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RESEARCH

The Imperfect Geometries of the Basilica S. Peter and Paul in Casalvecchio, Messina

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Abstract
The theme of this article is twofold: an instrumental survey, which increases the documentation of the Basilica Saints Peter and Paul in Casalvecchio, is accompanied by a reflection on the underlying geometries. The latter allowed some analysis on the distance between its theoretical form and realization, but also on the way of designing, tracing and constructing the church itself. The study, therefore, outlines a protocol for the morphological analysis of the Arab-Norman architecture built in north-eastern Sicily, and identifies the Basilica Saints Peter and Paul as a case study. The results of this research increase the available documentation of the building and support future enhancement and conservation actions. The protocol provides for: brief historical contextualization; instrumental survey; geometric and morphological analysis aimed at identifying: geometric patterns, units of measurement and proportional modules.

Keywords Design analysis · Free-handed vault construction · Practical geometry · XI century domes

Introduction

The research illustrates the case study of the church of Saints Peter and Paul in Casalvecchio, Messina. This building belongs to the so-called "Arab-Norman" architecture erected between 1090 and 1117 on the Ionian coast of eastern Sicily. The latter, characterized by modest dimensions and defensive functions, offer unusual formal solutions that reflect the geometric knowledge of the Arab workers of the time. In fact, in the Mediterranean basin, the Arab scholars of the eleventh century represented the most advanced point of mathematical knowledge (Raynaud 2012). They had incorporated the cultural heritage of ancient Greece and had

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developed its applicative potential. The profound knowledge of these themes reverberates in the architecture of Arabic origin in which is shown all the abstract beauty of geometry. However, it must be emphasized that the workers had access to exemplified, and often approximated, concepts described in the "practical geometry" manuals of the time. Among the few that have come down to us are that of Abū 'l-Wafā' al-Būzǧānī, *Fī mā yaḥtāǧu al-ṣāni' min al-a'māl al-handasiyya* (*About the geometric constructions indispensable to the craftsman*).[1] The descriptions of the tools used by the craftsmen and the strategies for verifying their correctness immediately give a sense of how approximate and imperfect the building tracing operations were.

The aim of the present research is to identify, with a deductive approach, what were the "ingenious procedures" (Bellosta 2002) that the planners and executors of the Basilica of Saints Peter and Paul used during the construction. Such as the drawing of the octagonal plan of the main dome achieved with only one circle as suggest by al-Būzǧānī (Calzolani 2017). The instrumental survey described in this paper, despite the limitations inherent to the Laser Scanner and the procedures, returns a virtual model very similar to the real building. But if we want to analyse the hypotheses of the original layout, the problem is complex, since the architecture in question presents various metric-angular irregularities, deriving not only from the inexperience in construction and from the measuring and tracing instruments, but also from factors that have taken over, at a later stage, such as conservation conditions, restorations or structural failures.

Is it possible then, to make assumptions if the building does not correspond to the presumed regularity of the project? The virtual model, which supports the traceability hypotheses, must take into account the irregularities and mediate the data obtained from the instrumental survey. The geometry that returns the theoretical form of the artefact formulates plausible hypotheses, consistent with the state of things and the history of the building.

The results of this research bring us closer to a deep understanding of Arab-Norman architecture and the cultural tradition that generated it. Furthermore, even if the Arab-Norman architecture has been studied for a long time from a historical and humanistic[2] point of view, currently there are few researches that try to verify the intuitions of past scholars with the aid of the most advanced survey methods.

[1] "First of all, the line for drawings and short strokes, because the longer lines were drawn using a taut string covered with a black patina; it then shows how to check if a line is straight, both by eye and by turning it over and verifying that the two traces obtained overlap. Followed by the compass (possibly equipped with a nut to fix the opening) and the wheel compass to draw large circles; then the square, for which it indicates different methods of construction and verification, based on the construction of the axis of a segment with the ruler and the compass, on the fact that the centre of the circle circumscribed by a right triangle is the midpoint of the hypotenuse, or that the median is half of the hypotenuse. Among these methods, the one he attributes to the craftsmen and which seems to have been in use since ancient times, is based on the Pythagorean theorem (…) He warns against the method (…), used by certain craftsmen, who to construct a regular polygon inscribed in a circle proceed by successive attempts with a compass (…). Regular polygons circumscribed to a circle are then constructed starting from inscribed polygons (…). Al-Būzǧānī presents different constructions of some of these polygons, some made with a fixed aperture compass (square, pentagon, octagon, decagon)." (Bellosta 2002).

[2] The abandoned church was rediscovered in the 1960s and became the subject of numerous studies. In this regard, see: Freshfield (1918), Bottari (1927), Calandra 2016), Bottari (1939), Basile (1975)

Methodology

The Arab-Norman architecture of eastern Sicily consist of small buildings, often designed to defend the territory. They were built by local workers, the Maghrebi Muslims, who have been residing in Sicily for over a century. The quality of the constructions, in terms of metric and formal precision, as many scholars point out, is poor. The cause of the imperfections certainly lies in the Eastern Byzantine tradition which, over the centuries, disregarded good Roman construction practices by favouring the decorative apparatus. In addition, the small buildings, located on the outskirts of the Empire, were built by local workers with little experience and simple tools.

The imperfections in the buildings surveyed in this area (including the case study of Saints Peter and Paul illustrated herein) trivially concern conspicuous imperfections in the tracing of right angles or in the sizing of the sides of the buildings. The Arab-Norman architecture owes the complexity of the architectural solutions to the Maghrebi matrix. In fact, in these buildings the ornamental component is not entrusted to stucco finishes or precious marble claddings but is inherent in the very structure of the building. Both the system of connections between the drums and the domes and the decoration with intertwined arches of the side facades have a decorative and structural function and are affected by the imperfections of the tracing. In this scenario, the instrumental survey is therefore essential in order to know the exact morphology of the building and to put forward hypotheses on the layout, on the proportional ratios and on the modules used.

The protocol therefore provides for two types of activities. The first through instrumental surveys, whose precision specifications will be illustrated below, documents the actual state of the asset and the actual morphology. The second, starting from the instrumental survey, analyses what has already been defined as “imperfect geometry” in the title of this intervention. In fact, the challenge is to interpret what was the geometric layout, or rather the theoretical shape of the building. The metric precision is entrusted to the survey while the geometric layout can only be approximated given the irregularity in the construction of the building and in the tools used for the layout. The strategy to identify the underlying geometries is conducted with a deductive procedure. The hypotheses formulated are verified on the basis of greater or lesser adherence to the “digital twin” obtained with the instrumental survey.

Historical Contextualization

The sacred buildings erected between the eleventh and the thirteenth centuries in the north-eastern cusp of Sicily have very similar dimensional and formal characteristics. The historical context in which they arise sees the coexistence of the Greek-Eastern culture, linked to the Byzantine tradition and the migratory flows of monks from Asia Minor; and the Islamic one, notably Aghlabid and then Fatimid. The Normans, who had just settled in Sicily, made use of local Arab and Byzantine workers, in possession of good building techniques, to rebuild the monasteries of the oriental rite now in decline. The monasteries were a territorial garrison, a link between the administrative power and the local population.

The so-called Arab-Norman architecture, in this part of Sicily, is characterized by the modest size of the religious buildings, the basilica plan and the presence of domes with free extradoses. The basilica plan, with the triple apses and the straight transept on the sides, is due to the choices of the monastic communities of the Eastern rite to which the structures were intended (Basile 1975: 46). The wall unit is characterized by the marked polychromies, the cloisonné construction technique, and the presence of intertwined arches. This decorative theme, of clear Arabic origin (Basile 1975: 46), blends, through the cloisonné technique, with oriental influences. Finally, extradoses of the domes are an explicit reference to the architectural language of the Islamic Fatimid mosques.

The Basilica of Saints Peter and Paul, from 1117, the last in chronological order to be built[3] in this area, summarizes the formal expectations that these three cultures have been able to express in the territory. The original church, built around 560, was completely destroyed by the Arabs and rebuilt in 1117. This date is certified by the Donation Act of Ruggiero II, dated 1116, written in Greek, preserved in the Vatican Code 8201, and translated into Latin in 1478.[4] The territory on which the Basilica lies is strongly characterized by the particular ground conditions of this mountainous region. The Peloritani Mountains descend sharply towards the coast line and are furrowed by deep streams that carve the slopes and give shape to the landscape. The rivers represent the ideal route both to connect the internal areas with the coast and to hide from enemy incursions. They are, therefore, the ideal place for the construction of the small monastic settlements of the Byzantine era (Fig. 1). In fact, the monastery, for centuries, was the fulcrum of the economic and religious life of the area. In 1794, due to adverse environmental conditions, the Agrò stream was infected by the miasma produced by intensive flax cultivation, and the abbey seat of the monastery was transferred to Messina.

Instrumental Survey

The tracing of the Arab-Norman buildings, as well as those of the Byzantine matrix, shows more marked “imperfections” than Western architecture. The instrumental survey undertaken in this paper is therefore an obligatory step to knowing the exact morphology of the work and to appreciate the distance between the conceptual model and physical realization. The survey of the Basilica was carried out in May 2018 with a Faro Focus 3D laser, 17 external and 18 internal scans were performed. The combined standard uncertainty of the instrument is $U_{cRanging} = 0.496$ mm. The individual scans were aligned, creating two separate groups: “internal” and “external”, the two groups were linked later. The ‘average tensions’ in the “internal” group are equal to 2.2704 mm while for the “external” group they are equal to

[3] The Arab-Norman basilicas built in north-eastern Sicily are in chronological order: S. Filippo in Demenna 1090, S. Maria Annunziata in Mili 1092, Basilica Saints Peter and Paul in Itala, 1092.

[4] From the Act of Donation, it can be deduced that Count Ruggiero II, during a trip near Sant’Alessio, was approached by the Basilian monk Gerasimo who asked for, and obtained, the resources to rebuild the monastery located in “fluvio Agrilea”.

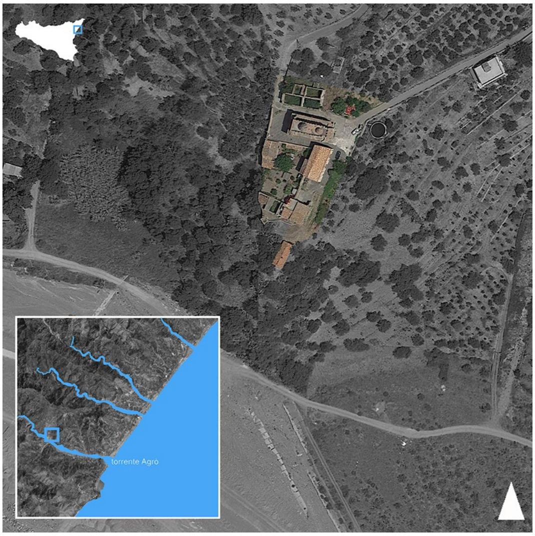

Fig. 1 Location of the Basilica of S. Peter and Paul in Casalvecchio, Messina

1.9922 mm. The 'average tension' in the alignment between the two groups is equal to 6.3607 mm.

The precision of the data relating to the instrumental survey is therefore equal to ± 6361 mm but the measurements reported in the restitution of the survey have a greater variability whose value depends, not only on the precision of the instrument used for the surveys, and from the detection methods (the position and number of scans carried out, the setting relating to the repetition of measurements and the number of points measured for each single scan) but above all for the state of conservation of the building and the materials used for construction. In fact, the church walls are not plastered, they are made of roughly hewn stone (lava stone which is very difficult to work), bricks and white sandstone (a stone that is easy to model but also very sensitive to wear and tear). The roughness that these materials create on the ideal surface of the walls can, certainly, generate an uncertainty of ± 10 mm.

It is important to underline that the tolerance in the tracing hypotheses must be wider than that allowed for the instrumental survey. In fact, the irregularities in the morphology of the building are conspicuous and vary according to the size. The two short sides of the church, measured inside the building, have a difference of 490 mm. In the span of the minor dome, the smaller sides differ by 30 mm and those greater by 60 mm. The tolerance of the correspondence between the tracing hypotheses and the data obtained from the instrumental survey will be highlighted, from time to time, taking into account the irregularities present in the portion of the building analysed.

The hypotheses of geometric analysis are focused on the elements that most characterize the building: the plan, longitudinal fronts, domes and related connections with the spans. The point clouds were imported into the CAD program so the morphologic complexity was analysed with surface and volumetric vector modelling (Figs. 2, 3).

Geometric Analysis

The Regulatory Layout of the Plan

The fortified Basilica Saints Peter and Paul,[5] facing east, has an unusual layout made up of typically oriental "pieces", *bema, naos* and *esonarthex* arranged to form a Latin layout. In the apse area the bema, flanked by *prothesis* and *diakonikon*, is marked by a small dome. The central body of the Basilica, divided into nine bays by the four columns that support the main dome, follows the typological arrangement of the naos. The access to the Basilica, to the west, is through the exonarthex. The latter, covered by a groin vault, is flanked laterally by two scalar towers probably completed by small domes (Calandra 2016: 38–39). The Basilica has a flat roof that houses a walkway protected with battlements.

The longitudinal axis of symmetry, essential for tracing the entire work, is not immediately identifiable due to the metric-angular alterations of the plan. To identify the axes of the Basilica, the construction of a geometric axis orthogonal to the south facade was first hypothesized. Tracing the axes that should have identified the position of the centre of the columns and the centre line of the central nave parallel to the south facade, we note that both the columns and the pillars of the bema appear not to be aligned (Fig. 4a). If, on the other hand, the main axis of the Basilica is placed orthogonally to the apse wall, the line that joins the centre of the columns on the south side is still parallel to the main axis, while the one that defines the alignment of the columns on the north side diverges (Fig. 4b).

By designating the main axis of the church as the conjunction of the midpoints of the apsidal wall and the main facade, it is noted that the internal spans have their midpoints coinciding with the axis just defined (Fig. 4c). It follows that it may be useful to exemplify the entire plan of the Basilica with quadrilaterals, very close to rectangles, which contain the perimeter walls (green), excluding the thickness of the walls (red) or the entire exonarthex (blue). The geometric figures obtained are gradually more regular (Fig. 4d).

For example, the internal space of the church is defined by a quadrilateral (the corners are not perfectly orthogonal) which has four sides of different dimensions (9.59 m, 15.03 m, 9.08 m, 15.10 m respectively). It is clear that the shape of the quadrilateral tends to a rectangle since the difference between the two long sides is only 70 mm while the short sides differ by 310 mm. The ratio between two adjacent sides oscillates between 1.57 (15.03 m/9.59 m) and 1.66 (15.1 m/9.08 m) If we assume that the theoretical shape of this quadrilateral is actually a rectangle then we must imagine angles at 90° and sides equal to two by two. To obtain an average measurement we can evaluate the medians of the quadrilateral corresponding to 9.34 m and 15.06 m. The median ratio is 1.612, very close to the golden ratio (Zanetto 2017: 152). This value leads us to make other considerations: we can use the two medians to verify if they are a multiple of the same measure which, in this case, would be the Sicilian Byzantine

[5] The average dimensions of the interior of the Basilica (15.03 m × 9.44 m), excluding the exonarthex, are proportional to those of the Basilica Saint. Peter and Paul in Itala (17 × 10.5 m) just a few kilometres away.

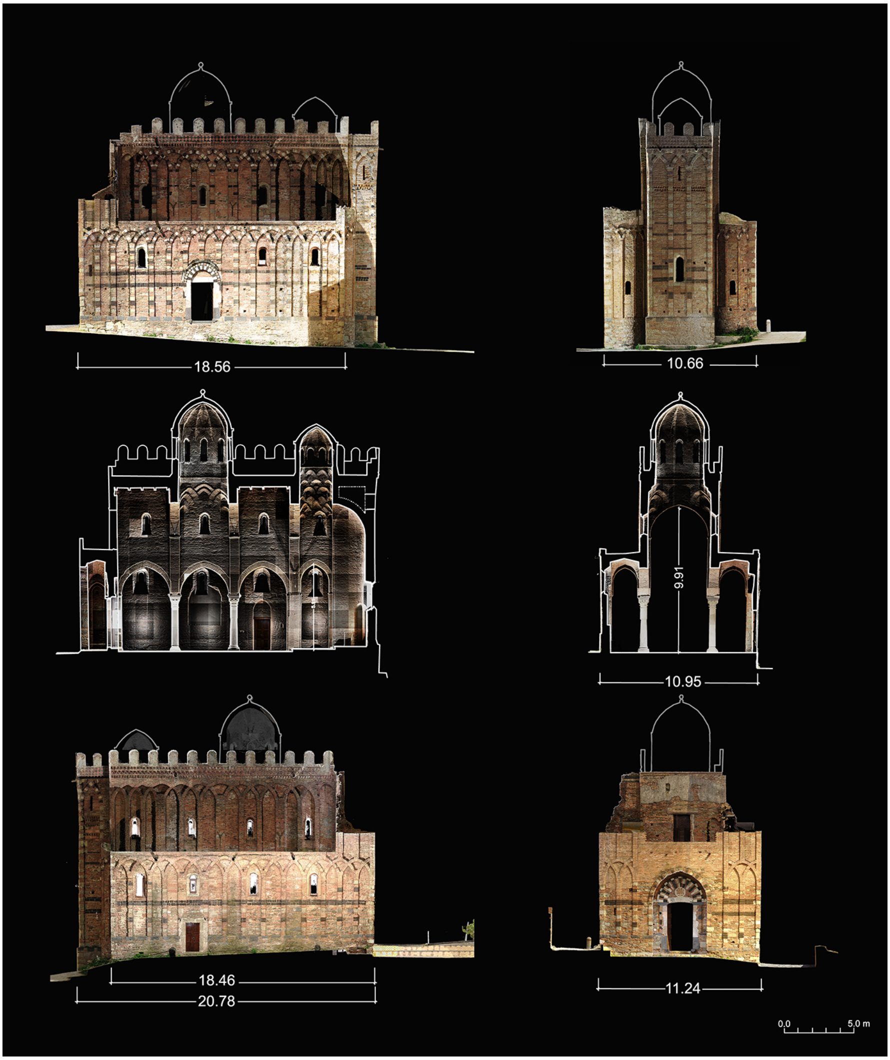

Fig. 2 Restitution of the instrumental survey and orthophotos: external elevations and sections

foot that oscillates in this area between 29 and 32 cm.[6] The measure that divides the medians into 30 and 48 units respectively (with a difference of 4 cm on the minor axis and 6 cm on the major axis) is equal to 31.25 cm, i.e. the Byzantine foot which is assumed to be the unit of measurement of this building (Fig. 4e).

[6] Pous (Foot) poûs (ποῦς) = 31.23 cm. Derived from the ancient Greek foot, the standard foot length in Byzantium seems to have been 31.23 cm, but in practice the length fluctuated between 30.8 and 32.0 cm.

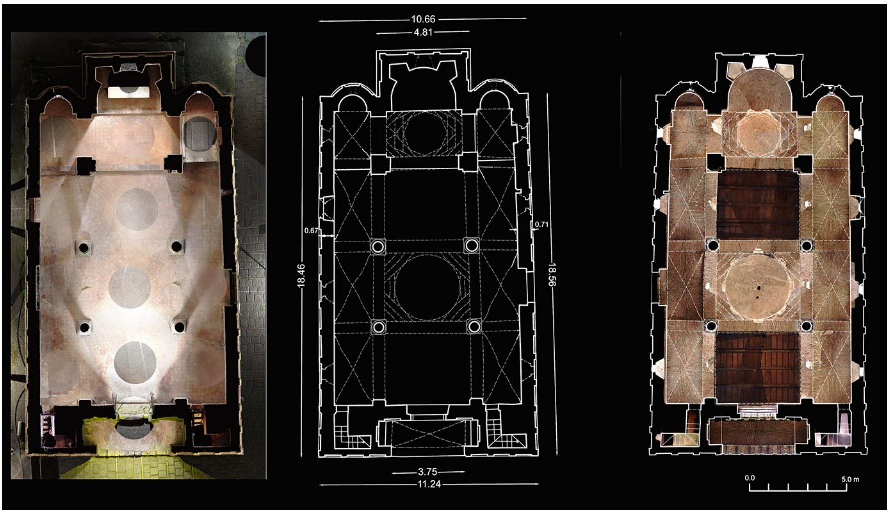

Fig. 3 Restitution of the instrumental survey: the orthophotos of the plan; the drawing of the plan; and the ortophoto of the reflected celling plan of the Basilica of S. Peter and Paul

In the quadrilateral,[7] that defines the interior of the Basilica deprived of the exonarthex and the apsidal part (naos), the adjacent sides have a ratio close to 4:3 (12.03 m/9.27 m = 1.3; while 12.09 m/9.59 m = 1.26). It can therefore be assumed that the space was traced with the knot rope technique that makes use of Pythagorean triangles (Fig. 5). By connecting the midpoint of the sides of the room two by two, 4 golden triangles are obtained, whose hypotenuses form an almost regular rhombus, with a side equal to 7.67 m (± 20 mm), formed by 5 modules of 1.53 m (the difference between the ideal Pythagorean triangle and those found in the church is 10 cm on catheti, equal to 1.5% of the length) (Fig. 6). By connecting the points that mark the 2/5 of the hypotenuse of the golden triangles the quadrilateral obtained contains, with a minimum deviation (10 cm in the South-Est base, 5 cm in the North-Est, and nothing in the others),[8] the bases of the columns. Figure 6c shows the ideal path drawn using perfectly orthogonal angles and homogeneous modules.

Given the precision of the dimension of the central rhombus (Fig. 6b), it can be assumed that the plan of the Basilica was traced starting from a single module equal to about 5 Byzantine feet, the so-called "double pace", whose length here oscillates between 1.51 m (on the greater cathetus) and 1.56 m (on the minor cathetus) probably due to the elasticity of the rope itself. The master of the work may have

[7] The dimensions of this compartment were identified by making a section a few centimetres from the height of the internal floor. In this way, is obtained a plan as close as possible to the building tracing plan.

[8] If the room was a rectangle, with perfect Pythagorean proportions, the circumference built on the base, diameter equal to 3 modules, and the circumference built on the midpoint of the major side, diameter equal to 2 modules, would be tangent exactly on the 2/5 of the segment that connects the centres of the aforementioned circumferences (Fig. 6c). In addition, this scheme also offers the perfect sizing of the base of the columns, very regular in shape and size. The plinth has the side equal to the radius of the circumference having a diameter equal to a Pace (Single) (βῆμα ἁπλοῦν) 2 + 1⁄2 Pous = 78.125 cm.

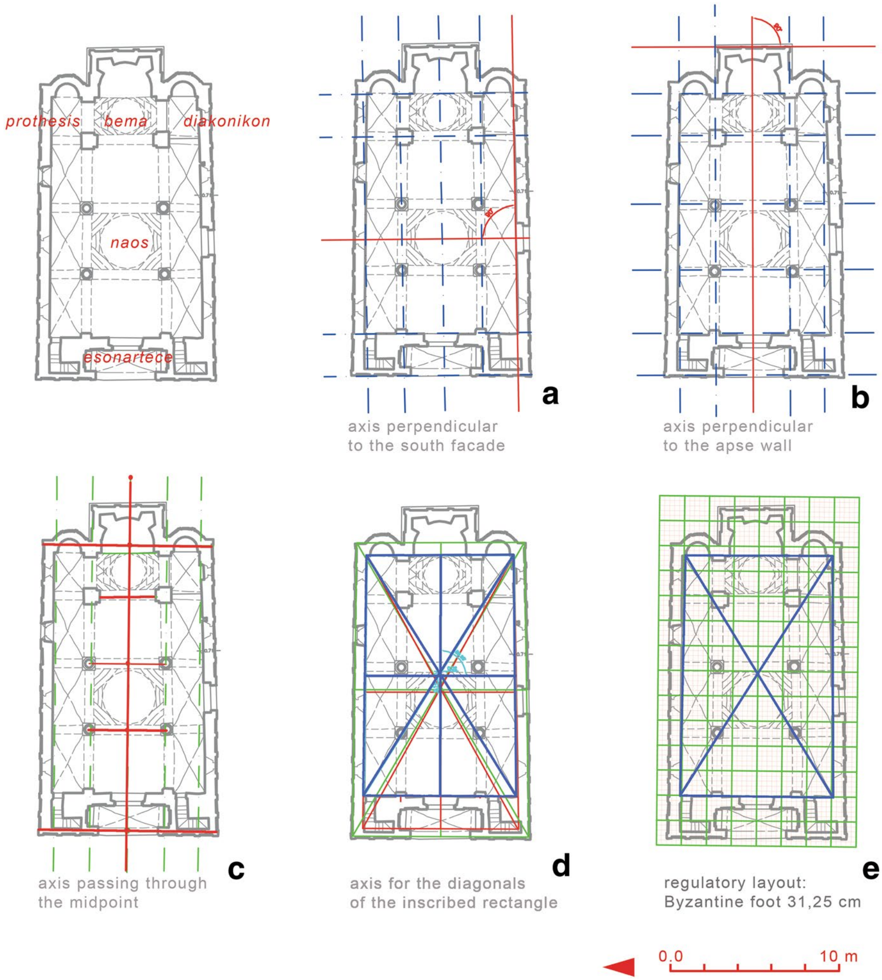

Fig. 4 Analysis of the geometries underlying the plan of the Basilica of S. Peter and Paul

traced the longitudinal axis, and on this he would have built the four Pythagorean triangles having a "double pace" as the module. From this figure, the green rhombus, all the geometry of the plan comes out.

The Wall Textures of the South and North Sides

On the outside, the basilica looks like a compact and stereometric volume, the only exception is the curved profile of the two side apses. The main facade, partly in ruins, has undergone considerable alterations. The portal, decorated with a pointed arch, is protected by the exonarthex. The rear façade, imposing and still intact, is divided into three parts: the central one, decorated with two crossed arches,

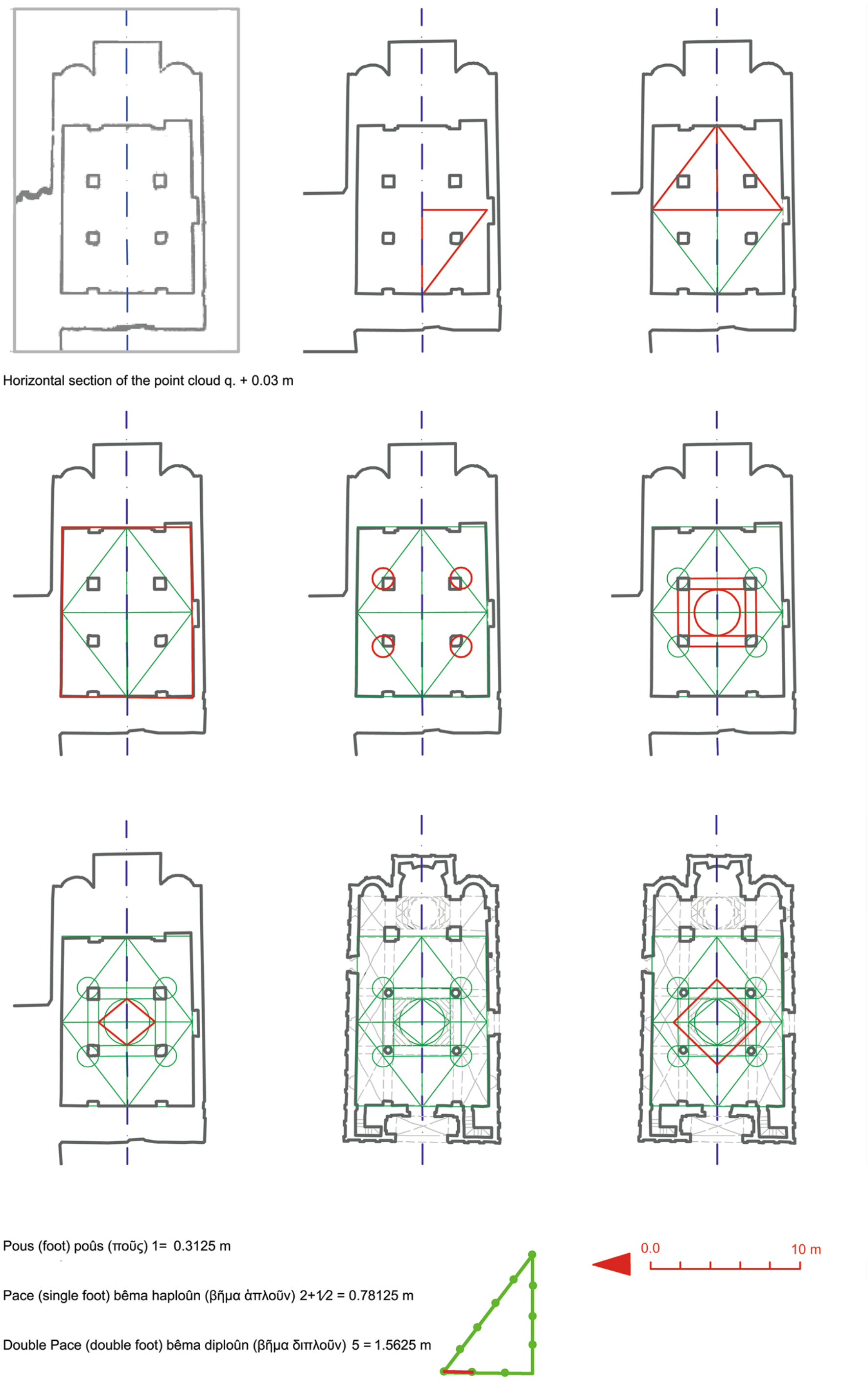

Fig. 5 Phases of the tracing of the Basilica

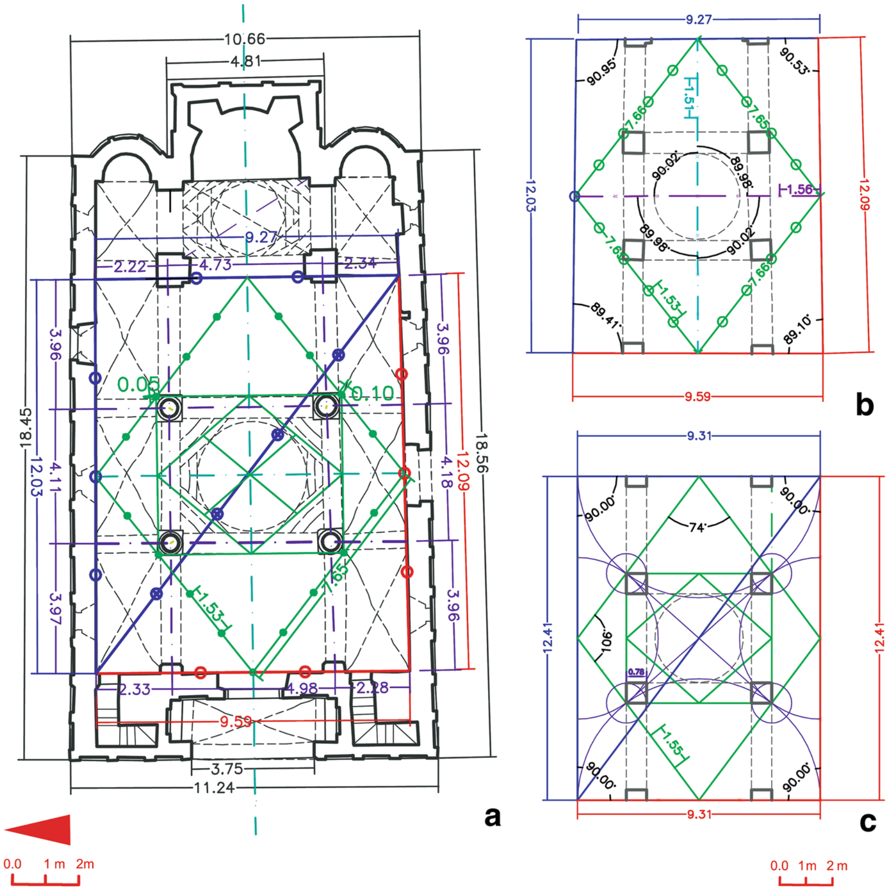

Fig. 6 **a** verification of the tracing of the Basilica; **b** regulator layout: central rhombus with sides of exactly 5 modules (153 cm); **c** the ideal path drawn using perfectly orthogonal angles and homogeneous modules (153 cm)

reaches 15 m and ends with large battlements; the two curved sides, decorated with intertwined blind arches, have different springing heights and are lacking in of the original crowning. The sides of the Basilica show the great difference in height between the central nave and the side aisles on the outside and appear horizontally divided into two parts.

The masonry is characterized by the contemporary use of brick[9] and rough stone (lava pumice, white limestone, yellow sandstone and Taormina stone). The

[9] "Bricks and tiles (…) were produced on site in kilns prepared for the occasion: this is testified by the fact that the ceramic body of the material is given by the firing, not always homogeneous and controlled, of local clays available in the surrounding area and that the sandy skeleton is given by sands with rounded edges taken from the shores of the nearest rivers and by fragments of recycled bricks" (Mamì 2008: 57).

construction technique, the cloisonné,[10] emphasizes the polychrome derived from the use of different materials and combines the arrangement of the bricks in stretcher, header and rowlock. The horizontal recesses, made with hewn blocks of lava pumice, emphasize large bands of bricks arranged in a herringbone pattern. All the external surfaces of the Basilica are decorated with intertwined arches that branch off from pilasters that are only 10 cm deep. The latter also have a structural function, to stiffen the walls and help to avoid peak load.[11]

The side elevations of the church have not undergone alterations and lend themselves well to a morphological and geometric study. The repetition of a single decorative element, the pointed arch, allows the verification of the formal and dimensional genesis of the wall partitions (Fig. 7).

The formal analysis of the elevations proceeds from the bottom up: the lava stone and brick base, perfectly horizontal, compensates for the difference in height present in the site and, from the south elevation, ideally continues on the right half of the main facade and on the rear one, wrapping the square wall of the apse. The base of the north facade, higher, cuts through the side door and shows, in the main facade, the difference in height between the two. The decorative partitions of the facade are simple pilasters crowned by intertwined pointed arches. The pilasters rest on the base and have a constant width of 33 cm with a very irregular rhythm. The irregularity could be mitigated, for example, by analysing the span of the arches that branch off from them. In this case the span to be taken into consideration is the sum of two bays plus the width of the central pilaster; even in this way, however, the measurements appear irregular and also trigger differences in the total number of arches present on the two longitudinal fronts. The first, facing south (18.56 m long) has two series of seven and a half arches; the second, north, (18.46 m long), has instead two series of seven arches. The arches of the north side are therefore, on average, wider than those on the south and, in some cases, they are round arches.

Both on the north and south sides, the span of the arches is irregular, ranging from a minimum of 1.88 m to a maximum of 2.32 m. Figure 7 identifies (in pink and blue) the arches that deviate most from the average size and shows how the irregularities of the design are not to be attributed, for example, to the position of the openings. In fact, the latter are not aligned with the internal bay but follow the order of the external fronts.

On the southern side of the building the span of the pilasters varies from a minimum of 77 cm to a maximum of 1.04 m; while on the north facade it is between 72 and 108 cm. The width of the pilasters, on the other hand, is constant, 330 mm, and is equal to the width of the bricks used in this building.[12]

By carefully observing the arrangement of the brick layer adjacent to the lava stone base, we can hypothesize that the irregularities are due to the arrangement of the bricks. In fact, among the most common measures, the one between 80 and 82 cm

[10] It is an oriental technique where the stone ashlars are horizontally and vertically interspersed with bricks; it literally means "partitioned" (Margani 2001).

[11] In the upper part of the sides, the pilasters double their width in correspondence with the internal support arches of the dome.

[12] The average size of the bricks found in this factory is 33 × 16.2x4.7 cm. (Todesco 2007: 160).

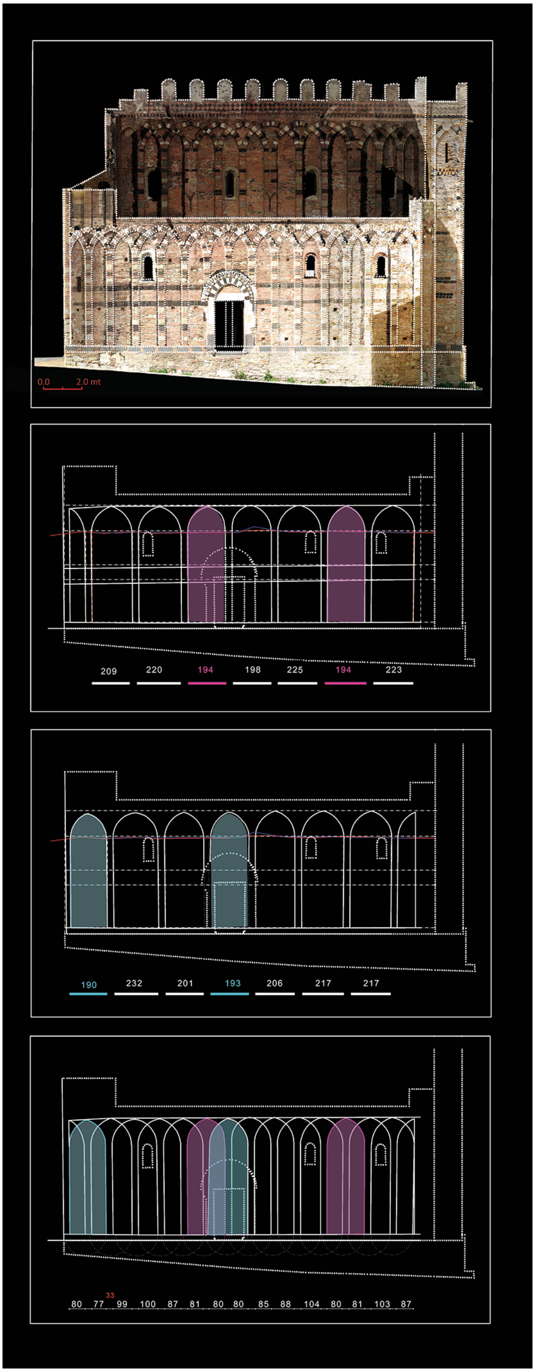

Fig. 7 South facade: analysis of the arches, identification of the underlying geometries

is obtained by placing two bricks arranged in stretcher mode and one in header; the one between 99 and 104 cm is made up of 3 bricks arranged in stretcher. The design of the two facades, determined by the rhythm of the blind arches, is therefore entrusted to the arrangement of the bricks. The resulting geometry emphasizes the arrangement of materials on the facade, giving lightness and complexity.

The Domes

The clear Arabic matrix of the domes of the Basilica is highlighted: by the modest dimensions; from the bricks arranged in successive rings of horizontal arches (also present in the semi-dome of the apse basin); from the system of fittings, squinches and trumpets, that connect the rectangular spans to the springing of the dome; from the extrados of the domes completely free; and finally, by the slender drum on which they rest. Furthermore, the Arabic term, used by historians who first analysed this type of architecture, refers here to the Muslim component present in Sicily at the time which was Fatimid, coming from nearby Tunisia. The formal language of the Arab masters of work in this part of Sicily was therefore inspired by the simplified and stereometric forms of Maghrebi mosques.[13] The construction techniques were instead orientated towards the simplicity of execution and the economy of the installation. Topics that were also well-known to the Byzantine workers, as shown by Choisy's studies on vaults without centering.[14] The theme of the constructions of vaults without ribs has recently been taken up by many scholars who, starting from instrumental surveys, have tried to identify the geometric matrix of complex vaults, taking up the thread of Choisy's considerations (Huerta 2009). Unfortunately, it is not possible to know the laying surface of the bricks as the dome is intact. Some authors argue that the modest-sized domes present in Sicily in this historical period, are actually pseudo-domes composed of projecting rings of bricks which are arranged on a tilting bed only in the terminal part. Others, such as Sanpaolesi (1971), propose that from a static point of view, the domes "support themselves as they grow, by virtue of their equipment and the shape of the whole. Because of this property they push much less or do not push at all on the supports and

[13] Many scholars have seen technical and formal correspondences between the miḥrāb dome of the Kairouan mosque and the central one of the Casalvecchio basilica. "The great dome of the miḥrāb has a "pumpkin" conformation, that is with convex and tapered segments (…) in addition to a decorative function the 24 ribs stiffen the structure, which becomes resistant in shape and its construction does not require the use of centering, as it is the result of the succession of concentric lobed rings, gradually smaller." (Antista 2016: 28).

[14] Choisy, argues that the construction of arches without centering was sought after by the Byzantines for the speed and cheapness of the results. (Choisy 1986: 62) The survey of the ribbed dome of the Church of the Holy Saviour in Chora, carried out during a restoration, allowed Choisy to identify, with certainty, the direction of the laying bed of the bricks. He notes that the generator of the laying surfaces does not converge at the centre of the dome but goes towards the opposite end of the base diameter. The overall inclination of the laying surfaces is therefore lower than that of the pushing domes and the bricks can be installed relying solely on the grip of the layers of mortar. It follows that the dome, built with concentric layers, is self-supporting.

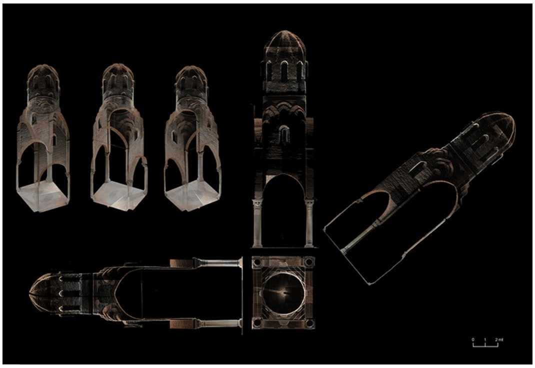

Fig. 8 Central dome, instrumental survey: plan, sections, and axonometric views

therefore this is an intrinsic and lasting virtue of their structure that distinguishes them as such" (Sanpaolesi 1971: 8–9). The presence of high drums confirms Sanpaolesi's hypotheses on self-supporting domes, as any lateral thrusts could not be compensated by the masonry of the drum which stops at the spring line of the dome itself.

The study on the vaults of the Basilica verifies the hypotheses advanced on the data coming from the instrumental survey. The analyses foresee the restitution of the survey, the search for regulating geometric paths and the verification of the latter on three-dimensional vector models (Fig. 8).

Main Dome

The main dome, barycentric to the naos of the Basilica, rests on a perfectly cylindrical drum and is connected with the rectangular span through four trumpets (Fig. 9). The vault, which is made up of 8 double-curved ribs, can be defined as a pleated dome (the terminology is not well defined, often the definition is ribbed or even pumpkin, or rather a "godron"[15]) and has a springing circumference with a radius of 1.68 m. The geometry of the dome seems to be affected by some imperfections due to construction techniques. The ribs are irregular both in the development of the profile and in the spatial orientation, however the vault appears well settled on the vertical plane.

Firstly, the dome springing plan was identified. This is marked by the presence of very small protruding corbels, only 2 cm deep, which hold the ridges of the coasts. By connecting each corbel with the diametrically opposite one, it can be verified that the edges of the ribs do not intersect each other in the geometric centre of the dome.

Near the windows, the vault tends to take the form of an "umbrella dome" even if, by making several horizontal sections (Fig. 10a), it can be seen that the profile

[15] "… just as the central dome […] slightly undulated in segments (like watermelon) is exceptional" (Calandra 2016: 41).

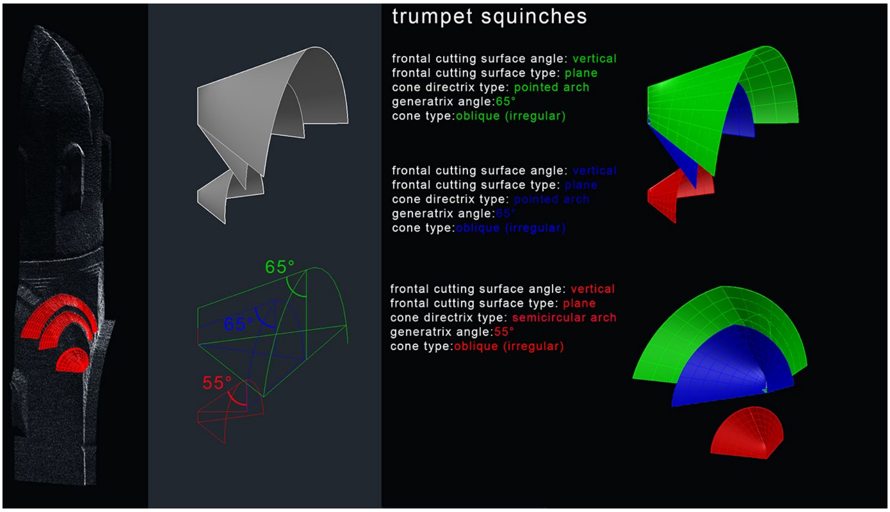

Fig. 9 Central dome, vector modelling of the connection system: trumpet squinches

of each rib never flattens completely, retaining an arrow of a few centimetres in the least concave point. The vertical faces, clearer in the apse dome, are here connected to the surfaces of the vault and prevent the immediate identification of the geometric structure.

To identify the construction techniques, we dissect one of the eight coasts with a bundle of planes belonging to the centre of the circumference that draws the profile of the coast itself. In this way we find many circumferences in which the radius decreases from the bottom to the vertex. By dissecting the dome, according to the horizontal bed joint planes of the bricks, a "flower" design is formed with gradually smaller, and less homogeneous, circumferences as one moves away from the springing plane (Fig. 10b, c).

The base octagon, inscribed in the circumference with a radius of 1.68 m (in green), is obtained by repeating a single circumference (0.91 m about 3 Byzantine feet, in red). In elevation, the edges of the ribs draw a pointed arch whose centres belong to the circumference in red. The latter also draws the basic octagon of the entire dome and generates the ideal profile. In fact, the arches that go beyond the green circle define the ideal shape of the dome in the plan. By dissecting the dome with a vertical plane, perpendicular to one of the sides of the octagon, we obtain an uncertain profile: the lower part is almost straight while the upper profile adheres to a circumference with a radius of 2.6 m. By carrying out repeated vertical sections, we obtain profiles similar to circumferences with a radius of 2.60 m (±50 mm) having non-coincident centres. Despite the irregularity, the circumferences seem to respond to a rule: their centres belong to an arch, marked in blue, inclined. The morphology of the dome, deduced from the numerous horizontal and vertical sections conducted on the cloud of points, does not immediately express an elementary geometric matrix. The question is: how was it made using simple tools (Fig. 11)? For this reason, the use of a mobile centering resting on the triangle in red,

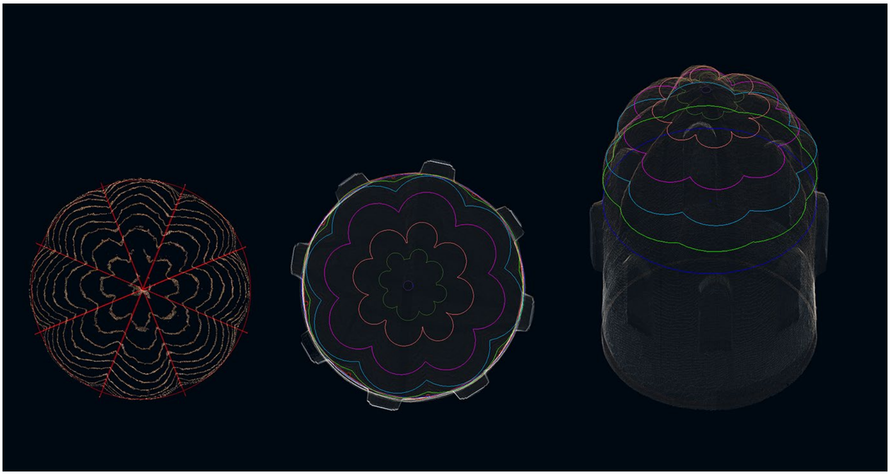

Fig. 10 Central dome: morphological analysis

was hypothesized. It, like a three-dimensional mainsail ("randa"), allows the dome to be built without ribs while controlling its geometry, albeit in an approximate way. The rib, hanging from the rod can rotate and oscillate.

The construction of the dome could have taken place like this: fixing a rod at the centre of the springing circumference; an isosceles triangle BAC is built, with sides AB and BC equal to 2.60 m (about 8 Byzantine feet) and the side AC equal to 2.23 m (the size of this side was determined by the discontinuity of the profile along the median section of the sail); a shape is constructed, fixed on the side AC, so as to form an arc of circumference having the centre at B and the radius BA; the vertex A of the triangle is fixed at the top of the vertical rod.

The triangle that holds the centering can rotate around the vertical rod and tilt to the horizontal plane. If we put the vertex B in the centres of the edges of the coasts, belonging to the springing plane, the CB side has an inclination of about 17° degrees to the springing plane. If point B is moved to the centre of the circumference that medially defines the sail, the CB side has an inclination of about 20° degrees and is positioned below the springing plane. Repeating the operation for radial, vertical planes, which divide a coast into eight parts, we will notice that point B describes, with some approximation (±30 mm), an arc of circumference with a radius of 0.91 m, inclined by about 30°. Basically, the geometry of each segment is defined by a circumference with a radius of 2.60 m that rotates around the axis of the dome and has its centre on an inclined arc that ends in the centres of the ridges. This small dome, thanks to a single mobile centering and the use of successive rings of horizontal arches, *opus isodomum*, could have been built, with some imperfections, without the use of centering in a simple and empirical way.

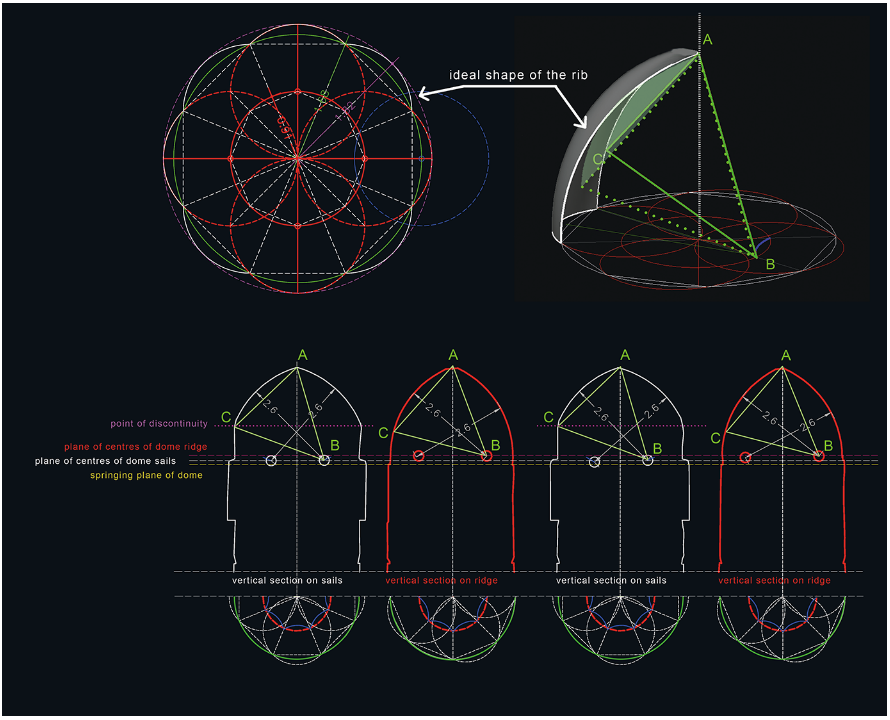

Fig. 11 Central dome: verification of the hypothesis of tracing

The Minor Dome

The span of the presbytery dome is covered by an "umbrella dome" on an octagonal base, illuminated by small pointed windows. The connection between the springing and the horizontal extension of the nave is unique in eastern Sicily (Fig. 12). In many studies, the elaborate solution adopted has been compared to muqarnas[16] or to a "system of pendentive and hanging alveoli" (Basile 1975: 27). In the present research, aided by an instrumental survey and by the orthophotos obtained, an attempt was made to recognize the geometric matrix underlying the construction of the complex connecting structure between the dome and the span. The span of the apse-dome has an extension of 3.97 m by 2.32 m and is defined at the base by four pointed arches.

Several hypotheses have been formulated by dissecting the connection system with horizontal planes but the imperfections of the building make it difficult to recognize the geometric matrix. The vector reconstruction of the morphology of the trumpets, verified by the point cloud in the CAD environment, provided valuable information about the configuration of the arches of the trumpets (Figs. 13, 14).

[16] "… the smaller dome rests on angular stalagmite solutions that extend only on two sides below to pass from the upper octagon to the lower rectangle." (Calandra 2016: 41).

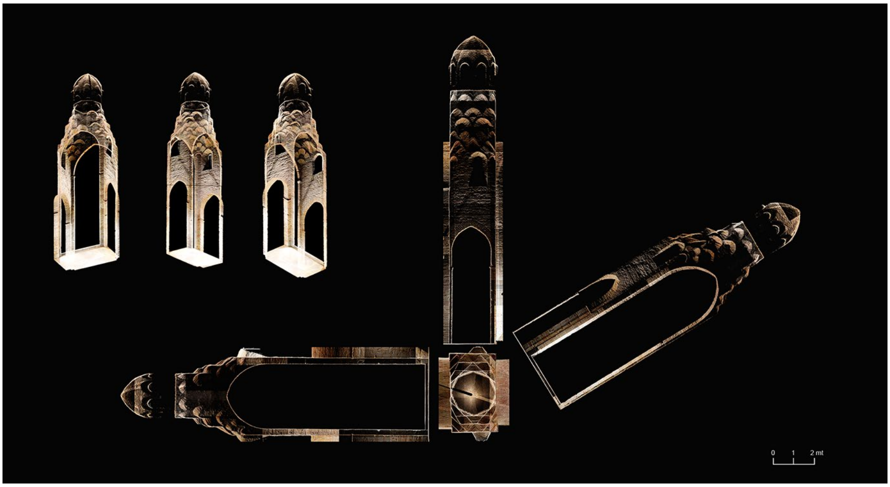

Fig. 12 Apsidal dome instrumental survey: plan, sections, and axonometric views

We made two hypotheses about the regulatory paths of the dome.[17] The first hypothesis originates from the base quadrilateral, very close to a rectangle (sides: 3.97 2.32; 3.91; 2.29 m, corners: 90.52°; 87.96°; 91.59°; 89.93°), and divides it according to the medians of the sides. Thus two axes, not orthogonal, are the basis of the whole geometric construction. The series of red and white squares increases the measurements of the side of the irrational number $\sqrt{2}$. The squares, obtained with the procedure just described, have slightly irregular sides and angles, nevertheless there is a perfect correspondence between the geometric pattern and the plan of the fittings. A small yellow circle underlines the correspondence between the drawing of the fittings detected and the hypothesized geometric layout. The tolerance accepted in this case is 20 mm (equal to the 0.5% of the major side). In the hypothesis in question, in the face of 30 intersections analysed, 27 fall within the hypothesized tolerance (Fig. 15a).

The second hypothesis overturns the procedure: the geometric layout originates from the quadrilateral perimeter. The square that surrounds quadrilateral is generated by overturning half of the sides of the base rectangle on the medians. This assumption allows a perfect geometric progression in the placement of the central square. The construction, effective with any rectangle with a relationship between the sides of 1: 3 (1: 7; 1:15 etc.) is less adherent to the survey than the first one. In fact, the octagon, defined by the two internal squares, red and blue, is 50 mm smaller than the real one and only 7 intersections on the 30 present have the hypothesized tolerance (Fig. 15b).

[17] See the entry *muqarba* (Bellosta 2002).

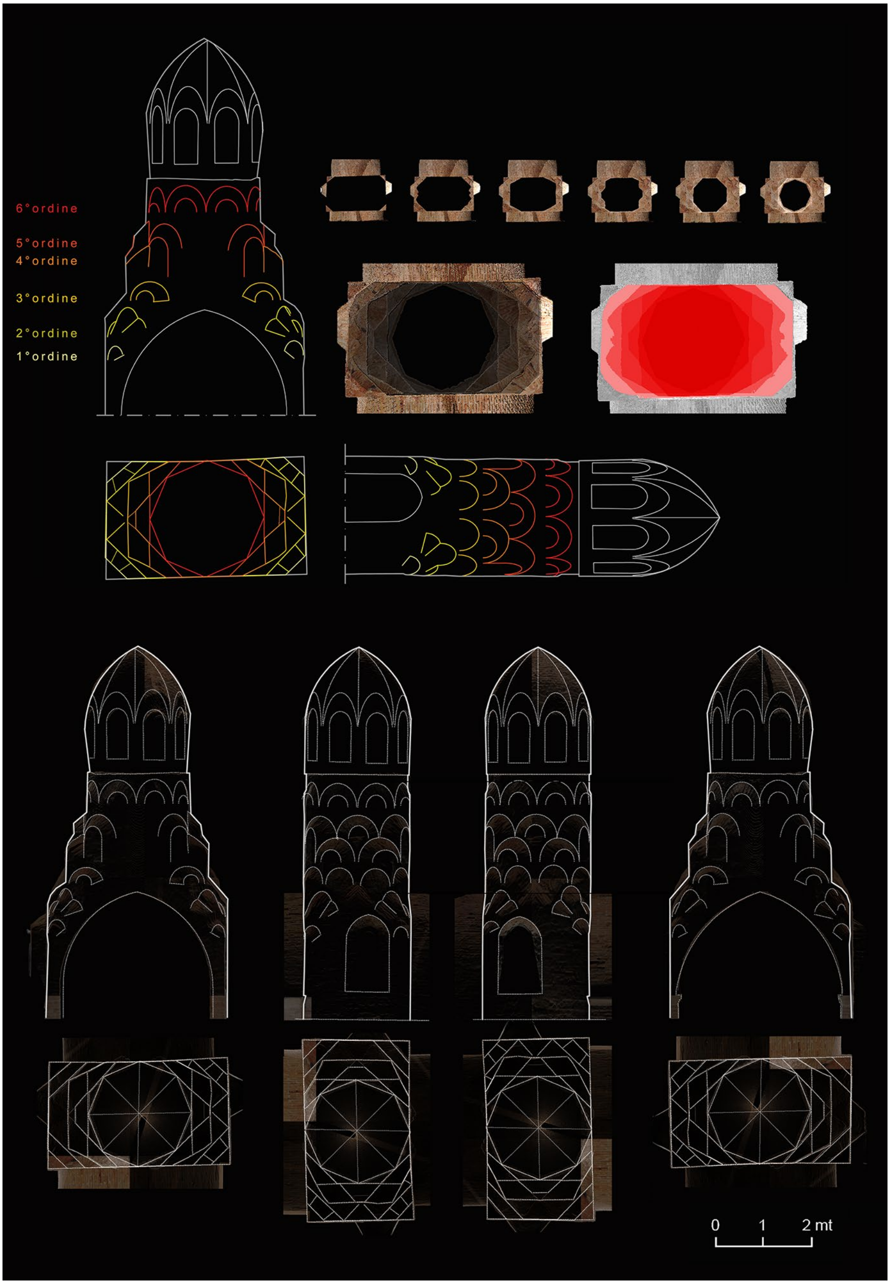

Fig. 13 Apsidal dome: drum span connection

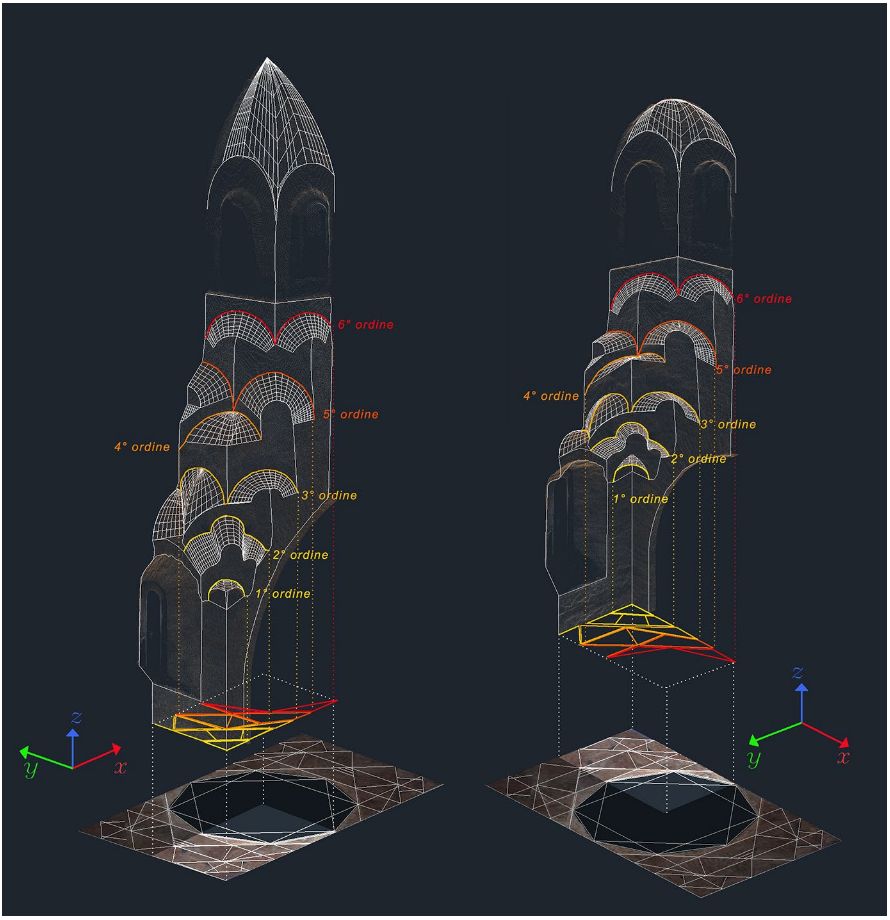

Fig. 14 Apsidal dome: vector modelling of the dome bay connection system

Conclusion

The tracing of buildings, as has often been reported by scholars, is typically carried out with simple tools such as ropes, pegs and rigid rods. The underlying geometry, while resting on scientific bases, is approximate and based on the use of modules, on the size of the sides, and on the use of midpoints.

The strategy adopted in this paper to identify the underlying geometries was to accept the existing "imperfections" and insert them in the hypothesized geometric patterns. In this way it was possible to verify some matches. For example, in the search for the regulating layout of the minor dome, the initial square built on the medians of the span, which are not orthogonal, is irregular as are all the figures that derive from it. In this way the correspondences and alignments of the whole path

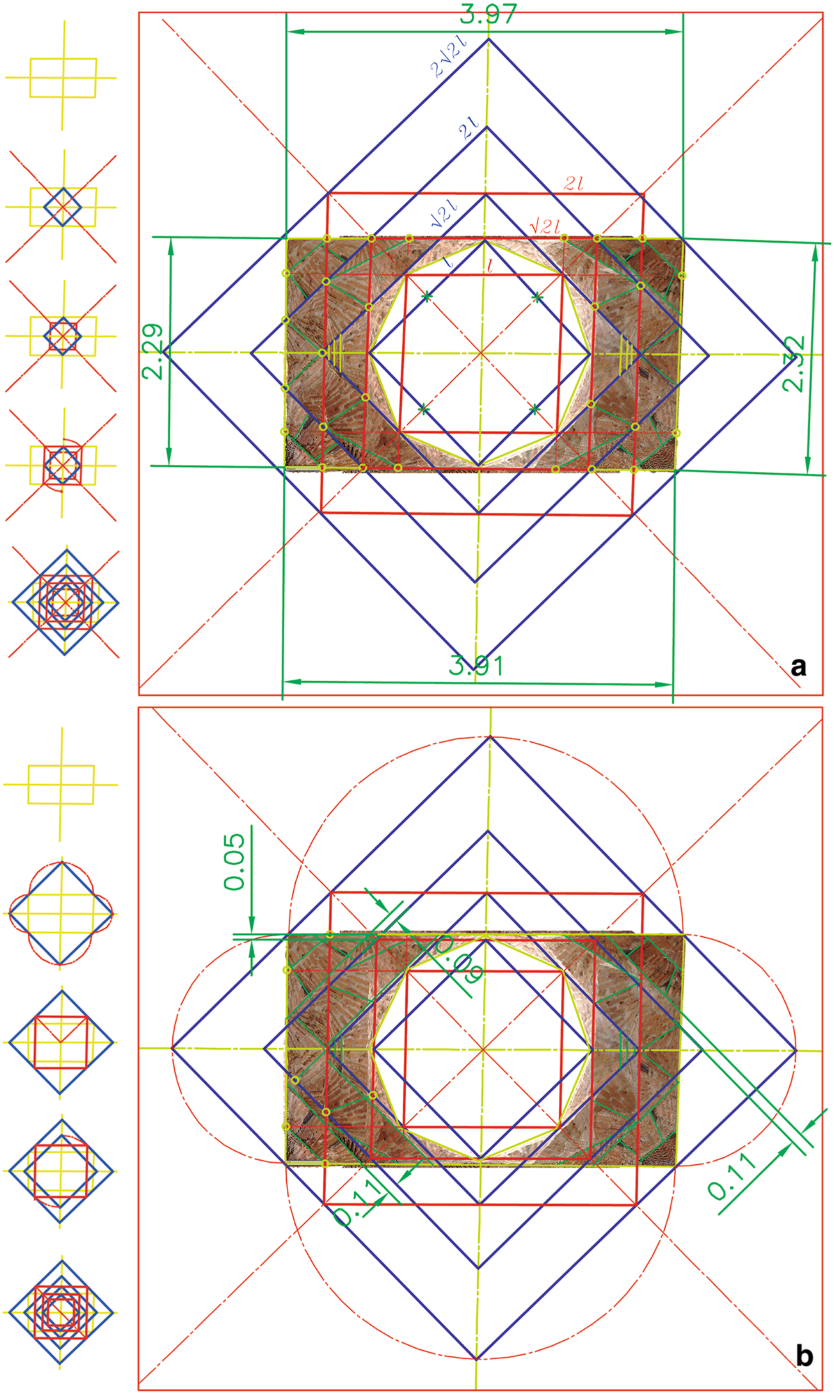

Fig. 15 Apsidal dome, hyposcopic view: underlying geometry analysis

are verified. In the main dome, the uncertain profile on the sails suggested a simple solution to trace a surface with a double curvature, with a single oscillating profile.

The layouts of this architecture originate from sophisticated and abstract mathematical concepts which, handed down over time and reworked by different cultures, have traded the *perfection of abstraction* for a *pragmatic imperfection*. The study, still in progress, brings a lot of information on the morphology and possible tracing of the Basilica, but at the same time offers ideas for further reflections. It would be interesting, for example, to analyse the formal and geometric correspondence of the other Arab-Norman churches present in the Ionian belt of eastern Sicily to identify similarities in morphology and tracing.

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References

Antista, Giuseppe. 2016. Le cupole in pietra d'età medievale nel mediterraneo (Sicilia e Maghreb), Palermo: Edizioni Caracol.

Arena, Marinella. Colistra, Daniele. 2018. The Church of Saints Peter and Paul in Itala (Messina). Survey, Graphic Analysis, Formal and Constructive Interpretation. In De-Sign Environment Landscape City_ 2018, ed. Giulia Pellegri, 113-124. Genova: GUP.

Basile, Ernesto. 1975. L'architettura della Sicilia normanna, Catania: Cavallotto.

Bellosta, Hélène. 2002. La civiltà islamica: antiche e nuove tradizioni in matematica. Geometria pratica. Storia della Scienza. Treccani. https://www.treccani.it/enciclopedia/la-civilta-islamica-antiche-e-nuove-tradizioni-in-matematica-geometria-pratica_%28Storia-della-Scienza%29/. Accessed 10 September 2021.

Bottari, Stefano. 1927. Nota sul Tempio normanno dei SS. Pietro e Paolo d'Agrò, Messina: Tip. D'Amico.

Bottari, Stefano. 1939. Chiese basiliane della Sicilia e della Calabria, Messina: Off. Graf. Principato.

Calandra, Enrico. 2016. Breve storia dell'architettura in Sicilia. Scicli: Edizioni di Storia e Studi Sociali.

Calzolani, Sergio. 2017. Il matematico Abu'l – Wafa al Buzjani http://www.geometriapratica.it/index.php/it/11-geometria-pratica/11-abu-l-wafa Accessed 25 January 2022.

Choisy, Auguste. 1883. L' art de bâtir chez les byzantins, Paris: Ed. Forni 1986. (I Ed. Paris: Librairie de la Societé Anonyme de Publications Périodiques,1883).

Freshfield, Edwin Hanson. 1918. Cellae trichorae and other christian antiquities..., vol. 2, London: Rixon & Arnold.

Huerta, Santiago. 2009. The Geometry and Construction of Byzantine vaults: the fundamental contribution of Auguste Choisy. In Auguste Choisy (1841-1909): L'architecture et l'art de bâtir, ed. J. Girón, S. Huerta, 289-305. Madrid: Instituto Juan de Herrera.

Mamì, Antonella. 2008. Le chiese basiliane della Sicilia orientale. Costruire in Laterizio 3: 56-61.

Margani, Giuseppe. 2001. Emergenze basiliane sulle pendici ioniche dei peloritani. In Naxos di Sicilia ed evidenze dai peloritani, ed. M. Costanza Lentini, 143-160. Bari: Edi Puglia.

Raynaud, Dominique. 2012. Abu al-Wafa' Latinus? A Study of Method. Historia Mathematica 39:34-83.

Sanpaolesi, Piero. 1971. Strutture a cupola autoportanti. Palladio III v.21:3-64.
Todesco, Fabio. 2007. Una proposta di metodo per il progetto di conservazione. La lettura stratigrafica della chiesa normanna di S. Maria presso Mili S. Pietro (Me), Roma: Gangemi Editore.
Wendland, David. 2004. Free-handed Vault Construction in the European Building Tradition: Vaulting Patterns in Half-Stone Vaults. Proceedings of the 13th international brick/block masonry conference, Amsterdam, July 4-7, 2004, eds. Dirk Martens and Ad Vermeltfoort, 011. Eindhoven: Technische Universiteit Eindhoven.
Zanetto, Serena. 2017. Tradizioni costruttive nell'Alto e Medio Adriatico (secoli VII-XI). Eredità e innovazione nell'alto medioevo, Firenze: All'insegna del Giglio.

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