

## Investigation of fractal-fractional order model of COVID-19 in Pakistan under Atangana-Baleanu Caputo (ABC) derivative

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### ABSTRACT

This manuscript addressing the dynamics of fractal-fractional type modified SEIR model under Atangana-Baleanu Caputo (ABC) derivative of fractional order  $\nu$  and fractal dimension  $\mu$  for the available data in Pakistan. The proposed model has been investigated for qualitative analysis by applying the theory of non-linear functional analysis along with fixed point theory. The fractional Adams–Bashforth iterative techniques have been applied for the numerical solution of the said model. The Ulam–Hyers (UH) stability techniques have been derived for the stability of the considered model. The simulation of all compartments has been drawn against the available data of covid-19 in Pakistan. The whole study of this manuscript illustrates that control of the effective transmission rate is necessary for stopping the transmission of the outbreak. This means that everyone in the society must change their behavior towards self-protection by keeping most of the precautionary measures sufficient for controlling covid-19.

### Introduction

Recently, the world is meeting with an outbreak of COVID-19 which has been tested firstly from a large city Wuhan in China. Nearly more than fifteen hundred thousand people (1.5 million) have died from this outbreak and about forty-four hundred thousand people have got the infection [1]. Much more of the human population has been recovered from the said disease. The death and recovery ratio is different in different countries of the world depending on the health situation and care measures taken by countries. In Pakistan, about 11514 people have died from this disease and nearly up to now 0.54 million people have been infected of which 4.9 million were recovered [2]. Every country and its government are trying to make different policies for controlling and reducing this outbreak. One important factor behind the spread of this disease is the interaction of infected people with uninfected ones in

society. Such type of immigration and interaction will increase the rate of spreading this pandemic. Therefore, since the beginning of the said pandemic up to now various steps like banned on air traffic, immigration, non-quarantine and social gathering in different areas, have been taken. This is in addition to enforcing some precautionary measures to minimize the much more loss of people. Every country of the world is trying to reduce the unneeded mobility of humans to minimize the rate of infections [3].

Much of the human population are afraid of Covid-19 because such pandemic in the past has become the cause of death for more than a hundred thousand humans. For every terrible outbreak different scientists and researchers trying to implement some precautionary measures and to discover their vaccine from the very beginning which may be applied in the future. Knowing the basic reasons for the pandemic need statistical data and some concepts of mathematical modeling. Such a

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**Table 1**  
Description of the parameters.

Parameters	Description
$\Lambda^*$	Recruitment rate
$\lambda^*$	Force of infection
$\alpha^*$	Rate of dissemination
$e^*$	Behavior change function
$\mu^*$	Natural death rate
$\theta^*$	Rate of recovery
$\eta^*$	The average effectiveness of existing self-preventive measures
$\delta^*$	Rate of death from virus
$\gamma^*$	Recovery rate of the people
$\epsilon^*$	Rate of shedding from exposed to environment
$\xi^*$	Rate of shedding from exposed to environment
$\psi^*$	Rate of decay

**Table 2**  
Estimated numerical values of the parameters.

Parameters	Data-I	Data-II
$\Lambda$	0.080	0.80
$\alpha$	0.000761	0.000761
$\mu$	0.080	0.0080
$\lambda$	0.00073	0.00073
$\delta$	0.00039	0.00039
$e$	1	1
$\theta$	0.00064	0.00064
$\epsilon$	0.75	0.0075
$\eta$	0.3998	0.5998
$\xi$	0.0023	0.0023
$\psi$	0.07862	0.07862
$\gamma$	0.000326	0.000236

concept of the mathematical formulation was applied for the first time in 1927. By applying the statistical data on different diseases will lead us to the construction of mathematical formula. The said formula will be then be applied for modeling different real-world problems. So for, different outbreaks have been molded by a single mathematical formula as can be study in [4–7].

Up to now, different scholars have investigated the recent pandemic caused by coronavirus by some methods of analysis as in [8–18]. Some other researchers formulated the said pandemic by mathematical models which provide future knowledge about the recent disease. The history along with present information of the said pandemic can help the policymakers to construct some good strategies for controlling it. Because the importance of modeling the aforesaid disease has been discussed in various research articles [19]. Constructing mathematical models involves an assumption about some parameters like immigration is one of the most important causes of spreading the disease will be included as a term in constructing models for it.

The model from which we will take our model fractal-fractional sense is given as under and can be seen in [20]

$$\begin{cases} \dot{S}(t) = \Lambda^* - (\lambda^* + \alpha^* e^* + \mu^*)S, \\ \dot{E}(t) = \alpha^* e^* S - (\theta^* + \mu^* + (1 - \eta^*)\lambda^*)E, \\ \dot{I}(t) = \lambda^* S + (1 - \eta^*)\lambda^* E - (\mu^* + \delta^* + \gamma^*)I, \\ \dot{R}(t) = \theta^* E + \gamma^* I - \mu^* R, \\ \dot{D}(t) = \delta^* I - \mu^* D, \\ \dot{M}(t) = \epsilon^* E + \xi^* I - \psi^* M, \end{cases} \quad (1)$$

where the parameter used in the above model with whole details are given below as: We will reconsider the model given in (1) by including arbitrary order derivative  $0 < \nu \leq 1$  and fractal dimension  $0 < \mu \leq 1$  as

given as (see Table 1 and 2)

$$\begin{cases} {}^{ABC}D_t^{\nu, \mu} S(t) = \Lambda^* - (\lambda^* + \alpha^* e^* + \mu^*)S, \\ {}^{ABC}D_t^{\nu, \mu} E(t) = \alpha^* e^* S - (\theta^* + \mu^* + (1 - \eta^*)\lambda^*)E, \\ {}^{ABC}D_t^{\nu, \mu} I(t) = \lambda^* S + (1 - \eta^*)\lambda^* E - (\mu^* + \delta^* + \gamma^*)I, \\ {}^{ABC}D_t^{\nu, \mu} R(t) = \theta^* E + \gamma^* I - \mu^* R, \\ {}^{ABC}D_t^{\nu, \mu} D(t) = \delta^* I - \mu^* D, \\ {}^{ABC}D_t^{\nu, \mu} M(t) = \epsilon^* E + \xi^* I - \psi^* M, \\ S(0) = S_0 \geq 0, \quad E(0) = E_0 \geq 0, \quad I(0) = I_0 \geq 0, \quad R(0) = R_0 \geq 0, \\ D(0) = D_0 \geq 0, \quad M(0) = M_0 \geq 0, \quad 0 < \nu, \mu \leq 1. \end{cases} \quad (2)$$

We have to analyze the Eq. (2) by using the derivative of fractal-fractional order in sense of ABC because it gives a much realistic result having a greater degree of freedom than integer-order. We consider the (2) having fractional-order  $\nu$  and fractal dimension  $\mu$  describing the situation lies between two integer values. The result will be achieved by having the whole density of every compartment converging quickly at low order.

The analysis of fractional order with fractal dimension describes the fractional order of the independent variables. Therefore, the fractal-fractional order differential equation converts both the order and dimension of the system to rational order. Due to this property, we can generalize the DE to any order of derivative and dimension. In [21], Atangana presented the fractal-fractional operators and showed the relation between Fractional and fractal calculus. For applications of fractal-fractional operators, see [22–25]. Because of this, modern calculus has gained more attraction as compared to classical calculus. Fractional calculus in which fractional global operators can model physical and biological problems relating to real-world phenomena with a very well degree of freedom. So for a large number of documents and research articles have been proposed by the scholars of fractional calculus. Various qualities of solution like qualitative and numerical analysis of non-integer order differential and integral equations have been presented in different books and monographs [26–47]. Some analysis of the recently pandemic covid-19 has been presented by several researchers in the form of fractional order differential equations, see [48].

Modern calculus has generalized the integer order calculus of differentiation and integration to rational or complex numbers, describing the situation between two integer values as in [49–51]. Up to now, various real-world problems have been modeled by integer-order differential equations, like population model, logistic population model, HIV, SEIR, TB, Cancer model, Predator–prey model, etc. Next, the researchers converted these equations to arbitrary order of differential equations which give much more real solution as in [52–62, 21, 63]. They have investigated these equations for existence and uniqueness by using properties and theorems of fixed point theory which may be seen in [64–67]. Some scholars use the Banach contraction theorem, topological degree theory and Leray-Schauder theorem [68]. The fractional differential equation has also very interesting for obtaining analytical as well as the numerical solution for different scholars and researchers. Finding the exact solution is very difficult, therefore, most of the scholars investigating FDEs for optimizing and approximate solution applying the pre-existing techniques. For numerical solution they applied Modified Euler techniques, Taylor’s series method, Adams Bash-Forth techniques, predictor–corrector method and different integral transforms along with wave-lets methods as in [26, 53, 69–72]. Because of the aforementioned facts, we will investigate our considered problem for qualitative and some numerical analysis using the concepts of fixed point theory and fractal-fractional Adams–Bashforth techniques.

This paper is organized as follows. In Section (1) we give some fundamental results. In Section (3) we give the theoretical results. Analytical and Numerical results are provided by applying the proposed techniques via Matlab in Section (4). Finally, a brief conclusion is

presented in Section (6). References are given at the end of the manuscript.

**Fundamental results**

**Definition 1.** [23] A continuous function  $z(t)$  in  $(a, b)$  with fractal dimension  $0 < \rho \leq 1$  and fractional order  $0 < \gamma \leq 1$  can be defined in ABC sense as

$${}^{ABC}D_{\rho, \gamma}^{\sigma} z(t) = \frac{ABC(\gamma)}{1-\gamma} \frac{d}{ds} \int_0^t z(s) \kappa_{\rho} \left[ \frac{-\sigma}{1-\gamma} (t-s)^{\gamma} \right] ds,$$

where  $ABC(0) = ABC(1) = 1$  is called the normalization constant.

**Definition 2.** [23] Consider a continuous function  $z(t)$  in  $(a, b)$  with fractal dimension  $0 < \rho \leq 1$  and fractional order  $0 < \gamma \leq 1$  in ABC sense can be defined as

$${}^{ABC}I_{\rho, \gamma}^{\sigma} z(t) = \frac{1-\gamma}{ABC(\gamma)} t^{\rho-1} z(t) + \frac{\gamma}{ABC(\gamma)\Gamma(\gamma)} \int_0^t (t-s)^{\gamma-1} s^{\rho-1} z(s) ds. \quad (3)$$

**Lemma 1.** [73] Let us define the solution for the given problem in view of  $0 < \gamma, \rho \leq 1$

$${}^{ABC}D_{\rho, \gamma}^{\sigma} z(t) = \rho t^{\rho-1} \Psi(t, z(t)), \quad t \in [0, T],$$

$$z(0) = z_0, \quad 0 < \gamma, \rho \leq 1,$$

is provided by

$$z(t) = z_0 + \frac{(1-\gamma)}{ABC(\gamma)} t^{\rho-1} \Psi(t, z(t)) + \frac{\gamma}{\Gamma(\gamma)ABC(\gamma)} \int_0^t (t-s)^{\gamma-1} s^{\rho-1} \Psi(s, z(s)) ds.$$

**Note:** To show the existence and uniqueness for the considered system, a Banach space can be defined as

$$Z = \Psi = H([0, T] \times \mathbb{R}^6, \mathbb{R}),$$

where  $\Psi = H[0, T]$  and the space norm is

$$\|W\| = \|z\| = \max_{t \in [0, T]} [|\mathbb{S}(t)| + |\mathbb{E}(t)| + |\mathbb{I}(t)| + |\mathbb{R}(t)| + |\mathbb{D}(t)| + |\mathbb{M}(t)|].$$

For showing our next results, we are going to present a fixed point theorem.

**Theorem 1.** [74] **statement:** A subset of  $Z$  be  $A$  which is convex and consider that the two operators  $F_1$  and  $F_2$  with

1.  $F_1(w) + F_2(w) \in A$  for each  $w \in A$ ;
2.  $F_1$  is contraction;
3.  $F_2$  is continuous and compact.

having the operator equation  $F_1 w + F_2 w = w$ , has one or more than one solution.

**Existence and uniqueness of model (2)**

In this section, we will find the existence and uniqueness and stability results, by using the fixed point theorem for the considered system (2). In this regard, the aforesaid need as the integral is differentiable, here we can rewrite the considered model (2) as

$$\begin{cases} {}^{ABC}D^{\rho}(\mathbb{S}(t)) = \rho t^{\rho-1} \mathcal{G}_1(\mathbb{S}(t), t) = \Lambda^* - (\lambda^* + \alpha^* e^* + \mu^*) \mathbb{S}, \\ {}^{ABC}D^{\rho}(\mathbb{E}(t)) = \rho t^{\rho-1} \mathcal{G}_2(\mathbb{E}(t), t) = \alpha^* e^* \mathbb{S} - (\theta^* + \mu^* + (1-\eta^*) \lambda^*) \mathbb{E}, \\ {}^{ABC}D^{\rho}(\mathbb{I}(t)) = \rho t^{\rho-1} \mathcal{G}_3(\mathbb{I}(t), t) = \lambda^* \mathbb{S} + (1-\eta^*) \lambda^* \mathbb{E} - (\mu^* + \delta^* + \gamma^*) \mathbb{I}, \\ {}^{ABC}D^{\rho}(\mathbb{R}(t)) = \rho t^{\rho-1} \mathcal{G}_4(\mathbb{R}(t), t) = \theta^* \mathbb{E} + \gamma^* \mathbb{I} - \mu^* \mathbb{R}, \\ {}^{ABC}D^{\rho}(\mathbb{D}(t)) = \rho t^{\rho-1} \mathcal{G}_5(\mathbb{D}(t), t) = \delta^* \mathbb{I} - \mu^* \mathbb{D}, \\ {}^{ABC}D^{\rho}(\mathbb{M}(t)) = \rho t^{\rho-1} \mathcal{G}_6(\mathbb{M}(t), t) = \epsilon^* \mathbb{E} + \xi^* \mathbb{I} - \psi^* \mathbb{M}. \end{cases} \quad (4)$$

In view of (4) and for  $t \in Z$ , then one can write

$${}^{ABC}D_{\rho, \gamma}^{\sigma} z(t) = \rho t^{\rho-1} \Psi(t, z(t)), \quad t \in [0, T],$$

$$z(0) = z_0, \quad 0 < \gamma, \rho \leq 1, \quad (5)$$

with solution

$$z(t) = z_0 + \frac{(1-\gamma)}{ABC(\gamma)} t^{\rho-1} \Psi(t, z(t)) + \frac{\gamma}{\Gamma(\gamma)ABC(\gamma)} \int_0^t (t-s)^{\gamma-1} s^{\rho-1} \Psi(s, z(s)) ds, \quad (6)$$

where

$$\begin{cases} z(t) = (\mathbb{S}, \mathbb{E}, \mathbb{I}, \mathbb{R}, \mathbb{D}, \mathbb{M})^T, \\ z_0(t) = (\mathbb{S}_0, \mathbb{E}_0, \mathbb{I}_0, \mathbb{R}_0, \mathbb{D}_0, \mathbb{M}_0)^T, \\ \Psi(t, z(t)) = (\mathcal{G}_i(\mathbb{S}, \mathbb{E}, \mathbb{I}, \mathbb{R}, \mathbb{D}, \mathbb{M}, t))^T, \quad i = 1, 2, 3 \dots 6. \end{cases} \quad (7)$$

Now, change the system (2) into the fixed point. Let us define mapping  $\mathcal{T} : \mathbf{V} \rightarrow \mathbf{V}$  given as:

$$\mathcal{T} z(t) = z_0 + \frac{(1-\gamma)}{ABC(\gamma)} t^{\rho-1} \Psi(t, z(t)) + \frac{\gamma}{\Gamma(\gamma)ABC(\gamma)} \int_0^t (t-s)^{\gamma-1} s^{\rho-1} \Psi(s, z(s)) ds. \quad (8)$$

Assume

$$\mathcal{T} = \mathbf{F} + \mathbf{G},$$

where

$$\mathbf{F}(z) = z_0(t) + \frac{(1-\gamma)}{ABC(\gamma)} t^{\rho-1} [\Psi(t, z(t))],$$

$$\mathbf{G}(z) = \frac{\gamma}{ABC(\gamma)\Gamma(\gamma)} \int_0^t (t-s)^{\gamma-1} s^{\rho-1} \Psi(s, z(s)) ds. \quad (9)$$

Now, we will show the qualitative analysis for the considered system by applying fixed point theory:

(U1) There will be a constants  $\mathcal{L}_{\Psi}, \mathcal{V}_{\Psi}, \vartheta$

$$|\Psi(t, z(t))| \leq \mathcal{L}_{\Psi} |z| + \mathcal{V}_{\Psi}.$$

(U2)  $\exists$  a constants  $\mathbb{L}_{\Psi} > 0 \ni$  for every  $z, \bar{z} \in Z$  as

$$|\Psi(t, z) - \Psi(t, \bar{z})| \leq \mathbb{L}_{\Psi} \|z - \bar{z}\|;$$

**Theorem 2.** The system (6) has at least one solution if (U1, U2) holds, then the system (2) has the same number of solution if  $\frac{(1-\gamma)}{ABC(\gamma)} t^{\rho-1} \mathbb{L}_{\Psi} < 1$ .

**Proof.** We have proven the theorem into two steps as:

**Step I:** Suppose  $\bar{z} \in A$  and  $A = \{z \in Z : \|z\| \leq \phi, \phi > 0\}$  is convex closed set. So the operator  $F$  defined in (9), we have

$$\begin{aligned} \|F(z) - F(\bar{z})\| &= \frac{(1-\gamma)}{ABC(\gamma)} t^{\rho-1} \max_{t \in [0, \bar{t}]} |\Psi(t, z(t)) - \Psi(t, \bar{z}(t))| \\ &\leq \frac{(1-\gamma)}{ABC(\gamma)} t^{\rho-1} \mathbb{L}_{\Psi} \|z - \bar{z}\|. \end{aligned} \quad (10)$$

Thus, the operator  $F$  is closed and hence contraction.

**Step-II:** Now we will prove the  $G$  is compact in comparison form, also we have to prove that  $G$  is continuous and bounded. It is clear that the operator  $G$  is defined on the whole domain as  $\mathbb{Y}$  is continuous, also for  $z \in A$ , as follows

$$\begin{aligned} \|G(z)\| &= \max_{t \in [0, \tau]} \left| \frac{\mathcal{I}\mathcal{Y}}{ABC(\varphi)\Gamma(\varphi)} \int_0^t (\tau - s)^{\varphi-1} s^{\varphi-1} \mathbb{Y}(s, z(s)) ds \right| \\ &\leq \frac{\mathcal{I}\mathcal{Y}}{ABC(\varphi)\Gamma(\varphi)} \int_0^t (s)^{\varphi-1} (1-s)^{\varphi-1} |\mathbb{Y}(s, z(s))| ds \\ &\leq \frac{\rho[\mathcal{L}_{\mathbb{Y}}|z| + \mathcal{I}_{\mathbb{Y}} T^{\varphi+\nu-1}] \mathbb{B}(\varphi, \rho)}{ABC(\varphi)\Gamma(\varphi)}. \end{aligned} \tag{11}$$

$$\begin{aligned} \|\mathcal{F}z - \mathcal{F}\bar{z}\| &\leq \frac{(1-\varphi)t^{\varphi-1}}{ABC(\varphi)} \max_{t \in [0, \tau]} |\mathbb{Y}(t, z(t)) - \mathbb{Y}(t, \bar{z}(t))| \\ &+ \frac{\mathcal{I}\mathcal{Y}}{ABC(\varphi)\Gamma(\varphi)} \max_{t \in [0, \tau]} \left| \int_0^t (t-x)^{\varphi-1} t^{\varphi-1} \mathbb{Y}(x, z(x)) dx - \int_0^t (t-x)^{\varphi-1} t^{\varphi-1} \mathbb{Y}(x, \bar{z}(x)) dx \right| \\ &\leq \Theta \|z - \bar{z}\|, \end{aligned} \tag{14}$$

Hence in view of (11) the operator  $G$  is bounded, for “equi-continuity” let  $t_1 > t_2 \in [0, \tau]$ , we have

$$\begin{aligned} |G(z(t_2)) - G(z(t_1))| &= \frac{\mathcal{I}\mathcal{Y}}{ABC(\varphi)\Gamma(\varphi)} \left| \int_0^{t_2} (t_2 - x)^{\varphi-1} x^{\varphi-1} \mathbb{Y}(x, z(x)) dx \right. \\ &- \left. \int_0^{t_1} (t_1 - x)^{\varphi-1} x^{\varphi-1} \mathbb{Y}(x, z(x)) dx \right| \\ &\leq \frac{\rho[\mathcal{L}_{\mathbb{Y}}|z| + \mathcal{I}_{\mathbb{Y}} T^{\varphi+\nu-1}] \mathbb{B}(\varphi, \rho)}{ABC(\varphi)\Gamma(\varphi)} [t_2^{\varphi} - t_1^{\varphi}]. \end{aligned} \tag{12}$$

As  $t_2 \rightarrow t_1$ , R.H.S. of (11) goes to zero. Also by continuous operator  $G$  so

$$|Gz(t_2) - Gz(t_1)| \rightarrow 0, \text{ as } t_2 \rightarrow t_1.$$

Hence, we showed that the operator  $G$  is bounded and continuous, so  $G$  is bounded and uniformly continuous. By “Arzelá-Ascoli theorem a subset  $z \in A$  of  $G$  is compact if and only if it is closed, bounded, and equicontinuous.” As  $G$  is relatively compact and completely continuous. From (2) and (6) we conclude that the system has at least one solution.  $\square$

**Uniqueness**

**Theorem 3.** With assumption (U2) and (6) has a unique solution so that consider the system (2) also has a unique solution if

$$\left[ \frac{(1-\varphi)t^{\varphi-1} \mathbb{X}_{\mathbb{Y}}}{ABC(\varphi)} + \frac{\rho[\mathbb{L}_{\mathbb{Y}} T^{\varphi+\nu-1}] \mathbb{B}(\varphi, \rho)}{ABC(\varphi)\Gamma(\varphi)} \right] < 1.$$

**Proof.** Suppose the operator  $\mathcal{F} : z \rightarrow z$  by

$$\begin{aligned} \mathcal{F}z(t) &= z_0(t) + [\mathbb{Y}(t, z(t)) - \mathbb{Y}_0(t)] \frac{(1-\varphi)t^{\varphi-1}}{ABC(\varphi)} \\ &+ \frac{\mathcal{I}\mathcal{Y}}{ABC(\varphi)\Gamma(\varphi)} \int_0^t (t-x)^{\varphi-1} t^{\varphi-1} \mathbb{Y}(x, z(x)) dx, \quad t \in [0, \tau]. \end{aligned} \tag{13}$$

Let  $z, \bar{z} \in z$ , then

and

$$\Theta = \left[ \frac{(1-\varphi)t^{\varphi-1} \mathbb{L}_{\mathbb{Y}}}{ABC(\varphi)} + \frac{\rho[\mathbb{L}_{\mathbb{Y}} T^{\varphi+\nu-1}] \mathbb{B}(\varphi, \rho)}{ABC(\varphi)\Gamma(\varphi)} \right]. \tag{15}$$

From (14) the operator  $\mathcal{F}$  is a contraction. So Eq. (6) has a unique solution. Thus the considered system (2) has a unique solution.  $\square$

**Ulam-Hyer stability**

Here, we will find the stability for the proposed system (2), let take a small change  $\Psi(t) \in C[0, T]$  and only satisfy,  $0 = \Phi(0)$  as.

- $|\Psi(t)| \leq \epsilon$  for  $\epsilon > 0$ ;
- ${}^{ABC}D_t^{\varphi, \rho} z(t) = \mathbb{Y}(t, z(t)) + \Psi(t)$ .

**Lemma 2.** The changed problem solution can be

$$\begin{aligned} {}^{ABC}D_t^{\varphi, \rho} z(t) &= \mathbb{Y}(t, z(t)) + \Phi(t), \\ z(0) &= z_0, \end{aligned} \tag{16}$$

satisfies the given relation

$$\begin{aligned} |z(t) - \left( z_0(t) + [\mathbb{Y}(t, z(t)) - \Phi_0(t)] \frac{(1-\varphi)t^{\varphi-1}}{ABC(\varphi)} + \frac{\mathcal{I}\mathcal{Y}}{ABC(\varphi)\Gamma(\varphi)} \int_0^t (t-x)^{\varphi-1} x^{\varphi-1} \mathbb{Y}(x, z(x)) dx \right)| \\ \leq \frac{\Gamma(\varphi)t^{\varphi-1} + \rho T^{\varphi+\nu-1}}{ABC(\varphi)\Gamma(\varphi)} \mathbb{B}(\varphi, \rho) \epsilon = \vartheta_{\varphi, \rho} \epsilon. \end{aligned} \tag{17}$$

**Theorem 4.** With the assumption (C2) together with Eq. (17), solution of the Eq. (6) is UH stable and consequently, the analytical solution for the proposed system is UH stable if  $\Theta < 1$ .

**Proof.** Suppose a unique solution be  $z \in z$  and  $\bar{z} \in z$  be any solution of Eq. (6), then

$$\begin{aligned}
 |z(t) - \bar{z}(t)| &= \left| z(t) - \left( z_0(t) + [\mathbb{Y}(t, \bar{z}(t)) - \mathbb{Y}_0(t)] \frac{(1-\varrho)}{\text{ABC}(\varrho)} t^{\varrho-1} + \frac{\mathcal{I}\mathcal{H}}{\text{ABC}(\varrho)\Gamma(\varrho)} \right. \right. \\
 &\quad \times \left. \left. \int_0^t (t-x)^{\varrho-1} x^{\varrho-1} \mathbb{Y}(x, \bar{z}(x)) dx \right) \right|, \\
 &\leq \left| z(t) - \left( z_0(t) + [\mathbb{Y}(t, z(t)) - \mathbb{Y}_0(t)] \frac{(1-\varrho)}{\text{ABC}(\varrho)} t^{\varrho-1} + \frac{\mathcal{I}\mathcal{H}}{\text{ABC}(\varrho)\Gamma(\varrho)} \right. \right. \\
 &\quad \times \left. \left. \int_0^t (t-x)^{\varrho-1} x^{\varrho-1} \mathbb{Y}(x, z(x)) dx \right) \right| + \left| \left( z_0(t) + [\mathbb{Y}(t, z(t)) - \mathbb{Y}_0(t)] \frac{(1-\varrho)}{\text{ABC}(\varrho)} t^{\varrho-1} \right. \right. \\
 &\quad \left. \left. + \frac{\mathcal{I}\mathcal{H}}{\text{ABC}(\varrho)\Gamma(\varrho)} \int_0^t (t-x)^{\varrho-1} x^{\varrho-1} \mathbb{Y}(x, z(x)) dx \right) \right. \\
 &\quad \left. - \left( z_0(t) + [(\mathbb{Y}(t, \bar{z}(t)) - \mathbb{Y}_0(t))] \frac{(1-\varrho)}{\text{ABC}(\varrho)} t^{\varrho-1} + \frac{\mathcal{I}\mathcal{H}}{\text{ABC}(\varrho)\Gamma(\varrho)} \int_0^t (t-x)^{\varrho-1} x^{\varrho-1} \mathbb{Y}(x, \bar{z}(x)) dx \right) \right|, \\
 &\leq \mathcal{L}_{\mathbb{Y}} \frac{(1-\varrho)}{\text{ABC}(\varrho)} t^{\varrho-1} \|z - \bar{z}\| + \frac{\rho T^{\varrho+\varrho-1} \mathcal{L}_{\mathbb{Y}}}{\text{ABC}(\varrho)\Gamma(\varrho)} \mathbb{B}(\varrho, \rho) \|z - \bar{z}\|, \\
 &\leq \mathcal{L}_{\mathbb{Y}} \varrho + \Theta \|z - \bar{z}\|.
 \end{aligned} \tag{18}$$

From (18), we can write as

$$\|z - \bar{z}\| \leq \frac{\mathcal{L}_{\mathbb{Y}} \varrho}{1 - \Theta} \|z - \bar{z}\|. \tag{19}$$

From (19), we concluded that the solution of (6) is UH stable and consequently generalized Ulam-Hyers Stable by using  $\mathbb{Y}_z(\varepsilon) = \mathcal{L}_{\mathbb{Y}} \varrho \varepsilon$ ,  $\mathbb{Y}_z(0) = 0$ , which shows that the solution of the proposed problem is Ulam-Hyers stable and also generalized Ulam-Hyers stable.  $\square$

### Numerical solution

In this section, we will find the numerical solutions for fractal-arbitrary order of the system (2), by using ABC derivative due to famous fractal-fractional AB technique. With the help of some iterative scheme, we obtain the approximate solution for the considered model. For this, we have to use the fractal-fractional AB techniques [58] for

$$\begin{aligned}
 \mathbb{S}(t_{c+1}) - \mathbb{S}(0) &= \frac{(1-\varrho)}{\text{ABC}(\varrho)} (t_{c+1}^{\varrho-1}) [\mathcal{I}_1(\mathbb{S}(t_c), t_c)] + \frac{\mathcal{I}\mathcal{H}}{\text{ABC}(\varrho)\Gamma(\varrho)} \int_0^{t_{c+1}} (t_{c+1}-x)^{\varrho-1} x^{\varrho-1} \mathcal{I}_1(\mathbb{S}(x), x) dx, \\
 &= \frac{(1-\varrho)}{\text{ABC}(\varrho)} (t_{c+1}^{\varrho-1}) [\mathcal{I}_1(\mathbb{S}(t_c), t_c)] + \frac{\mathcal{I}\mathcal{H}}{\text{ABC}(\varrho)\Gamma(\varrho)} \sum_{r=0}^c \int_r^{t_{r+1}} (t_{c+1}-x)^{\varrho-1} x^{\varrho-1} \mathcal{I}_1(\mathbb{S}(x), x) dx.
 \end{aligned}$$

obtaining an approximate solution for the plotting of the system (2). Thus, we go further with (4) can be write as:

$$\begin{aligned}
 \mathbb{S}(t_{c+1}) &= \mathbb{S}(0) + \frac{(1-\varrho)}{\text{ABC}(\varrho)} (t_{c+1}^{\varrho-1}) [\mathcal{I}_1(\mathbb{S}(t_c), t_c)] + \frac{\mathcal{I}\mathcal{H}}{\text{ABC}(\varrho)\Gamma(\varrho)} \sum_{r=0}^c \left( \frac{\mathcal{I}_1(\mathbb{S}(t_c), t_c)}{\Delta} \right. \\
 &\quad \times \left. \int_r^{t_{r+1}} (t-t_{r-1})(t_{r+1}-t)^{\varrho-1} t_r^{\varrho-1} dt - \frac{\mathcal{I}_1(\mathbb{S}(t_c), t_c)}{\Delta} \int_r^{t_{r+1}} (t-t_r)(t_{c+1}-t)^{\varrho-1} t_r^{\varrho-1} dt \right),
 \end{aligned}$$

$$\left\{ \begin{aligned}
 \text{ABC}^{\varrho}(\mathbb{S}(t)) &= \rho t^{\varrho-1} \mathcal{I}_1(\mathbb{S}(t), t), \\
 \text{ABC}^{\varrho}(\mathbb{E}(t)) &= \rho t^{\varrho-1} \mathcal{I}_2(\mathbb{E}(t), t), \\
 \text{ABC}^{\varrho}(\mathbb{I}(t)) &= \rho t^{\varrho-1} \mathcal{I}_3(\mathbb{I}(t), t), \\
 \text{ABC}^{\varrho}(\mathbb{R}(t)) &= \rho t^{\varrho-1} \mathcal{I}_3(\mathbb{R}(t), t), \\
 \text{ABC}^{\varrho}(\mathbb{D}(t)) &= \rho t^{\varrho-1} \mathcal{I}_5(\mathbb{D}(t), t), \\
 \text{ABC}^{\varrho}(\mathbb{M}(t)) &= \rho t^{\varrho-1} \mathcal{I}_6(\mathbb{M}(t), t),
 \end{aligned} \right. \tag{20}$$

where  $\mathcal{I}_i, i = 1, 2, 3, \dots, 6$  are defined in (4) By applying antiderivative of fractional order and fractal dimension to the 1<sup>st</sup> equation of (4) using ABC form, we get

$$\begin{aligned}
 \mathbb{S}(t) - \mathbb{S}(0) &= \frac{(1-\varrho)}{\text{ABC}(\varrho)} t^{\varrho-1} [\mathcal{I}_1(\mathbb{S}(t), t)] \\
 &\quad + \frac{\mathcal{I}\mathcal{H}}{\text{ABC}(\varrho)\Gamma(\varrho)} \int_0^t (t-x)^{\varrho-1} x^{\varrho-1} \mathcal{I}_1(\mathbb{S}(x), x) dx.
 \end{aligned}$$

Set  $t = t_{c+1}$  for  $i = 0, 1, 2, \dots$ ,

The approximate function be  $\mathcal{I}_1$  on the interval  $[t_r, t_{r+1}]$  through the interpolation polynomial as follows

$$\mathcal{I}_1 \cong \frac{\mathcal{I}_1}{\Delta} (t - t_{r-1}) - \frac{\mathbf{R}_1}{\Delta} (t - t_r)$$

which implies that

$$\mathbb{S}(t_{c+1}) = \mathbb{S}(0) + \frac{(1-\varrho)}{\text{ABC}(\varrho)}(I_{c+1}^{\varrho-1})[\mathcal{I}_1(\mathbb{S}(t_c), t_c)] + \frac{\varrho\vartheta}{\text{ABC}(\varrho)\Gamma(\vartheta)} \sum_{r=0}^c \left( \frac{I_r^{\varrho-1} \mathcal{I}_1(\mathbb{S}(t_r), t_r)}{\Delta} I_{r-1,\varrho} - \frac{I_{r-1}^{\varrho-1} \mathcal{I}_1(\mathbb{S}(t_{r-1}), t_{r-1})}{\Delta} I_{r,\varrho} \right). \tag{21}$$

$$I_{r,\varrho} = \int_r^{t_{r+1}} (t-t_r)(t_{c+1}-t)^{\varrho-1} dt, \\ = -\frac{1}{\varrho}[(t_{r+1}-t_r)(t_{c+1}-t_{r+1})^\varrho] \\ - \frac{1}{\varrho(\varrho-1)}[(t_{c+1}-t_{r+1})^{\varrho+1} - (t_{c+1}-t_r)^{\varrho+1}],$$

Calculating  $I_{r-1,\varrho}$  and  $I_{r,\varrho}$  we get

$$I_{r-1,\varrho} = \int_r^{t_{r+1}} (t-t_{r-1})(t_{c+1}-t)^{\varrho-1} dt, \\ = -\frac{1}{\varrho}[(t_{r+1}-t_{r-1})(t_{c+1}-t_{r+1})^\varrho - (t_r-t_{r-1})(t_{c+1}-t_r)^\varrho] \\ - \frac{1}{\varrho(\varrho-1)}[(t_{c+1}-t_{r+1})^{\varrho+1} - (t_{c+1}-t_r)^{\varrho+1}],$$

put  $t_r = r\Delta$ , we get

and

substituting the values of (22) and (23) in (21), we get

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$$I_{r-1,\varrho} = -\frac{\Delta^{\varrho+1}}{\varrho}[(r+1-(r-1))(c+1-(r+1))^\varrho - (r-(r-1))(c+1-r)^\varrho] \\ - \frac{\Delta^{\varrho+1}}{\varrho(\varrho-1)}[(c+1-(r+1))^{\varrho+1} - (c+1-r)^{\varrho+1}], \\ = \frac{\Delta^{\varrho+1}}{\varrho(\varrho-1)}[-2(\varrho+1)(c-r)^\varrho + (\varrho+1)(c+1-r)^\varrho - (c-r)^{\varrho+1} + (c+1-r)^{\varrho+1}], \\ = \frac{\Delta^{\varrho+1}}{\varrho(\varrho-1)}[(c-r)^\varrho(-2(\varrho+1)-(c-r)) + (c+1-r)^\varrho(\varrho+1+c+1-r)], \\ = \frac{\Delta^{\varrho+1}}{\varrho(\varrho-1)}[(c+1-r)^\varrho(c-r+2+\varrho) - (c-r)^\varrho(c-r+2+2\varrho)], \tag{22}$$


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$$I_{r,\varrho} = -\frac{\Delta^{\varrho+1}}{\varrho}[(r+1-r)(c+1-(r+1))^\varrho] - \frac{\Delta^{\varrho+1}}{\varrho(\varrho-1)}[(c+1-(r+1))^{\varrho+1} - (c+1-r)^{\varrho+1}], \\ = \frac{\Delta^{\varrho+1}}{\varrho(\varrho-1)}[-(\varrho+1)(c-r)^\varrho - (c-r)^{\varrho+1} + (c+1-r)^{\varrho+1}], \\ = \frac{\Delta^{\varrho+1}}{\varrho(\varrho-1)}[(c-r)^\varrho(-(r+1)-(c-r)) + (c+1-r)^{\varrho+1}], \\ = \frac{\Delta^{\varrho+1}}{\varrho(\varrho-1)}[(c+1-r)^{\varrho+1} - (c-r)^\varrho(c-r+1+\varrho)], \tag{23}$$


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$$\mathbb{S}(t_{c+1}) = \left\{ \begin{array}{l} \mathbb{S}(0) + \frac{(1-\varrho)}{\text{ABC}(\varrho)}(I_{c+1}^{\varrho-1})[\mathcal{I}_1(\mathbb{S}(t_c), t_c)] + \frac{\varrho\vartheta}{\text{ABC}(\varrho)\Gamma(\vartheta)} \sum_{r=0}^c \left( \frac{I_r^{\varrho-1} \mathcal{I}_1(\mathbb{S}(t_r), t_r)}{\Delta} \right. \\ \times \left[ \frac{\Delta^{\varrho+1}}{\varrho(\varrho-1)}[(c+1-r)^\varrho(c-r+2+\varrho) - (c-r)^\varrho(c-r+2+2\varrho)] \right] \\ \left. - \frac{I_{r-1}^{\varrho-1} \mathcal{I}_1(\mathbb{S}(t_{r-1}), t_{r-1})}{\Delta} \left[ \frac{\Delta^{\varrho+1}}{\varrho(\varrho-1)}[(c+1-r)^{\varrho+1} - (c-r)^\varrho(c-r+1+\varrho)] \right] \right) \end{array} \right\}. \tag{24}$$

Similarly the rest of the classes E, I, R, D and M we can find the same numerical scheme as

$$\mathbb{E}(t_{c+1}) = \left\{ \begin{aligned} & \mathbb{E}(0) + \frac{(1-\varrho)}{\text{ABC}(\varrho)}(t_{c+1}^{\varrho-1})[\mathcal{S}_2(\mathbb{E}(t_c), t_c)] + \frac{\varrho}{\text{ABC}(\varrho)\Gamma(\varrho)} \sum_{r=0}^c \left( \frac{t_r^{\varrho-1} \mathcal{S}_2(\mathbb{E}(t_r), t_r)}{\Delta} \right. \\ & \times \left. \left[ \frac{\Delta^{\varrho+1}}{\varrho(\varrho-1)} [(c+1-r)^\varrho(c-r+2+\varrho) - (c-r)^\varrho(c-r+2+2\varrho)] \right] \right. \\ & \left. \left. - \frac{t_{r-1}^{\varrho-1} \mathcal{S}_2(\mathbb{E}(t_{r-1}), t_{r-1})}{\Delta} \left[ \frac{\Delta^{\varrho+1}}{\varrho(\varrho-1)} [(c+1-r)^{\varrho+1} - (c-r)^\varrho(c-r+1+\varrho)] \right] \right) \right\}. \end{aligned} \right. \tag{25}$$

$$\mathbb{I}(t_{c+1}) = \left\{ \begin{aligned} & \mathbb{I}(0) + \frac{(1-\varrho)}{\text{ABC}(\varrho)}(t_{c+1}^{\varrho-1})[\mathcal{S}_3(\mathbb{I}(t_c), t_c)] + \frac{\varrho}{\text{ABC}(\varrho)\Gamma(\varrho)} \sum_{r=0}^c \left( \frac{t_r^{\varrho-1} \mathcal{S}_3(\mathbb{I}(t_r), t_r)}{\Delta} \right. \\ & \times \left. \left[ \frac{\Delta^{\varrho+1}}{\varrho(\varrho-1)} [(c+1-r)^\varrho(c-r+2+\varrho) - (c-r)^\varrho(c-r+2+2\varrho)] \right] \right. \\ & \left. \left. - \frac{t_{r-1}^{\varrho-1} \mathcal{S}_3(\mathbb{I}(t_{r-1}), t_{r-1})}{\Delta} \left[ \frac{\Delta^{\varrho+1}}{\varrho(\varrho-1)} [(c+1-r)^{\varrho+1} - (c-r)^\varrho(c-r+1+\varrho)] \right] \right) \right\}. \end{aligned} \right. \tag{26}$$

$$\mathbb{R}(t_{c+1}) = \left\{ \begin{aligned} & \mathbb{R}(0) + \frac{(1-\varrho)}{\text{ABC}(\varrho)}(t_{c+1}^{\varrho-1})[\mathcal{S}_4(\mathbb{R}(t_c), t_c)] + \frac{\varrho}{\text{ABC}(\varrho)\Gamma(\varrho)} \sum_{r=0}^c \left( \frac{t_r^{\varrho-1} \mathcal{S}_4(\mathbb{R}(t_r), t_r)}{\Delta} \right. \\ & \times \left. \left[ \frac{\Delta^{\varrho+1}}{\varrho(\varrho-1)} [(c+1-r)^\varrho(c-r+2+\varrho) - (c-r)^\varrho(c-r+2+2\varrho)] \right] \right. \\ & \left. \left. - \frac{t_{r-1}^{\varrho-1} \mathcal{S}_4(\mathbb{R}(t_{r-1}), t_{r-1})}{\Delta} \left[ \frac{\Delta^{\varrho+1}}{\varrho(\varrho-1)} [(c+1-r)^{\varrho+1} - (c-r)^\varrho(c-r+1+\varrho)] \right] \right) \right\}. \end{aligned} \right. \tag{27}$$

$$\mathbb{D}(t_{c+1}) = \left\{ \begin{aligned} & \mathbb{D}(0) + \frac{(1-\varrho)}{\text{ABC}(\varrho)}(t_{c+1}^{\varrho-1})[\mathcal{S}_5(\mathbb{D}(t_c), t_c)] + \frac{\varrho}{\text{ABC}(\varrho)\Gamma(\varrho)} \sum_{r=0}^c \left( \frac{t_r^{\varrho-1} \mathcal{S}_5(\mathbb{D}(t_r), t_r)}{\Delta} \right. \\ & \times \left. \left[ \frac{\Delta^{\varrho+1}}{\varrho(\varrho-1)} [(c+1-r)^\varrho(c-r+2+\varrho) - (c-r)^\varrho(c-r+2+2\varrho)] \right] \right. \\ & \left. \left. - \frac{t_{r-1}^{\varrho-1} \mathcal{S}_5(\mathbb{D}(t_{r-1}), t_{r-1})}{\Delta} \left[ \frac{\Delta^{\varrho+1}}{\varrho(\varrho-1)} [(c+1-r)^{\varrho+1} - (c-r)^\varrho(c-r+1+\varrho)] \right] \right) \right\}. \end{aligned} \right. \tag{28}$$

$$\mathbb{M}(t_{c+1}) = \left\{ \begin{aligned} & \mathbb{M}(0) + \frac{(1-\varphi)}{\text{ABC}(\varphi)} (t_{c+1}^{-1}) [\mathcal{G}_6(\mathbb{M}(t_c), t_c)] + \frac{\varphi}{\text{ABC}(\varphi)\Gamma(\varphi)} \sum_{r=0}^c \left( \frac{t_r^{-1} \mathcal{G}_6(\mathbb{M}(t_r), t_r)}{\Delta} \right. \\ & \times \left. \left[ \frac{\Delta^{\varphi+1}}{\varphi(\varphi-1)} [(c+1-r)^\varphi (c-r+2+\varphi) - (c-r)^\varphi (c-r+2+2\varphi)] \right] \right. \\ & \left. \left. - \frac{t_{r-1}^{-1} \mathcal{G}_6(\mathbb{M}(t_{r-1}), t_{r-1})}{\Delta} \left[ \frac{\Delta^{\varphi+1}}{\varphi(\varphi-1)} [(c+1-r)^{\varphi+1} - (c-r)^\varphi (c-r+1+\varphi)] \right] \right) \right\}. \end{aligned} \right. \quad (29)$$

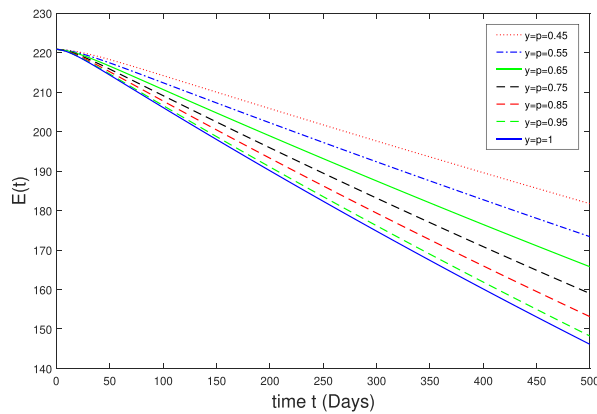
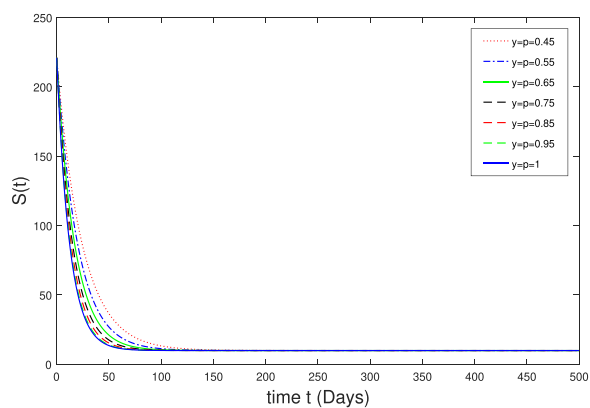


Fig. 1. Dynamical simulation of  $\mathbb{S}(t)$  and  $\mathbb{E}(t)$  at different arbitrary order of derivatives for data-I.

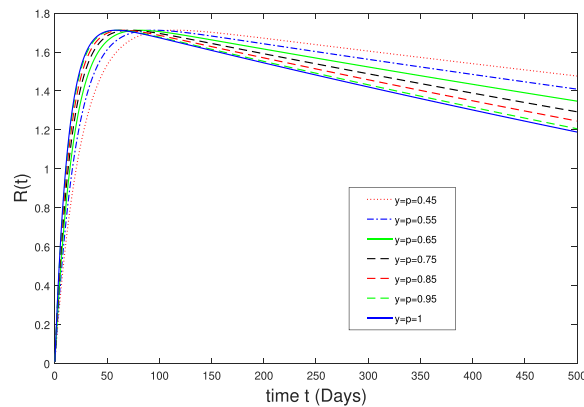
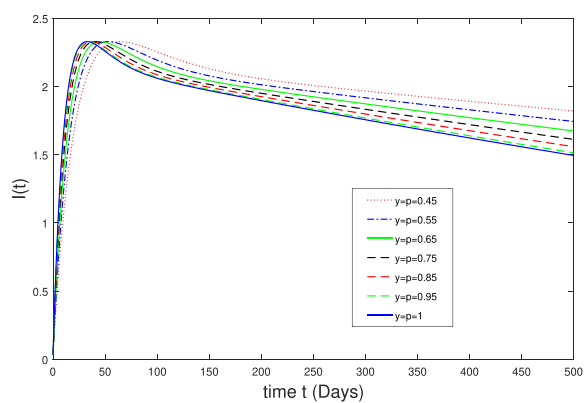


Fig. 2. Dynamical simulation of  $\mathbb{I}(t)$  and  $\mathbb{R}(t)$  at different arbitrary order of derivatives for data-I.

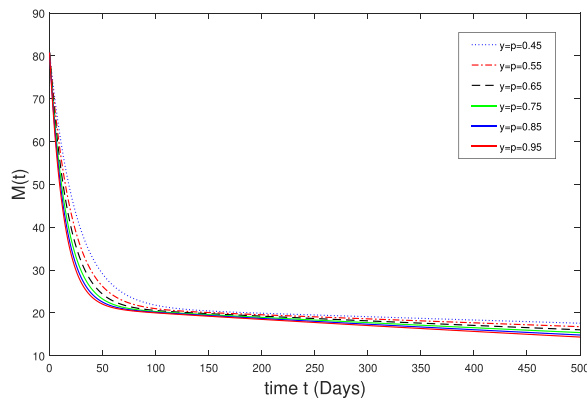
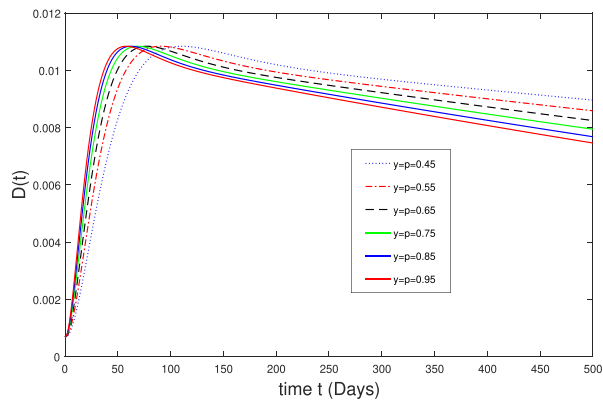


Fig. 3. Dynamical simulation of  $\mathbb{D}(t)$  and  $\mathbb{M}(t)$  at different arbitrary order of derivatives for data-I.

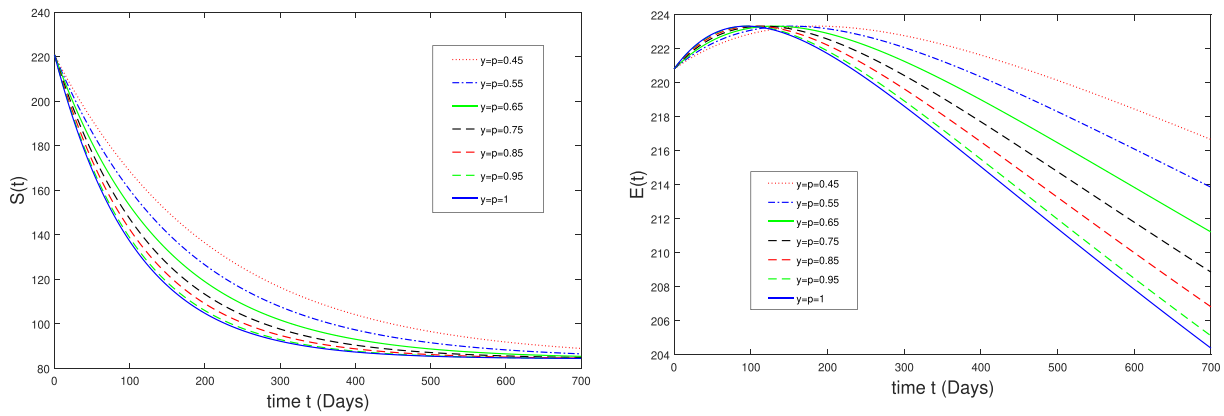


Fig. 4. Dynamical simulation of  $\mathbb{S}(t)$  and  $\mathbb{E}(t)$  at different arbitrary order of derivatives for data-II.

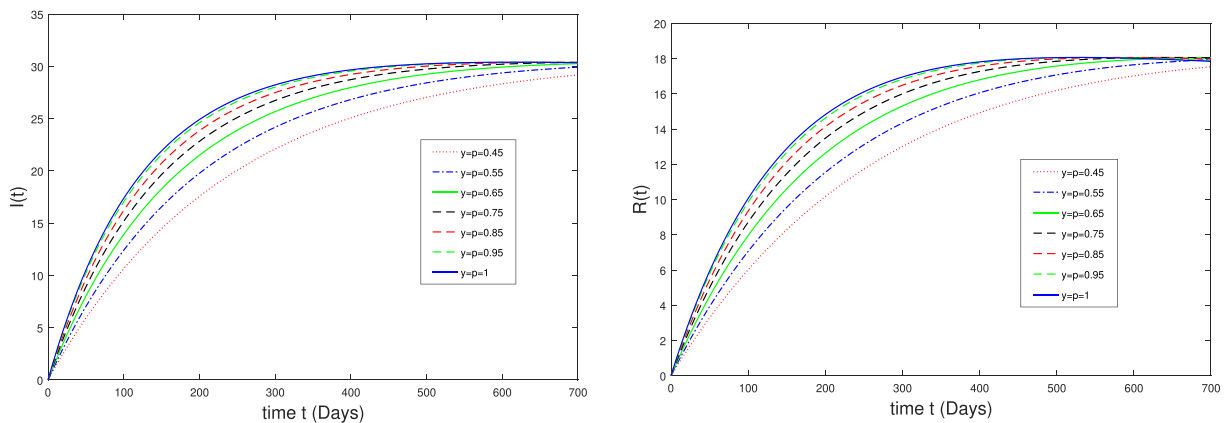


Fig. 5. Dynamical simulation of  $\mathbb{I}(t)$  and  $\mathbb{R}(t)$  at different arbitrary order of derivatives for data-II.

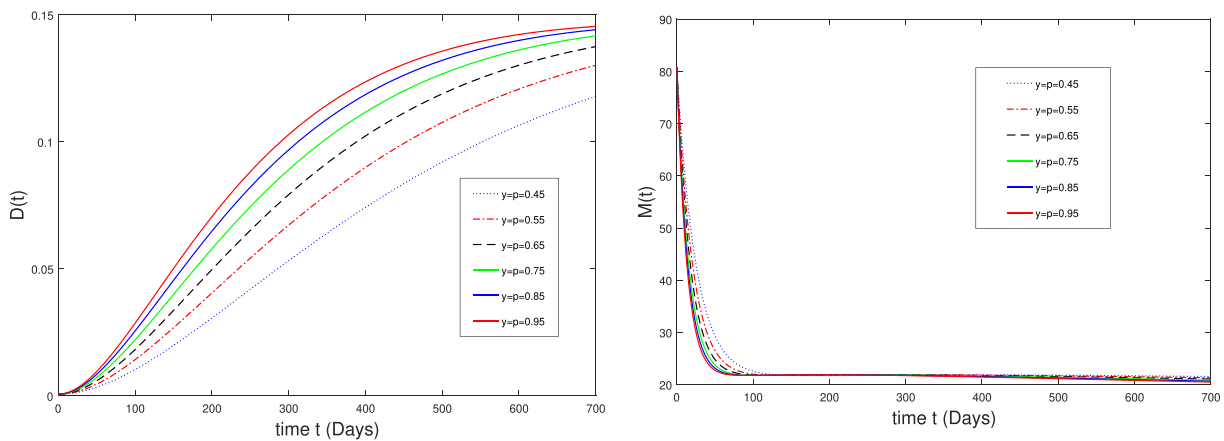


Fig. 6. Dynamical simulation of  $\mathbb{D}(t)$  and  $\mathbb{M}(t)$  at different arbitrary order of derivatives for data-II.

**Simulations results**

We now take the values for the considered system (2) in the following table. The data have been taken for Pakistan. The initial compartment cases of the given system for Pakistan are about  $\mathbb{S}(t) = 220.892$ ,  $\mathbb{E}(t) = 220.81857$ ,  $\mathbb{I}(t) = 0.03208$ ,  $\mathbb{R}(t) = 0.008555$ ,  $\mathbb{D}(t) = 0.000706$ ,  $\mathbb{M}(t) = 80.7777$  millions.

Figs. 1–3 are the numerical simulation for the Data-I while Figs. 4–6 are for data-II respectively. The left panel of Fig. 1 shows rapid decay with time as different contaminated materials of covid-19 are faced by the susceptible individuals to moves to the exposed and infected classes.

Up to the 100 days they decline and after that they become stable at different fractional order  $\nu$  and fractal dimension  $\mu$ . In the right panel of Fig. 1 is the representation of Exposed class for various fractional-order  $\nu$  and fractal dimensions  $\mu$ . This class also declines as they transfer to other classes. In the left panel of Fig. 2 one can see that the infection reaches its peak value of about 2.25 and then become slightly decreased to reach the field of stability at different fractional-order and fractal dimension. The declines and stability lie in the infection because of the self-protection terms included in the model. As the Infection increases the recovery also increases with the passage of time and reaches up to the maximum value of 1.7 and then becomes stable as shown in the right

panel of Fig. 5. We can also see in all the simulations that increasing the fractional-order values will converges to the integer-order values 1. In the left panel of Fig. 6 is the simulation death occurs due to the covid-19 which also reached to its peak value of 0.01 and the become declines towards stability at different fractal dimension and fractional order. The decrease and stability occur in the death cases because of declines in infection and other precautionary measures for clean environment which declines the contaminated material  $M(t)$  as shown in the right panel of Fig. 3.

Figs. 4–6 are the simulation for data-II showing the similar situation as for data-I. The simulation of data-II shows more infection and more recovery as compared to data-I.

## Conclusion

In this manuscript, the dynamical behavior of fractal-fractional modified non-linear SEIR type system has been discussed having non-integer order and fractal dimension under ABC derivative. The investigation of the model shown that how the covid-19 infection may be controlled and stabilized. The Qualitative analysis has been achieved with the help of some well-known theorems of non-linear functional analysis. The simulation of the model by fractal-fractional Adams–Bashforth techniques have been shown the dynamical behavior for each compartment of both sets of data at different fractional-order and fractal dimension. The graphical representation of the consider model gives the whole density of each compartment lying between two different integer values. The memory time for stability and convergency is short for low fraction order and fractal dimension. From the study we recommend for every individual to change their behavior for facing covid-19 by following WHO SOP's such as keeping face masks, keeping social distance, washing their hands with soap and an alcohol-based sanitizer, and disinfecting the surface. These mechanisms will decline and stop the spread of the virus from the contaminated environment to uninfected individuals and from infected individuals to the surrounding. It is also essential to create awareness and disseminate information for societies to keep themselves from the virus, which can reduce the pandemic threshold of the infections. As a whole, we conclude that the manuscript deals with the covid-19 situation in Pakistan at any stage for both sets of data at different fractional-order and fractal dimensions.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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