 2014.

[12] O. M. Bucci, T. Isernia, and A. F. Morabito, "Optimal synthesis of directivity constrained pencil beams by means of circularly symmetric aperture fields," *IEEE Antennas and Wireless Propagation Letters*, vol. 8, pp. 1386-1389, 2009.

[13] A. F. Morabito, A. R. Laganà, and T. Isernia, "Optimizing power transmission in given target areas in the presence of protection requirements," *IEEE Antennas and Wireless Propagation Letters*, vol. 14, pp. 44-47, 2015.

[14] D. A. M. Iero, T. Isernia, A. F. Morabito, I. Catapano, and L. Crocco, "Optimal constrained field focusing for hyperthermia cancer therapy: a feasibility assessment on realistic phantoms," *Progress in Electromagnetic Research*, vol. 102, pp. 125-141, 2010.

[15] O. M. Bucci, L. Caccavale, and T. Isernia, "Optimal far-field focusing of uniformly spaced arrays subject to arbitrary upper bounds in nontarget directions," *IEEE Transactions on Antennas and Propagation*, vol. 50, n. 11, pp. 1539-1554, 2002.

[16] A. F. Morabito, "Synthesis of maximum-efficiency beam arrays via convex programming and compressive sensing," *IEEE Antennas and Wireless Propagation Letters*, vol. 16, pp. 2404-2407, 2017.

[17] T. Isernia and G. Panariello, "Optimal focusing of scalar fields subject to arbitrary upper bounds," *Electronics Letters*, vol. 34, n. 2, pp.162-164, 1998.

[18] R. Vescovo, "Reconfigurability and beam scanning with phase-only control for antenna arrays," *IEEE Transactions on Antennas and Propagation*, vol. 56, n. 6, pp. 1555-1565, 2008.

[19] A. F. Morabito and P. Rocca, "Reducing the number of elements in phase-only reconfigurable arrays generating sum and difference patterns," *IEEE Antennas and Wireless Propagation Letters*, vol. 14, pp. 1338-1341, 2015.

This is the post-print version of the following article: L. T. P. Bui, N. Anselmi, T. Isernia, P. Rocca, and A. F. Morabito, "On bandwidth maximization of fixed-geometry arrays through convex programming," Journal of Electromagnetic Waves and Applications, vol. 34, n. 5, Pages 581-600, 2020.
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https://www.tandfonline.com/doi/pdf/10.1080/09205071.2020.1724832?needAccess=true
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[20] M. Alvarez Folgueiras, J. A. Rodriguez González, and F. Ares Pena, "Optimal compromise among sum and difference patterns in monopulse antennas: use of subarrays and distributions with common aperture tail," *Journal of Electromagnetic Waves and Applications*, vol. 23, n. 17-18, pp. 2301-2311, 2009.

[21] J. A. Rodriguez, A. Trastoy, J. C. Bregains, F. Ares, and G. Franceschetti, "Beam reconfiguration of linear arrays using parasitic elements," *Electronics Letters*, vol. 42, no. 3, pp. 131-133, 2006.

[22] A. F. Morabito, A. Massa, P. Rocca, and T. Isernia, "An effective approach to the synthesis of phase-only reconfigurable linear arrays," *IEEE Transactions on Antennas and Propagation*, vol. 60, n. 8, pp. 3622-3631, 2012.

[23] D. F. Kelley and W. L. Stutzman, "Array antenna pattern modeling methods that include mutual coupling effects," *IEEE Transactions on Antennas and Propagation*, vol. 41, n. 12, pp. 1625–1632, 1993.

[24] O. M. Bucci, C. Gennarelli, and C. Savarese, "Representation of electromagnetic fields over arbitrary surfaces by a finite and nonredundant number of samples," *IEEE Transactions on Antennas and Propagation*, vol. 46, n. 3, pp. 351-359, 1998.

[25] T. Isernia, O. M. Bucci, and N. Fiorentino, "Shaped beam antenna synthesis problem: feasibility criteria and new strategies," *Journal of Electromagnetic Waves and Applications.*, vol. 12, pp. 103-137, 1998.

[26] T. Isernia and A. F. Morabito, "Mask-constrained power synthesis of linear arrays with even excitations," *IEEE Transactions on Antennas and Propagation*, vol. 64, n. 7, pp. 3212-3217, 2016.

[27] A. F. Morabito, L. Di Donato, and T. Isernia, "Orbital angular momentum antennas: understanding actual possibilities through the aperture antennas theory," *IEEE Antennas and Propagation Magazine*, vol. 60, n. 2, pp. 59-67, 2018.

[28] A. F. Morabito, A. Di Carlo, L. Di Donato, T. Isernia, and G. Sorbello, "Extending spectral factorization to array pattern synthesis including sparseness, mutual coupling, and mounting-platform effects," *IEEE Transactions on Antennas and Propagation*, vol. 67, n. 7, pp. 4548-4559, 2019.

[29] G. Bellizzi and O. M. Bucci, "On the optimal synthesis of sum or difference patterns of centrosymmetric arrays under arbitrary sideLobe constraints," *IEEE Transactions on Antennas and Propagation*, vol. 66, n. 9, pp. 4620-4626, 2018.

[30] A. F. Morabito, R. Palmeri, V. A. Morabito, A. R. Laganà, and T. Isernia, "Single-surface phaseless characterization of antennas via hierarchically ordered optimizations," *IEEE Transactions on Antennas and Propagation*, vol. 67, n. 1, pp. 461-474, 2019.

[31] M. C. Tang, T. Shi, and R. W. Ziolkowski, "Planar ultrawideband antennas with improved realized gain performance," *IEEE Transactions on Antennas and Propagation*, vol. 64, n. 1, pp. 61-69, 2016.

[32] J. T. Logan, R. W. Kindt, M. Y. Lee, and M. N. Vouvakis, "A new class of planar ultrawideband modular antenna arrays with improved bandwidth," *IEEE Transactions on Antennas and Propagation*, vol. 66, n. 2, pp. 692-701, 2018.

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DOI: 10.1080/09205071.2020.1724832

[33] M. Serhir and D. Lesselier, “Wideband reflector-backed folded bowtie antenna for ground penetrating radar,” *IEEE Transactions on Antennas and Propagation*, vol. 66, n. 3, pp. 1056-1063, 2018.

[34] T. Goel and A. Patnaik, “Novel broadband antennas for future mobile communications,” *IEEE Transactions on Antennas and Propagation*, vol. 66, n. 5, pp. 2299-2308, 2018.

[35] O. M. Bucci, T. Isernia, A. F. Morabito, S. Perna, and D. Pinchera, “Aperiodic arrays for space applications: An effective strategy for the overall design,” in *Proc. 3rd Eur. Conf. Antennas Propag. (EuCAP)*, Berlin, Germany, Mar. 2009, pp. 2031–2035, Art. no. 5068016.

[36] A. F. Morabito, “Power synthesis of mask-constrained shaped beams through maximally-sparse planar arrays,” *Telkomnika (Telecommunication Computing Electronics and Control)*, vol. 14, n. 4, pp. 1217-1219, 2016.

This is the post-print version of the following article: L. T. P. Bui, N. Anselmi, T. Isernia, P. Rocca, and A. F. Morabito, "On bandwidth maximization of fixed-geometry arrays through convex programming," Journal of Electromagnetic Waves and Applications, vol. 34, n. 5, Pages 581-600, 2020.
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