A Compressive-Sensing Inspired Procedure for Array Antennas Diagnostics by a Small Number of Phaseless Measurements

A. F. Morabito, R. Palmeri, and T. Isernia

***Abstract*—A new approach to array antennas diagnostics by means of amplitude-only far-field measurements is proposed. The procedure takes advantage from the Compressive Sensing theory and is able to deal with linear, planar, and conformal fixed-geometry arrays. In case of low faulty-elements percentages, the overall problem is reduced to a convex programming optimization. The minimum number of measurements allowing success of recovery of either real or complex excitations is identified through an extensive set of numerical results. Interesting outcomes concerning the impact of the noise on measured data, the criterion adopted to choose the measurement points, and the 'trivial ambiguities' inherent to the solution of phase retrieval problems are also provided.**

***Index Terms*—Antenna arrays, compressed sensing, fault diagnosis, phase retrieval.**

I. Introduction and Motivations

Large efforts have been devoted in the literature to the problem of characterizing a radiating source by using measurement set-ups as simple as possible, and to reduce the measurement time (see for instance [1],[2] and references therein). In particular, one can individuate at least three different main areas of interesting research.

First, a large body of work has been devoted to the problem of performing the diagnostics of an antenna (or near-field to far-field transformations) by using the least possible number of measurements. A relevant contribution on such a topic has been provided by Prof. Bucci and co-workers in [1]. Basing on such a cornerstone result, a number of possible configurations and solutions have been then proposed and developed by Gennarelli, Savarese and co-workers (see [2] and references therein).

A second very interesting area of research has also arisen in the last three years as a consequence of the recently introduced Compressive Sensing (CS) theory [3]. In fact, this latter allows the on-off diagnostics of an array antenna by using a number of measurements (say M) much lower than the number of array elements (say N) [4]-[6]. Of course, because of the underlying CS theory, M is supposed to anyway be larger or even much larger than the number of 'off' (i.e., faulty) elements (say S). Contributions on such a topic consider both cases of near-field [4] and far-field [5],[6] *amplitude and phase* measurements.

By following a different goal, phaseless measurements have also been the subject of intensive research (see for instance [7]-[15]). In fact, measuring a *complex* far-field pattern requires a stable phase reference and accurate positioning of the probe in each measurement point, so that measurements may become problematic [11],[12].

As a matter of fact, phaseless measurements have been proposed as an effective alternative to amplitude and phase measurements in applications ranging from NF-FF transformations and antenna metrology [7]-[10] to array excitations retrieval [12] and on-site diagnostics of reflectors for radio astronomy [15]. Very recent contributions on the topic, witnessing its growing interest in submillimeter and terahertz bands, also include [16]-[18]. Finally, the possibility to perform measurements 'on site' by means of properly-equipped Unmanned Aerial Vehicles (UAV) [19] also adds interest to phaseless measurements, which require less accuracy in the probes positioning.

The specific problem of identifying faulty elements in an array by far-field phaseless measurements has also been considered [13],[14]. In particular, the readers are deferred to [13] as far as the problem of the uniqueness of a solution (and the avoidance of 'false solutions') is concerned. Notably, all possible independent intensity measurements are used in [13], while the exploitation of a number of measurements less or much less than the number of array elements was first suggested and explored in [14], which dates to prior of the diffusion of the CS paradigm. The latter possibility is particularly interesting. In fact, besides the obvious savings in terms of measurement time and of cost and complexity of the set-up, one has the possibility to perform measurements on site (and eventually during normal operations).

For all the above, in view of the expected advantages in terms of performances and understanding, possible exploitation of CS for *phaseless* antenna diagnostics is of interest. On the other side, CS theory and procedures have been developed and well assessed for the case of *linear* problems, which is not the case at hand.

By using simple arguments, we are able to show in the following that array antenna diagnostics by using a small number of far-field phaseless measurements can be effectively performed by means of CS-inspired techniques. As a consequence, we also introduce a solution procedure which reduces the overall problem to a Convex Programming (CP) one in case of low faulty-elements percentages. The procedure is able to deal with linear, planar, and conformal fixed-geometry arrays. It is worth to note that, excluding a small set of cases wherein the nominal excitations distribution exhibits particular properties (see below), the approach is insensitive to the 'trivial ambiguities' [20] inherent to the solution of phase retrieval problems. In both cases of real and complex array excitations, the *minimum* number of measurements allowing a successful diagnostics is identified for different levels of noise on the data. Criteria for choosing measurement points are also provided.

The paper is organized as follows. Section II is devoted to introduce the rationale of the proposed approach and to show how the non-linear problem at hand can still be solved by using the same simple CS tools which are used in case of linear problems. Then, in Section III, the performance of the

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proposed technique is analyzed and discussed in the case of linear equispaced arrays. Conclusions follow.

II. The Proposed Approach

Let us suppose to know the 'nominal' far field expected to be radiated by an Array Under Test (AUT) of sources when all elements are correctly working. In particular, let us suppose amplitude and phase of the nominal array factor are known. By the sake of simplicity, let us also consider the case where the array is linear and all elements are identical, equally oriented, and uniformly spaced (with a constant distance equal to d), so that the problem can be conveniently analyzed in terms of an array factor[1]. Also, let us denote by N and θ the overall number of array elements and the observation angles with respect to the array axis, respectively. Then, the array factor associated to the 'nominal' or 'expected' field is formally given by

$$F^E(u)=\sum_{n=1}^{N} a_n^E e^{jnu} \qquad (1)$$

where $a_1^E,...,a_N^E$ are the known 'nominal' or 'expected' complex excitations and $u=\beta d\cos\theta$ (denoting by $\beta=2\pi/\lambda$ the wavenumber).

In case some element is not correctly working, the actual array factor will be given by

$$F^A(u)=\sum_{n=1}^{N} a_n^A e^{jnu} \qquad (2)$$

where $a_1^A,...,a_N^A$ are the 'actual' excitations. As a consequence, the signal given by the difference between the 'nominal' and the 'actual' array factors, say ΔF, will be given by

$$\Delta F(u)=F^E(u)-F^A(u)=\sum_{n=1}^{N} \Delta a_n e^{jnu} \qquad (3)$$

with $\Delta a_n = a_n^E - a_n^A$ for n=1,…,N.

If the number of faulty elements, i.e., the number of elements whose excitation is different from the nominal one, say S, is small with respect to N, ΔF is intrinsically S-*sparse* [6] when represented in terms of $\Delta a_1,\ldots,\Delta a_N$. Such a circumstance has been already considered and exploited in a number of papers recently appeared in these Transactions performing the array diagnostics by means of amplitude and phase measurements [5],[6]. While reducing the complexity and cost of measurements, these techniques still require an accurate phase measurement and hence a stable phase reference and an accuracy in positioning the probes in the order of a small fraction of the wavelength. Hence, in order to provide a further simplification, it makes sense to consider the case where only *phaseless* measurements (which are less sensitive to positioning errors, and do not require a phase reference) are available. Then, let us denote by P^E and P^A the 'nominal' and 'actual' power patterns, i.e., the square amplitudes of F^E and of F^A, respectively. By straightforward algebra, in a generic measurement point u_m one achieves:

$$P^A(u_m)-P^E(u_m)=-2\mathrm{Re}\left[F^E(u_m)\Delta F^*(u_m)\right]+\left|\Delta F(u_m)\right|^2 \qquad (4)$$

wherein * denotes complex conjugation.

[1] As it will be easily understood, the overall approach can be easily applied to any kind of linear, planar or even conformal arrays.

As ΔF is *sparse*, by using M different sampling points $u_1,\ldots,u_M$, the CS theory and results can be of interest. On the other side, (4) is a non-linear relation, so that standard CS theory and procedures cannot be applied in a straightforward fashion. In fact, while the first term at the right-hand member is linear with respect to ΔF, the second one is indeed quadratic.

However, a useful circumstance comes into help. In fact, as long as one has a low number of faulty elements (which is anyway needed for applying CS) the energy of the signal ΔF is small with respect to the energy of F^E, so that, in the average, the second term at the right-hand member of (4) will be small with respect to the first one. Moreover, one could have a further lowering of the weight of the quadratic term in (4) by performing the M measurements in those points where the signal F^E (which is known) has a large intensity. By so doing, equation (4), particularized in a proper series of measurement points, can be considered to be 'almost linear' in terms of the unknown signal ΔF, so that the procedures suggested by CS literature for linear problems are indeed of interest.

By taking advantage from all the above, the diagnostics problem from phaseless measurements can be formulated as:

$$\min_{\Delta a_1,\ldots,\Delta a_N} \left\|\underline{\Delta a}\right\|_1 \qquad (5)$$

subject to:

$$\left\|\underline{\Delta x}\right\|_2 \le \varepsilon \qquad (6)$$

being $\underline{\Delta a}=[\Delta a_1 \ldots \Delta a_N]$ and $\underline{\Delta x}=[\Delta x_1 \ldots \Delta x_M]$, with

$$\Delta x_m = \frac{P^A(u_m)-P^E(u_m)+2\,\mathrm{Re}\left[F^E(u_m)\Delta F^*(u_m)\right]-\left|\Delta F(u_m)\right|^2}{|F^E(u_m)|} \qquad \text{for } m=1,\ldots,M \qquad (7)$$

wherein the subscripts 1 and 2 respectively stand for ℓ_1 and ℓ_2 norms. As discussed in a number of papers (see for instance [5],[6]), minimization of functional (5) is meant to enforce sparsity, while condition (6) ensures fulfillment of the (normalized) data equation (4) within a given tolerance ε, and the reason of the adopted normalization by $|F^E(u_m)|$ is given in below and in Appendix I. As well known, an accurate recovery is granted for the corresponding linear problems provided a number of conditions on M, S, N (as well as on the location of measurement points are fulfilled).

Success of the proposed procedure is related to the properties of the matrix underlying the linear term of (4). In particular, some conditions guaranteeing the so called Restricted Isometry Property (RIP) [3] or a low mutual coherence between the sensing and representation basis [21] should be verified. It is interesting to note (see Appendix I) that in case of pencil beams (where the nominal field is real [22]) the matrix underlying the linear term of (4) has exactly the same components as in the Discrete Cosine Transform (DCT) and the Discrete Sine Transform (DST), so that a small mutual coherence is indeed guaranteed [21]. Hence, the proposed framework is expected to work fine in case of pencil beams or real-field patterns. As the additional factors which come into play do not seem to modify the coherence properties in a significant fashion, the approach is however expected to succeed even in case of complex field patterns, which is confirmed in Section III by a large number of numerical examples.

As far as the requirements on the ratio M/S are concerned, it is well known [5] that at least M=2S *complex* measurements are required to identify S faulty elements. As one complex measurement is equivalent to two real measurements, it is expected that at least M=4S *phaseless* measurements are required to perform the exact recovery of an S-sparse signal. As discussed in [23], this is indeed the case. At the same time, the location of measurement points should obey a number of rules. First, one should avoid to deal with very close measurement points, as very close samples would carry the same information, i.e., would give rise to equations almost linearly dependent each from the other [1]. Second, according to CS theory, measurements should be taken in some random fashion (in order to ensure some kind of RIP) [5]. Third, as already discussed above, measurements should correspond to points wherein P^E is large, in such a way that (4) is nearly linear in terms of the unknowns. Under such hypothesis, constraint (6) practically results quadratic (and convex) in terms of the unknowns and hence, being the cost functional (5) convex, the overall problem is a CP one. Although the second and third requirements may seem conflicting, in choosing the measurement points one can still perform random choices amongst all those locations where $|F^E|$ exceeds a given threshold, so that both requirements can be preserved.

As long as we want to deal with an 'on-off' diagnostics problem, which is the case considered in the remainder of the paper, an even more powerful formulation can be given. In fact, the unknown 'actual' excitation of each element can be written as the multiplication of the 'nominal' excitation by an unknown real coefficient, which will be equal to 0 or 1 in case of a fully defective or a fully working element, respectively. In such a way, one can take advantage from the expected range of values of these real coefficients. Then, by adopting as unknown of the problem the vector containing these real coefficients (say $\underline{\tau}=[\tau_1,\ldots,\tau_N]$) instead of $\underline{\Delta a}$, one can give a second formulation of the problem as follows (wherein $\underline{I}$ is a vector of N ones):

$$\min_{\tau_1,\ldots\tau_N} \left\| \underline{I} - \underline{\tau} \right\|_1 \tag{8}$$

subject to:

$$\left\| \underline{\Delta x} \right\|_2 \leq \varepsilon \tag{9}$$

$$a_n^A = \tau_n a_n^E \text{ for } n=1,\ldots,N \tag{10}$$

$$0 \leq \tau_n \leq 1 \text{ for } n=1,\ldots,N \tag{11}$$

In fact, by virtue of (10) it will result $\Delta a_n = a_n^E (1-\tau_n)$ for n=1,…,N, and hence if $\underline{\Delta a}$ is sparse then ($\underline{I}$-$\underline{\tau}$) is sparse as well, so that CS can still be profitably applied. Moreover, constraints (11) will speed up the overall procedure and improve robustness by restricting the search space to the set wherein the optimal (admissible) solutions are certainly located. Notably, constraints (10),(11) are linear in terms of the unknowns, so that their consideration does not affect the effectiveness of the solution procedure. Last, but not least, formulation (8)-(11) allows to deal with half the number of unknowns with respect to problem (5)-(7) (wherein one looks for N complex unknowns, which are equivalent to 2N real quantities). This leads to an at-least-halved computational burden (since CP optimizations can be performed in polynomial time) as well as to better performances.

As discussed in the Appendix 2, the approach is also insensitive to the 'trivial ambiguities' [20] usually involved in the solution of phase retrieval problems (but for very peculiar cases, wherein the procedure is anyway able to identify a binary alternative for the solution).

III. NUMERICAL ASSESSMENT

Very many numerical simulations have been performed in order to identify, in different scenarios of actual interest, the minimum number of phaseless measurements required for the successful identification of faulty elements. In particular, three different sets of hypotheses concerning the measurement set-up have been considered. By the sake of clarity, in the following a separate Subsection is devoted to each of them.

For each AUT, we tested the performance of the approach by repeatedly varying M and S and evaluating (as in [5]) the Mean Squared Error (MSE) defined by :

$$MSE = \frac{1}{N}\sum_{n=1}^{N} \left| a_n^A - a_n^R \right|^2 \tag{12}$$

(where a_n^R is the estimated value of the n-th excitation). In particular, for each pair (M, S), we solved problem (8)-(11) 100 times and considered 'successful' only those simulations achieving a MSE lower than a given threshold MSE_{MAX}. Moreover, for each value of S, we also analyzed the Rate of Success of Excitations Recovery (RSER) [5], i.e., the number of successful diagnostics experiments over the overall number of simulations, as a function of the ratio M/S. The S failures have been represented by zero-amplitude excitations randomly selected with uniform distribution among the N coefficients. All results concern linear equispaced arrays with a λ/2 inter-element spacing. Each simulated retrieval required in the average 2.0 seconds to be performed by a calculator having an Intel Core i7-3537U 2.50 GHz CPU and a 10 GB RAM.

Each simulation has been performed by separately adopting formulation (5)-(7) and formulation (8)-(11). By the sake of brevity, in the following we report just the outcomes achieved through the second of these two alternatives, which has led to slightly better performances both in terms of RSER and computational time.

III.1 Randomly-selected measurement points in the absence of noise on the data

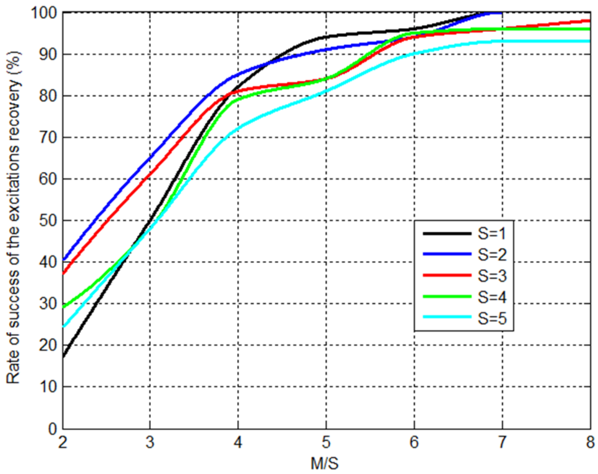
In this Subsection we analyze performances in case of noiseless data when selecting the measurement points by means of a random (uniformly-distributed) sampling of u variable[2] and choosing MSE_{MAX}=-30dB. In a first class of numerical experiments, we used an AUT composed by N=20 elements and chose the nominal excitations as the Chebyshev coefficients providing a -20 dB equiripple far-field pattern[3]. In

[2] According to the reasonings made in Section II about linearization of relation (4), only those u values corresponding to a nominal power pattern's normalized value larger than -25 dB have been randomly sampled.

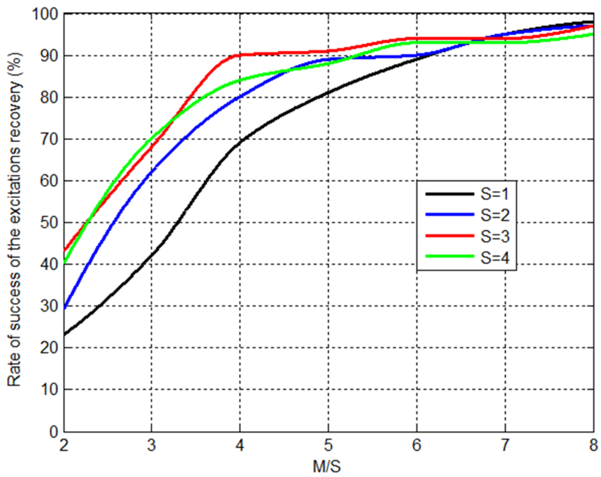
[3] For each simulation involving Chebyshev nominal excitations, we calculated the MSE as the minimum value achieved by substituting into (12) the excitations corresponding to the retrieved field and to its complex conjugate, respectively. This has been necessary to counteract the potential occurrence of the only possible trivial ambiguity (see Appendix 2).

a second class of simulations, in order to validate the approach also in case of complex reference excitations, we considered an AUT composed by N=16 elements and used as nominal excitations the coefficients reported at page 1131 of [24] (given by Elliott to generate a cosecant power pattern).

Results pertaining to the two groups of simulations are summarized in Fig. 1. As it can be seen, for a percentage of faulty elements S/N roughly between 5% and 25%, the minimum number of measurements allowing an 'almost sure' successful reconstruction (i.e., RSER>90%) is M=6S in both cases of real and complex nominal excitations. In any case, M=8S phaseless measurements have been sufficient to guarantee a RSER close to 100%.



(a)



(b)

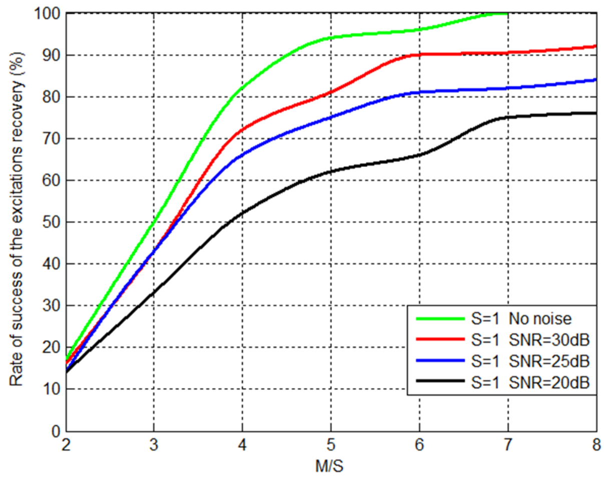
Fig. 1. Results achieved in case of noiseless data by selecting the measurement points as a random sampling of the u variable and using Chebyshev (a) and Elliott (b) nominal excitations. Intersections amongst curves are due to the fact that an increase of S induces, for a fixed M/S value, a growth of both M (which has a positive effect on the RSER) and $\|\underline{\Delta F}\|_2$ (which has a negative effect on the RSER).

It is also worth noting that, whatever the number of failures, the RSER (expected) growth for increasing values of M is very steep until M approaches 4S, and then grows smoother. Such a circumstance agrees with the theoretical results in [23], and it is very similar to the one shown in [5].

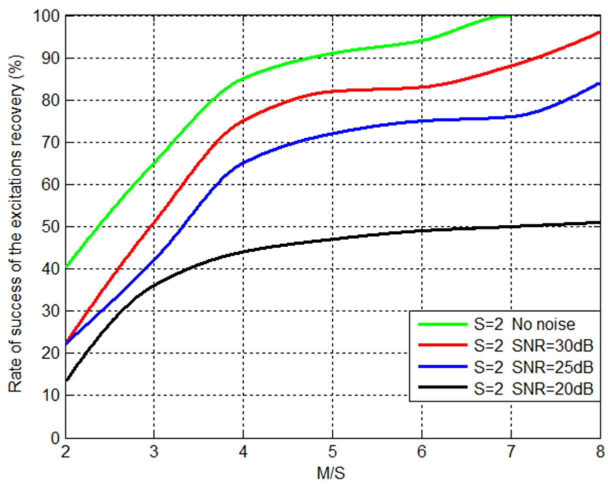
As expected, all curves are monotonic in terms of M/S, Also, for large values of M/S, performances become worse for increasing S. Such a circumstance can be attributed to the fact that increasing S means increasing the non-linear term of (4), thus departing from the optimal conditions for use of CS tools.

III.2 Randomly-selected measurement points in the presence of noise on the data

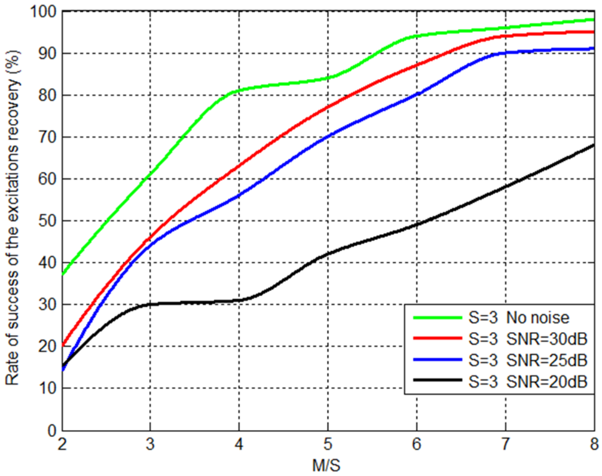
We repeated the simulations carried out in the previous Subsection (just for the Chebyshev excitations set) by corrupting with different levels of noise the 'measured' value of $P^A(u_m)$, for m=1,…M. In particular, we enforced a Signal to Noise Ratio (SNR) respectively equal to 30 dB, 25 dB, and 20 dB. A MSE_{MAX} value equal to -30 dB, -25 dB, and -20 dB has been respectively considered in these three cases. Fig. 2 reports the results achieved by simulating S=1, 2, 3 failures. As it can be seen, in all three cases the RSER exhibits a behavior similar to the one shown in Fig. 1. The figure shows that the approach has a good tolerance to noise, being able to guarantee RSER≈95% in many instances (see curves with M=8S, SNR=30dB, and S equal to 2 or 3).



(a)



(b)



(c)

Fig. 2. Chebyshev nominal excitations and random sampling of the u variable: results achieved in case of measured data corrupted by different levels of noise for S=1 (a), S=2 (b), and S=3 (c).

III.3 An assessment of performance in a realistic scenario

We repeated the simulations carried out in Subsection III.1 (just for the Chebyshev excitations set and S=1, 2) by a-priori excluding the whole main-beam region (i.e., $|u|\leq 0.35$) from the domain which is randomly sampled in order to determine the measurement points. The aim of these experiments is to test the proposed approach for a 'on site' diagnostics while the array is anyway performing its job. Such a case is of interest in very many cases, including radar arrays [25]. This new measurement strategy obviously makes harder the overall diagnosis problem, as it does not include high-amplitude field samples able to lower the weight of the quadratic term in (4) and hence to raise the applicability of the CS theory.

The achieved results are summarized in Fig. 4. As it can be seen, despite the harder working conditions, for S=1 the approach guarantees performances identical to the ones shown in Subsection III.1. The diagnostics technique keeps performing very well also for S=2, guaranteeing a RSER≥90% for $M/S\geq 5$.

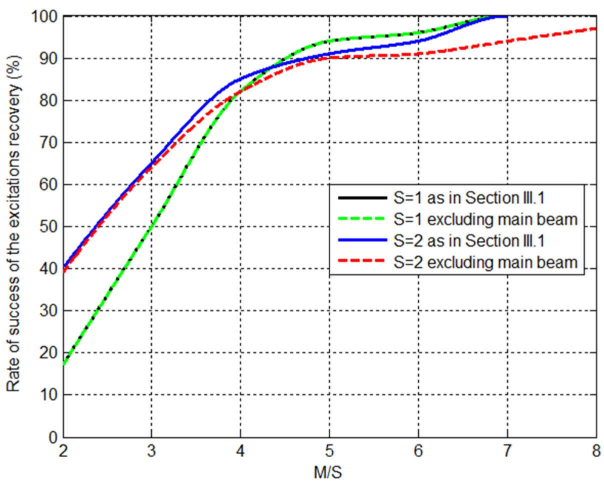


Fig. 4. Comparison of the RSER values achieved by including (continuous lines) and excluding (dashed lines) the main-beam region from the domain which is randomly sampled in order to determine the measurement points.

IV. Conclusions

A new approach to the on-off diagnostics of linear, planar, and conformal fixed-geometry array antennas by means of a small number of phaseless far-field measurements has been presented. Despite dealing with a non-linear problem, the central engine of the proposed technique has been devised in such a way to exploit at best the Compressive Sensing theory, with consequent advantages in terms of both accuracy and computational burden. In fact, under reasonable conditions the overall problem still reduces to a Convex Programming one.

The approach has shown good reconstruction capabilities even in the presence of noise on far-field data, and the performed tests suggest that on site diagnostics while the array is performing its usual job is also possible.

Appendix I

The aim of this Appendix is to show that the matrix relating the unknown $\underline{\Delta a}$ to the first term at the right-hand member of (4) is in the form of a DCT minus a DST as long as the nominal field is real. To this end, let us first note that:

$$2\,\mathrm{Re}\left[F^E(u_m)\Delta F^*(u_m)\right]= \\ = F^E(u_m)\Delta F^*(u_m)+F^{E*}(u_m)\Delta F(u_m) \tag{13}$$

Then, by decomposing the n-th unknown in term of its real and imaginary parts, i.e., $\Delta a_n=\Delta r_n+j\Delta i_n$, for n=1,…,N, and substituting (3) into (13), one achieves:

$$2\,\mathrm{Re}\left[F^E(u_m)\Delta F^*(u_m)\right]= \\ =\sum_{n=1}^{N}\begin{Bmatrix} F^E(u_m)[\cos(nu_m)-j\sin(nu_m)]\Delta r_n + \\ -jF^E(u_m)[\cos(nu_m)-j\sin(nu_m)]\Delta i_n + \\ +F^{E*}(u_m)[\cos(nu_m)+j\sin(nu_m)]\Delta r_n + \\ +jF^{E*}(u_m)[\cos(nu_m)+j\sin(nu_m)]\Delta i_n \end{Bmatrix} \tag{14}$$

Therefore, since

$$\frac{\mathrm{Re}\left[F^E(u_m)\right]}{\left|F^E(u_m)\right|}=\cos\alpha_m \qquad \frac{\mathrm{Im}\left[F^E(u_m)\right]}{\left|F^E(u_m)\right|}=\sin\alpha_m \tag{15}$$

wherein α_m is the phase of $F^E(u_m)$, (14) entails that:

$$\frac{\mathrm{Re}\left[F^E(u_m)\Delta F^*(u_m)\right]}{\left|F^E(u_m)\right|}=\sum_{n=1}^{N}\left[\cos(\varphi_{m,n})\Delta r_n-\sin(\varphi_{m,n})\Delta i_n\right] \tag{16}$$

with $\varphi_{m,n}=nu_m-\alpha_m$. By virtue of (16), the relation between the unknowns and the linear term in (7) can be written as:

$$\underline{y}=\underline{\underline{\Psi}}_1\underline{\Delta r}-\underline{\underline{\Psi}}_2\underline{\Delta i} \tag{17}$$

where:

$$\underline{y}=\begin{bmatrix}\dfrac{\mathrm{Re}\left[F^E(u_1)\Delta F^*(u_1)\right]}{\left|F^E(u_1)\right|}\\ \vdots \\ \dfrac{\mathrm{Re}\left[F^E(u_M)\Delta F^*(u_M)\right]}{\left|F^E(u_M)\right|}\end{bmatrix} \qquad \underline{\Delta r}=\begin{bmatrix}\Delta r_1\\ \vdots\\ \Delta r_N\end{bmatrix} \qquad \underline{\Delta i}=\begin{bmatrix}\Delta i_1\\ \vdots\\ \Delta i_N\end{bmatrix} \tag{18}$$

$$\underline{\underline{\Psi}}_1 = \begin{bmatrix} \cos\varphi_{1,1} & ... & \cos\varphi_{1,N} \\ \vdots & ... & \vdots \\ \cos\varphi_{M,1} & ... & \cos\varphi_{M,N} \end{bmatrix} \quad \underline{\underline{\Psi}}_2 = \begin{bmatrix} \sin\varphi_{1,1} & ... & \sin\varphi_{1,N} \\ \vdots & ... & \vdots \\ \sin\varphi_{M,1} & ... & \sin\varphi_{M,N} \end{bmatrix}$$

Therefore, as long as the nominal field is real, i.e., α_m is either equal to 0 or π, so that matrices in (18) respectively are a DCT and a DST matrix [26] (but for signs, which do not modify coherence properties), Q.E.D.

APPENDIX II

By the sake of simplicity, let us refer to the case of linear or planar sources. By leaving aside the obvious ambiguity deriving from a phase constant, retrieving a component of the far field of a source having a limited extension by knowing just its squared amplitude distribution is known to be subject to 'trivial ambiguities' [20] as follows:

i. two different fields being the complex conjugate each of the other have identical square amplitude distributions. In the case at hand, such an ambiguity corresponds to sources which are reversed (with respect to their axes) and complex conjugate each of the other;
ii. two different fields differing by a linear phase term have identical squared amplitude distributions. In the case at hand, such an ambiguity corresponds to sources which are shifted with respect to the actual ones;
iii. any combination of the two ambiguities above may occur.

Because of the fact that the proposed approach looks for 'maximally-sparse' [6] solutions, it can be easily shown that trivial ambiguities cannot occur (but for very particular cases) for the present diagnostics technique. In fact, as far as the first possible ambiguity is concerned, let us consider an AUT whose actual far field is $F^{A1}(u)$, which yields:

$$\Delta F^{A1}(u) = F^E(u) - F^{A1}(u) \qquad (19)$$

and where ΔF^{A1} is a sparse signal. The possible ambiguous field solution, i.e., $F^{A2}=(F^{A1})^*$, would give rise to a perturbation ΔF^{A2} given by:

$$\Delta F^{A2}(u) = F^E(u) - F^{A1*}(u) \qquad (20)$$

Then, by straightforward manipulation:

$$\Delta F^{A2}(u) = \Delta F^{A1*}(u) + 2j\,\mathrm{Im}\{F^E(u)\} \qquad (21)$$

As a consequence, unless the second term at the right-hand member of (21) is null, ΔF^{A2} is not a sparse signal, so that the proposed CS technique, which looks for maximally-sparse solutions, will allow to avoid such a kind of ambiguity. Unfortunately, this trivial ambiguity can still occur as long as the nominal field is real, i.e., in case of hermitian excitations. However, in case the ambiguity occurs the technique is still able to identify a binary alternative for the solution (see Subsection III.1), so that it keeps useful in any case.

As far as the second ambiguity is concerned, the requirement for sparsity comes again into play. In fact, such a kind of ambiguity corresponds to a shift of the source. As a consequence, unless the 'nominal' excitations distribution is constant and the AUT has a single faulty element either in the first or last location on its axis, a translation of the source will induce a different amount of sparsity on the final solution. Therefore, the adopted CS framework will be able to distinguish the actual solution of the problem from the one associated to the ambiguity.

Obviously, the same kind of arguments also apply to the (composite) third possible kind of trivial ambiguity.

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