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Research paper

An enhanced fuzzy IDOCRIW-COCOSO multi-attribute decision making algorithm for decisive electric vehicle battery recycling method

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ARTICLE INFO

Keywords: Electric vehicle Entropy CILOS CoCoSo q-rung picture fuzzy

ABSTRACT

An adaptation to electric mobility quickens waste management tasks for recyclers to end-to-end processing of marketed electric vehicle batteries. Especially lithium-ion batteries play a prominent role in electrifying the world for e-transport technology innovation. This research offers a multi-attribute decision-making (MADM) structure for finding the best performance e-vehicle recycling techniques. The structured algorithm combines an advanced stratified MADM strategy with e-transportation recycling techniques. The optimal algorithm evaluates the results of qualitative attributes and alternatives using a weighted-ranking MADM approach. The importance of attributes is calculated using a blending of dual objective-weighted approaches: entropy and CILOS methods, viz., the aggregated IDOCRIW approach. The ranking of alternatives is determined through the COCOSO method in a hesitation environment. The q-rung orthopair picture fuzzy set (q-ROPFS) is used to cope with uncertainty and vagueness in decision analysis. The feasibility and robustness of the suggested algorithm were validated through different MADM methods and by altering crucial ranking-dependent parameters in the problem.

1. Introduction

The world is targeting sustainable transportation due to the impact on greenhouse gas emissions and climate change. As per the IAE (International Energy Agency), nearly 37% of CO2 is emitted through fossil fuel vehicles, and 61.2% of world oil is consumed in the transportation sector. An overuse of fossil fuels is causing two main problems, namely the energy crisis and environmental problems. Fossil fuel is non-renewable, has no long-term usage, and creates environmental pollution as well as reducing air quality and increasing greenhouse gas emissions due to that emission of carbon dioxide. This motivated me to find a sustainable and green alternative for road transportation. The industries had focus on minimizing the consumption of fossil fuels by replacing non-fossil fuel vehicles. Non-fossil alternative fuels are liquefied petroleum gas (LPG), ethanol, methanol, biodiesel, propane, natural gas, and electricity. From these non-fossil fuel-based alternatives, electric vehicles (EVs) are operated by rechargeable batteries and e-motors, which have low running-maintenance costs, low emissions, high efficiency, and free noise pollution. The transformation to e-mobility significantly reduces traffic congestion and provides a healthier lifestyle to people. Consequently, today, fossil fuel-based industries are switching to zero-emission vehicle production. The developed nation has executed different strategies to encourage sales and production of e-vehicles. As per the 2017 survey, the sale of EVs surpassed more than one million ve-

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https://doi.org/10.1016/j.rineng.2024.102272

Received 17 February 2024; Received in revised form 23 March 2024; Accepted 13 May 2024

Available online 17 May 2024

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Table 1

EVs	Electric vehicles
LB	Lithium-ion battery
EVLB	Electric vehicle lithium-ion battery
q-ROPFS	q-rung orthopair picture fuzzy set
q-ROFNS	q-rung orthopair fuzzy numbers
TFS	Triangular fuzzy set
DM	Decision Matrix
EVLBRM	Electric vehicle lithium-ion battery recycling methods
IDOCRIW	Integrated Determination of Objective CRIteria Weights
COCOSO	Combined Compromise Solution
IVIFS	Interval valued intuitionistic fuzzy set
MCDM	Multi-criteria decision making
MADM	Multi-attribute decision making

hicles worldwide. According to the EV manufacturing report for 2020, the EV has been rapidly growing for the last two decades, with 10 million vehicles on the overall world road. During the COVID-19 period. EV registration expanded by 41%. When these vehicles end their useful life, they will be recycled. During the EV recycling process, the wastage of EV batteries provides more burden to the growing EV industries and vehicles around the world. This stage presents critical challenges for the battery recycling and storage process in the world-wide EV manufacturing industry. EV waste shall also indicate the valuable materials because the components of an EV battery are unavailable, and arranging the resources is a serious problem in many countries. When compared with other batteries, lithium-ion batteries (LB) dominate the EV markets because of their high-energy, long-life span, power density, and reduced swapping costs. In this era, more than 90% of EVs are powered by LB, with peak growth in industrial speculation. Specifically, the worldwide manufacturing of EVLB is anticipated to increase five times between 2021 and 2023, reaching 5,500 GWH. The LB recycling process is difficult due to the complex, structured combination of cathode, anode, electrolyte, and energy collectors. As per the IAE estimation, the EV generated 5 lakh tons of LB waste and was guessed to generate million tons of waste in 2040. In this critical environmental crisis, the identification of superlative alternatives among different LB recycling processes is significant. MADM is a conceptualization that is implemented to choose the best appropriate one among already-determined alternatives by analyzing them in view of many attributes. The MADM approach uses two different pathways, as conventional data and uncertainty data are generally used to rank the alternative. In terms of handling uncertainty, the conventional MADM approaches are deemed insufficient. Therefore, here we propose the MADM with fuzzy sets to deal with uncertainty in the decision-making process. Many researchers are dealing with the problem with conventional data. The data for the withholding application is inconsistent in many LB recycling research papers. When dealing with Zadeh's theory (Fuzzy set) provides a more accurate solution with uncertainty data. Hence, the proposed algorithm employed the MADM method and a fuzzy environment for LB recycling method selection. (See Table 1.)

2. Literature review

The concept of uncertainty, introduced by Lotfi Zadeh in 1965 [1] under the name of fuzzy set theory, is used to handle uncertainty within the range of zero to one. In 1985, Gorzalczancy [2] extended the fuzzy set to an interval-valued fuzzy set, allowing for more space compared to traditional fuzzy sets. Atanassov [3] later developed intuitionistic fuzzy sets (IFS) by adding a non-membership grade to classical fuzzy sets, with the condition that the sum of the membership and non-membership functions does not exceed one. From the IFS, several researchers have expanded the fuzzy space and created new fuzzy sets with unique conditions, such as the Pythagorean fuzzy set (PyFS) [4], q-rung orthopair fuzzy set (q-ROFS) [5], and Fermatean fuzzy set (FFS) [6]. When q = 1 in IFS, q = 2 in PFS, and q = 3 in FFS,

which is a common expansion of the q-ROFS. q-ROFS is the general format of IFS, PFS, and FFS. Besides the expansion of IFS, some fuzzy sets have specific properties and conditions. Torra [7] introduced a hesitant fuzzy set consisting of all possible membership values, while Cuong and Kreinovich [8] developed a picture fuzzy set (PFS) with membership, abstain membership, and non-membership functions.

2.1. Motivation of the research

- · Below are some recent papers for discussion that relate to q-rung orthopair and picture fuzzy set. [9] employed q-ROFS and q-ROFS weighted mean aggregated. Narayanamoorthy et al. [10] employed q-ROFNS with q = 3 as a FFS and devised an ideal score function. Some researchers are expanding the q-ROFNS with different fuzzy sets. [11] Extended q-ROFSs with interval-valued fuzzy sets provide more space and are easy to access with uncertainty. [12] Combined q-ROFNs with Torra's hesitant fuzzy set into q-ROHFS to provide a set of all finite q-ROFNS solved complexity problems. This paper [13] enlarged the q-ROFSs space fused with rough set over two different environments utilized. [14] discussed the linguistic q-rung orthopair fuzzy set with the qth value q = 1 and 2 and defined the heronian mean operator for solving decision-making problems. Riaz and others [15] used the q-ROF m-polar fuzzy set to manage multi-polarity and membership/non-membership functions for the application of agri-robotic framing selection. [16] Using PFS for product design evaluation through improvised TOPSIS and the grey relational analysis method with entropy-based criterion evaluation. A comprehensive approach to addressing inconsistent information and pattern recognition problems has been developed in this paper [17], which includes a detailed explanation of distance measures. In this paper we employed q-rung orthopair picture fuzzy set (q-ROPFs) are used to deal with uncertain and imprecise data in the three dimenentinal pattern in the form of qth value. Parthasarathy et al. (2024) [57] used a q-rung orthopair picture fuzzy set for the selection of green energy sources, considering five alternative and fifteen criteria through the CRITIC and EXPROM-II methods. The q-ROPFS data are solved through multi-attribute decision-making (MADM) techniques.
- MADM is the procedure of making an advantageous judgement through a countable number of pre-specified alternatives under multiple and usually conflicting attributes. These techniques are classified into two categories: the weighted MADM approach and the ranking MADM approach. Many weighted and ranking approaches have been developed, namely objective weighted methods as [18], [19], [20], [21], [22] are based on the mathematical computation of decision matrices and subjective weighting methods are [23], [24], [25], [26], [27], [28] are gathering opinions from decision experts.
- In recent days, some papers have combined objective and subjective priority methods to get a more precise result for weight in the MCDM problem. [29] This study combines objective Method based on MEREC(Method based on the removal effects of criteria) and subjective Stepwise Weight Assessment Ratio Analysis (SWARA) methods for weight determination. Tej Singh et al. (2024) [59] applied the CRITIC approach to finding the weight of criteria for the application of automotive brake friction composites in reinforced selection; afterwards, the paper used the MARCOS approach for ranking the alternative. Debnath et al. [60] used Grey Decision-Making Trial and Evaluation Laboratory (Grey DEMATEL) model to evaluate the difficulties encountered when implementing sustainable production methods within the apparel manufacturing sector. Narayanamoorthy and others [30] used the weighted calculating aggregation for determining the best solid waste disposal method through an interval-valued q-rung orthopair fuzzy weighted operators. The minimum number of studies available for the incorporation of dual objective and dual subjective weighted

Table 2

EV literature survey.

Author and Reference	Problem	MADM Methods	Data
Karasan et al. (2018) [37]	Selecting sustainable location for an EV charging station in Turkey	IF-DEMATEL and IF-TOPSIS	IVIFS
Wang et al. (2020) [38]	Site selection for the battery swapping station in Beijing, China	DEMATEL and MULTIMOORA	TFS
Ghosh et al. (2021) [39]	GIS-based site selection of an EV charging station in Howrah, India	F-AHP,F-TOPSIS and F-COPRAS	Hexagonal fuzzy data
Loganathan et al. (2021) [40]	Selecting the best lithium-ion battery for electric vehicles	WSM	Crisp
Ecer (2021) [41]	Identifying the best EV among ten variants of alternatives	MARCOS, MAIRCA, COCOSO, ARAS, and COPRAS	Crisp
Loganathan et al. (2021) [42]	Selection of EVLBRM	Equal weight and WPM is used to rank the three alternatives	Crisp
Ashok and others (2022) [43]	Identifying the common barriers and pathways for EV	PROMOTHEE method	Crisp
Shaurya and Ramesh (2022) [44]	EV charging technology selection	AHP-VIKOR methods	TFS
Puska and et al. (2023) [45]	Best electric car selection	Double normalised MEREC and CRADIS	Crisp
Boskovic et al. (2023) [46]	Last mile delivery EV selection	AROMAN	Crisp
Wei and Zhou (2023) [47]	EV supplier selection	BWM and VIKOR methods	TFS
Bassel and others (2023) [48]	Sustainable location selection for EV charging in Egypt	DEMATEL-COPRAS	Type-2 neutrosophic
Dwivedi and Sharma (2023) [49]	EV selection	Entropy and TOPSIS	Crisp
Gupta and others (2023) [50]	Risk factors for electric vehicle charging infrastructure selection	AHP	TFS
Parthasarathy et al. (2024) [34]	EV charging technology selection	IOSWA and MARCOS approach	IVPq-ROFS
Our Study	Identifying the optimal recycling method for EVLB based on three alternatives and eleven attributes	Aggregated IDOCRIW and COCOSO method is used	q-ROPFS

techniques An IDOCRIW (Integrated Determination of Criteria Objective Weight) [31] is a combination of dual objective methods employed in the research. The IDOCRIW method was developed by Zavadskas and Podvezko in 2016. This technique combines the Entropy and Criterion Impact LOSs (CILOS) methods to determine a relative impact loss and attribute weights. It is used in MADM and involves a normalization step, degree of entropy calculation, the creation of a square matrix, and the determination of the relative impact loss and weight system matrix in the weighted calculation process.

· Based on the ranking approaches here, we employed the Combined Compromise Solution (COCOSO) method [32], which assists three aggregation operators. The COCOSO is an amalgamation of simple additive weighting (SAW) and aggressive weighted product modeling (WPM), and in the final utility value solving equation, the parameter fixative stage is more effective than the SAW and WPM methods. [33] hybrid SWARA and COCOSO for the problem of site selection for logistics centres and validation compared with MADM methods in the triangular fuzzy database. Torkayesh et al. [58] framed COCOSO ranking with BWM and level-based weight assessment (LBWA) for the case study of considering seven countries in eastern Europe in terms of economic support and their development, mainly focused on the healthcare segment. This paper [35] carries interval rough numbers data based on the objective SWARA method and COCOSO method for evolving sustainable railway transportation systems in West Africa, with a spearman correlation coefficient employed for comparative analysis. This paper [36] signifies the amalgamation of CRITIC with the linear weighted comprehensive technique and COCOSO with PYFS to find the solution to the 5G industry evaluations. Dwivedi and Sharma [61] explore the application of Shannon entropy and the COCOSO decision-making framework to achieve sustainable development goals in Indian union territories. The conclusions indicate that Chandigarh has successfully implemented the Sustainable Development Goals, yielding the best outcomes among other states.

• From the Table 2 and exiting literature survey, no research has done in EV recycling process selection through three alternatives with eleven parameters. Many research based on electric vehicle charging site selection and identifying the best EV among different varieties of EV. This is one of the innovative research projects, EVLB selection through uncertainty manner in the form of q-ROPFS.

2.2. Research gaps

- The majority of research is conducted through individual objective, individual subjective, or combination objective and subjective weighted approaches.
- The q-ROFS only deals with the two-dimensional space having belongingness and non-belongingness grades. PFS with three-dimensional space without qth dimensional space.
- The existing study fails to combine the q-ROPFS with the IDOCRIW method; moreover, the q-ROPFS deals with MEREC, entropy, and CILOS methods.
- The existing literature on the EVLB recycling selection problem involves the maximum five attributes and fails to elaborately explain the application.
- More EV background decision science research dealing with location selection, EV selection, risk factors, and EV delivery selection.
- We utilized three distinct normalization processes, which fall into three categories, for the problem at hand, despite the fact that many MADM problems can be solved with just a single normalization process.
- When comparing to other MADM methods. The COCOSO method is not easily affected by the weighted method preference and the neutralized-based approach based on the weighted and ranking directives.

2.3. Contribution

- q-ROPFS have a lower likelihood of increasing decision-making uncertainty, resulting in increased precision and advantages. Mainly, q-ROPFS are insufficient to define a neutral membership grade. This is the main motive to address the problem in q-ROPFS.
- The paper utilized a combined objective-weighted method for criterion evaluation. This helped to analyses the weight of the criterion from different perspectives and finally combine the weights into a unique value. The IDOCIRW method is more acceptable than individual objective, subjective, and combination of them. The objective approach provides exact valuation and mathematical calculation directly through a ranking decision matrix. The extended IDOCRIW reduces computational time compared with other weighted methods.
- Here we discussed the in-depth analysis of the EVLB recycling technique. It helps to improve the adaptation of EVs and decrease waste generation from EVs. For EVLB process selection, crisp data is used in all MCDM papers, and here we used q-ROPFS to cope with the uncertainty of the decision-maker's thoughts.
- The COCOSO method is not easily affected by the weighted method preference and the neutralized-based approach based on the weighted and ranking directives.
- The article developed a new formulation of the IDOCRIW and COCOSO approaches in the form of q-ROPFS to provide effective solution MADM.
- This is the first research that thoroughly examines EVLB recycling selection based on eleven decision making attributes under uncertainty environment.

3. Preliminaries

Definition 1. The q-rung orthopair fuzzy set (q-ROFS) [5] (\mathfrak{Q}) on non-void sets \mathbb{D} is defined as

$$\mathfrak{Q} = \{(\tau, \alpha_{\mathfrak{Q}}(\tau), \gamma_{\mathfrak{Q}}(\tau) | \tau \in \mathbb{D}\}$$
(1)

where $\alpha_{\mathfrak{Q}}(\tau) : \mathbb{D} \to [0,1]$ and $\gamma_{\mathfrak{Q}}(\tau) : \mathbb{D} \to [0,1]$ indicates membership and non-membership grade of $\tau \in \mathbb{D}$. $h_{\mathfrak{Q}}(\tau) = 1 - ((\alpha_{\mathfrak{Q}}^{q} + \gamma_{\mathfrak{Q}}^{q}))^{1/q}$ represents indeterminacy grade of $\tau \in \mathbb{D}$.

Definition 2. The Picture fuzzy set (PFS) [7] (\mathfrak{P}) on nonempty set \mathbb{D} is defined by

$$\mathfrak{P} = \{(\tau, \alpha_{\mathfrak{P}}(\tau), \xi_{\mathfrak{P}}(\tau), \gamma_{\mathfrak{P}}(\tau) | \tau \in \mathbb{D}\}$$
(2)

where $\alpha_{\mathfrak{P}}(\tau) : \mathbb{D} \to [0,1], \xi_{\mathfrak{P}}(\tau) : \mathbb{D} \to [0,1]$ and $\gamma_{\mathfrak{P}}(\tau) : \mathbb{D} \to [0,1]$ indicate truth, neutral and falsity membership grade of $\tau \in \mathbb{D}$ and hesitancy grade is $h_{\mathfrak{P}}(\tau) = 1 - (\alpha_{\mathfrak{P}}(\tau) + \xi_{\mathfrak{P}}(\tau) + \gamma_{\mathfrak{P}}(\tau)).$

Definition 3. The q-rung orthopair picture fuzzy set (q-ROPFS) [56] $(q\mathfrak{P})$ is nonvoid set on \mathbb{D} defined by

$$\mathfrak{q}\mathfrak{P} = \{(\tau, \alpha_{\mathfrak{a}\mathfrak{B}}(\tau), \xi_{\mathfrak{a}\mathfrak{B}}(\tau), \gamma_{\mathfrak{a}\mathfrak{B}}(\tau)) / \tau \in \mathbb{D}\}$$
(3)

where $\alpha_{q\mathfrak{P}}^{q}(\tau) : \mathbb{D} \to [0,1]$, $\xi_{q\mathfrak{P}}^{q}(\tau) : \mathbb{D} \to [0,1]$ and $\gamma_{q\mathfrak{P}}^{q}(\tau) : \mathbb{D} \to [0,1]$ represents positive, neutral and negative grade of $\tau \in \mathbb{D}$ and refusal grade is $h_{q\mathfrak{P}}(\tau) = (1 - (\alpha_{q\mathfrak{P}}^{q}(\tau) + \xi_{q\mathfrak{P}}^{q}(\tau) + \gamma_{q\mathfrak{P}}^{q}(\tau)))^{(0.5*q)}$. The general representation of q-ROPFS (q \mathfrak{P}) is $q\mathfrak{P} = (\alpha_{q\mathfrak{P}}, \xi_{q\mathfrak{P}}, \gamma_{q\mathfrak{P}})$.

Definition 4. Let $\mathfrak{q}\mathfrak{P}_1 = (\alpha_{\mathfrak{q}\mathfrak{P}_1}, \xi_{\mathfrak{q}\mathfrak{P}_1}, \gamma_{\mathfrak{q}\mathfrak{P}_1}), \ \mathfrak{q}\mathfrak{P}_2 = (\alpha_{\mathfrak{q}\mathfrak{P}_2}, \xi_{\mathfrak{q}\mathfrak{P}_2}, \gamma_{\mathfrak{q}\mathfrak{P}_2}), \ \mathfrak{q}\mathfrak{P}_3 = (\alpha_{\mathfrak{q}\mathfrak{P}_3}, \xi_{\mathfrak{q}\mathfrak{P}_3}, \gamma_{\mathfrak{q}\mathfrak{P}_3})$ be three q-ROPFSs [56], then

- 1. $\mathfrak{q}\mathfrak{P}_1 \oplus \mathfrak{q}\mathfrak{P}_2 = \mathfrak{q}\mathfrak{P}_2 \oplus \mathfrak{q}\mathfrak{P}_1$
- 2. $\mathfrak{q}\mathfrak{P}_1 \otimes \mathfrak{q}\mathfrak{P}_2 = \mathfrak{q}\mathfrak{P}_2 \otimes \mathfrak{q}\mathfrak{P}_1$
- 3. $\lambda (q \mathfrak{P}_1 \oplus q \mathfrak{P}_2) = \lambda q \mathfrak{P}_1 \oplus \lambda q \mathfrak{P}_2$
- 4. $(\lambda_1 + \lambda_2) q \mathfrak{P}_3 = \lambda_1 q \mathfrak{P}_3 \oplus \lambda_2 q \mathfrak{P}_3$

$$\begin{aligned} & 5. \quad \left(\mathfrak{q}\mathfrak{P}_1\otimes\mathfrak{q}\mathfrak{P}_2\right)^{\lambda} = \mathfrak{q}\mathfrak{P}_1^{\lambda}\otimes\mathfrak{q}\mathfrak{P}_2^{\lambda}, \lambda > 0 \\ & 6. \quad \mathfrak{q}\mathfrak{P}_3^{\lambda_1}\otimes\mathfrak{q}\mathfrak{P}_3^{\lambda_2} = \mathfrak{q}\mathfrak{P}_3^{(\lambda_1+\lambda_2)}, \lambda_1, \lambda_2 > 0 \end{aligned}$$

Definition 5. The score and accuracy function of the q-rung orthopair picture fuzzy set (q-ROPFS) [56] $\mathfrak{q}\mathfrak{P}_1 = (\alpha_{\mathfrak{q}\mathfrak{P}_1}, \xi_{\mathfrak{q}\mathfrak{P}_1}, \gamma_{\mathfrak{q}\mathfrak{P}_1})$ is

$$\begin{split} & \mathbb{S}(\mathbf{q}\mathfrak{P}_1) = \alpha^q_{\mathbf{q}\mathfrak{P}_1} - \xi^q_{\mathbf{q}\mathfrak{P}_1} - \gamma^q_{\mathbf{q}\mathfrak{P}_1}, \\ & \mathbb{S}(\mathbf{q}\mathfrak{P}_1) = \alpha^q_{\mathbf{q}\mathfrak{P}_1} + \xi^q_{\mathbf{q}\mathfrak{P}_1} + \gamma^q_{\mathbf{q}\mathfrak{P}_1}, \\ & \mathbb{A}(\mathbf{q}\mathfrak{P}_1) = \alpha^q_{\mathbf{q}\mathfrak{P}_1} + \xi^q_{\mathbf{q}\mathfrak{P}_1} + \gamma^q_{\mathbf{q}\mathfrak{P}_1}, \\ & \mathbb{A}(\mathbf{q}\mathfrak{P}_1) \in [0, 1] \end{split}$$
(4)

4. Proposed algorithm

This section presents the q-ROPFS-IDOCRIW-COCOSO methodology. The entries of q-ROPFS are defuzzed by the score function of q-ROPFS. First, Shannon's entropy method is extended in the q-ROPFS environment to calculate objective weight. Again, the CILOS method is employed and lengthened in the q-ROPFS to determine the objective weight of attributes. Subsequently, we aggregated the entropy [19] and CILOS [20] weights into a single IDOCRIW weight. The importance of attributes is revealed through the IDOCRIW approach [31]. Finally, q-ROPFS-COCOSO [32] orders the alternative. The COCOSO algorithm is an expansion of the WPM and SAW methods. The combination of q-ROPFS, IDOCRIW, and COCOSO has multiple advantages. Combining entropy with the CILOS approach neglects the drawbacks of the entropy method, which avoids human interference. It provides a more precise solution when compared with the standalone MCDM technique. The q-ROPFS-IDOCRIW-COCOSO technique increases the accuracy and reliability of the problem compared to other prior work. The algorithm for the proposed methodology steps is listed below. Fig. 1 displays the proposed algorithm's step-to-step methodology.

Step 1: Form an initial decision matrix based on the decision maker's opinion in the form of the described picture q-rung orthopair, which includes a set of alternatives and attributes.

$$\tilde{D} = \begin{bmatrix} (\tilde{\xi}_{11}, \tilde{\rho}_{11}, \tilde{\phi}_{11}) & \dots & (\tilde{\xi}_{1n}, \tilde{\rho}_{1n}, \tilde{\phi}_{1n}) \\ (\tilde{\xi}_{21}, \tilde{\rho}_{21}, \tilde{\phi}_{21}) & \dots & (\tilde{\xi}_{2n}, \tilde{\rho}_{2n}, \tilde{\phi}_{2n}) \\ \dots & \ddots & \dots \\ (\tilde{\xi}_{m1}, \tilde{\rho}_{m1}, \tilde{\phi}_{m1}) & \dots & (\tilde{\xi}_{mn}, \tilde{\rho}_{mn}, \tilde{\phi}_{mm}) \end{bmatrix}$$

Step 2: Defuzzification of the q-ROPFS using the score function described below,

$$S(\tilde{D}) = \xi^{q}_{\tilde{D}} - \rho^{q}_{\tilde{D}} - \phi^{q}_{\tilde{D}}, S(\tilde{D}) \in [-1, 1].$$
(6)

Step 3: Normalization of the score functioned q-ROPFS matrix is resolved by using the below equation:

$$\bar{D}_{ij} = \frac{d_{ij}}{\sum_{j=1}^{q} \tilde{d}_{ij}}.$$
(7)

Step 4: The degree of entropy is calculated as

$$\tilde{E}_{j} = -\frac{1}{\ln n} \sum_{i=1}^{n} \bar{d}_{ij} \cdot \ln \bar{d}_{ij} \text{ and } j = 1.$$
(8)

Step 5: The degree of entropy deviation especially calculated as

$$\hat{\delta} = 1 - \tilde{E}_j. \tag{9}$$

Step 6: The entropy weight is calculated as

$$\bar{\omega}_j = \frac{\delta_j}{\sum_{i=1}^q \hat{\delta}_j}.$$
(10)

Step 7: Transforming negative attributes into positive attributes in the decision matrix using the below equation and afterwards normalizing the transforming matrix depending on step three

$$\hat{d}_{ij} = \frac{\min_i d_{ij}}{d_{ij}}; \quad i, j \in \{1, \dots, n\}.$$
(11)

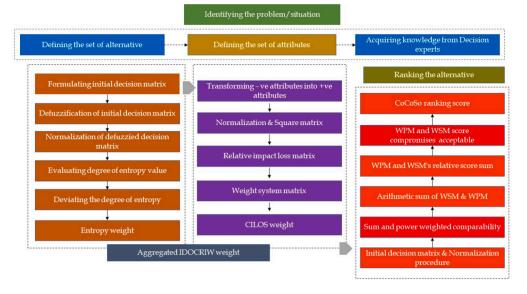


Fig. 1. Overview of proposed algorithm.

Step 8: The square matrix is computed as follows:

$$b_i = max_i \tilde{d}_{ij} = b_{s_i} j, \tag{12}$$

where $b_{s_i}j$ indicates the maximum value of the column-wise attributes employed from the judgmental matrix with s_i from the decision matrix with s_i to shape a square matrix. In equations $b_{ij} = b_{s_i}j$ and $b_{jj} = b_j$.

Step 9: The relative impact loss value is determined as follows based on the knowledge gathered through the step-by-step matrix mentioned above:

$$r_{ij} = \frac{b_{jj} - b_{ij}}{b_{jj}}, r_{jj} = 0 \text{ and } i, j \in \{1, 2, \dots m\},$$
(13)

here r_{ij} denotes the relative impact loss of the alternative for justifying the best alternative.

Step 10: Taking the value of r_{ij} , then the weight system matrix is calculated as follows:

$$\mathcal{F} = \begin{pmatrix} -\sum_{i=1}^{n} \mathcal{R}_{i1} & \mathcal{R}_{12} & \cdots & \mathcal{R}_{1n} \\ \mathcal{R}_{21} & -\sum_{i=1}^{n} \mathcal{R}_{i2} & \cdots & \mathcal{R}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{R}_{n1} & \mathcal{R}_{n2} & \cdots & -\sum_{i=1}^{n} \mathcal{R}_{in} \end{pmatrix}_{n \times n}.$$

Step 11: The priority of attributes is evolved through the solution below the equations:

$$\mathcal{F}\tilde{\beta}^T = 0. \tag{14}$$

Step 12: The values of entropy weight and CILOS weight obtained from the equations (10) and (14) are aggregated into a single value by using the equation, which is known as the aggregated IDOCRIW weight of the attributes.

$$\beta_j = \frac{\bar{\beta} \cdot \tilde{\psi}}{\sum_{i=1}^n \bar{\beta} \cdot \tilde{\psi}},\tag{15}$$

where β_j represents aggregated IDOCRIW weight, $\bar{\beta}$ represents entropy weight, and $\bar{\omega}$ indicates CILOS weight.

Step 13: The compromise normalization procedure is applied in the SF-PQROFS for ranking direction using the below equation:

$$\bar{D}_{ij} = \begin{cases} \frac{\bar{D}_{ij} - \bar{D}_j^-}{\bar{D}_j^+ - \bar{D}_j^-}, & j \in B\\ \frac{\bar{D}_j^+ - \bar{D}_j^-}{\bar{D}_j^+ - \bar{D}_j^-}, & j \in C \end{cases}$$
(16)

$$\bar{D}_i^- = \min \bar{D}_{ij}$$
 and $\bar{D}_i^+ = \max \bar{D}_{ij}$,

where B and C represent beneficial and non-beneficial attributes.

Step 14: The sum and power of weighted comparability for each alternative are calculated as follows, X_i and Y_i respectively,

$$X_{i} = \sum_{i=1}^{n} (\bar{D}_{ij})^{\omega_{j}},$$
(17)

where X_i value is obtained based on the grey relation approach.

$$Y_{i} = \sum_{i=1}^{n} (\bar{D}_{ij})^{\omega_{j}},$$
(18)

where Y_i value is attained based on the multiplicative WASPAS technique.

Step 15: In this step, the subsequent aggregation equations are used to calculate the relative significance of the alternatives. Three appraisal scores are used, which are calculated using the below formulas:

$$\widehat{\mathbf{x}}_{i\alpha} = \frac{X_i + Y_i}{\sum_{i=1}^{n} (X_i + Y_i)}.$$
(19)

This Eqn. (19) demonstrates the arithmetic mean of the summation of WPM and WSM.

$$\mathfrak{K}_{i\beta} = \frac{X_i}{\min_i X_i} + \frac{Y_i}{\min_i N_i}.$$
(20)

This formula (Eqn. (20)) demonstrates the way WPM and WSM's relative score sum compares to the best.

$$\Re_{i\gamma} = \frac{\lambda(X_i) + (1 - \lambda)(Y_i)}{\lambda \max_i X_i + (1 - \lambda)\max_i Y_i}; \ 0 \le \lambda \le 1.$$
(21)

The above Eqn. (21) produces a WPM and WSM score compromises acceptable.

Step 16: An alternative's overall ranking score is determined based on the $\hat{\mathbf{x}}_i$ (Eqn. (22)) values:

$$\mathfrak{K}_{i} = (\mathfrak{K}_{i\alpha} * \mathfrak{K}_{i\beta} * \mathfrak{K}_{i\gamma})^{\frac{1}{3}} + \frac{1}{3}(\mathfrak{K}_{i\alpha} + \mathfrak{K}_{i\beta} + \mathfrak{K}_{i\gamma}).$$
(22)

5. Case study

The highly demanding market for EVs continuously witnesses a rise, and production of EV equipment is also increasing. While the production of LB faces challenges on account of the high material costs of cobalt and lithium, the environmental problem is also considered in the situation. The production of new batteries has an impact on the

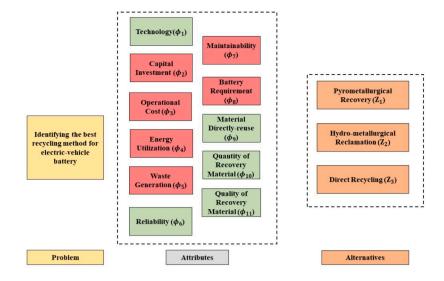


Fig. 2. Overview of the problem.

environment. The best way to overcome this situation is to reuse and recycle EVLBs. This would solve the demand for cobalt, lithium, and other components to produce EVLBs. Though reusing the old batteries may be cost-effective and help procure highly concentrated metals, the safety hazards associated with the process make it nearly impossible. Here, three recycling methods were discussed: pyrometallurgical, hydrometallurgical, and direct recycling methods, which are elaborated below. The representation of alternatives and attributes is illustrated in Fig. 2.

5.1. Description about alternatives

- **Pyrometallurgical recovery** (Z_1) : The used battery materials are melted at high temperatures to convert the metal oxides into alloy metals that are initiated in the cathode, including copper, nickel, iron, and cobalt. Metal alloy segments, slags, and gases are the spinoffs of these actions. Organic substances made from the electrolyte and binder apparatuses cause gases to be produced at low temperatures. The polymers in the battery were burned and dissolved. Metal alloys produced by this process can be operated in the cement processing division. Additionally, they can be used for remanufacturing batteries because the modules and cells are exposed to high temperatures in the presence of a reductant during reclamation. The advantage of this process is that there are few safety hazards. It helps in the extraction of high concentrations of expensive materials such as cobalt, nickel, and copper. Whole cells can be used to reduce the risk of exposure to highly volatile substances. Owing to its easy maintenance, this is a more reliable method and offers low production costs. The extracted material can also be used in cement. The drawback of this process is that the production of toxic gases poses a high risk to the environment; it incurs high energy costs; not all metals can be recovered; and it is a follow-up process.
- **Hydro-metallurgical reclamation** (Z_2): Using this technique, battery cells are crushed and then dissolved in an aqueous solution to leach and recover the metal components from the cathode material. To aid the recovery of metals after leaching, the pH of the solution was adjusted to cause a sequence of precipitation processes. After cobalt was separated, sulphate, hydroxide, carbonate, and lithium were separated. Before delivering the battery for this procedure, mechanical shredding was performed as a prerequisite phase. The advantages of hydrometallurgical reclamation are that high-purity materials can be extracted, resulting in comparatively lower greenhouse gas emissions, and that they do not require high-temperature settings. The disadvantages of this process are that

it requires a higher level of solvents, the extracted materials can be easily contaminated, the delamination process takes more time, and it requires secondary waste treatment.

Direct recycling (Z_3) : A process that uses sound energy to separate particles from a chemical mixture. The direct recycling approach is appropriate for re manufacturing low-value cathodes because it does not require lengthy or expensive purification procedures. The production process for cathode oxides adds to high cathode costs, increased energy usage, and carbon dioxide emissions. It has high maintenance costs and is not reliable. It also poses the risk of forming hazardous byproducts during the methods of removing electronic binder. It could lead to the piling of batteries. The efficiency of this process depends on the condition of the battery. The parts of the battery should be adeptly sorted for this process to be done. This makes the process more complicated and increases operational costs. This process also falls short when it comes to reliability. Furthermore, the pyrolysis process, which is used to remove electronic binder, carries the risk of forming harmful by-products.

5.2. Description about attributes

- **Technology** (ϕ_1) : It involves setting up industries, machines, and equipment that improve productivity and quality. It is a positive criterion.
- **Capital investment** (ϕ_2) : It includes the initial cost associated with the location establishment and the procurement of machinery for the commencement of business. If the capital investment is high, it affects the possibility of setting up a new business, and hence it is a negative criteria.
- **Operational cost** (ϕ_3) : Operational cost is the capital spent on the raw materials, labour, and other expenses that occur while running an industry. It is one of the key factors that influences the growth of the business. Minimising the operational cost greatly increases the profit. Therefore, it is a negative criterion.
- Energy utilisation (ϕ_4) : Electricity is a vital source for running machinery. Conservation of energy without affecting productivity should be taken into account, as it not only reduces expenses significantly but also aids in the preservation of our environment.
- Waste generation (ϕ_5): Waste refers to the unwanted residual or useless quantity that remains after the process is complete. If more waste is released into the environment, it affects the ecosystem, rendering it damaged. A productive technique that reduces waste generation must be employed to get good results.

Table 3

Picture	q-rung	orthopair	linguistic
scale.			

q-ROPFNs
(0.95, 0.3, 0.2)
(0.85, 0.13, 0.12)
(0.65, 0.33, 0.22)
(0.45, 0.53, 0.33)
(0.25, 0.73, 0.43)

Table 4

q-rung orthopair picture fuzzy linguistic decision matrix.

Z_i/ϕ_j	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7	ϕ_8	ϕ_9	ϕ_{10}	ϕ_{11}
Z_1											
$Z_2 Z_3$										SH M	

- **Reliability** (ϕ_6): It is important that the process not be easily prone to failure. High reliability and effectiveness positively impact the success of a process.
- Maintainability (ϕ_7) : Maintenance refers to the procedures involved in sustaining a process. A process with high maintenance is susceptible to more breakdowns, and the repair costs are also high. It causes disruption in resources and production and causes environmental problems.
- Battery requirement (ϕ_8): Different battery models are composed of different materials. As a result, the battery's components must be considered because they have an impact on how the process is used. Some techniques are applicable to different battery kinds, while others are only applicable to specific models. It is therefore a negative criterion.
- Material directly reusable (\$\phi_9\$): Directly reusing materials instead of putting them through costly recycling processes could considerably lower the cost. As a result, it is a favourable criterion.
- **Recovery material-Quality** (ϕ_{10}): The choice of the best recycling technique is greatly influenced by the quality of the recovered material. A recycling procedure that results in high-quality material is ideal. The procedure cannot be used again if the product is of poor quality.
- Recovery material-quantity (ϕ_{11}) : Utilising a costly procedure that only extracts a tiny amount of material is not advisable. The procedure is ideal when there is a large amount of recovered material. Therefore, it is a beneficial criterion.

6. Result and discussion

6.1. Numerical example

In this section, we analyse the mathematical solution for the electric vehicle battery recycling process through the integrated proposed IDOCRIW and CoCoSo methodologies with unique expert opinion in the system of q-ROPFS. In this integrated MCDM problem, incorporate three alternatives and eleven attributes for categorising the outcomes in an accurate manner. The picture q-rung orthopair linguistic scale defined in Table 3 for expressing the DE opinion in a qualitative manner. The data acquisition gathered based on the defined linguistic scale of q-rung orthopair picture fuzzy set (q-ROPFS) from the decision experts opinion.

Step 1: In this step, the scored q-ROPFS employed using the equation. (6) and score-functioned value described in the Table 5, based on the experts decision matrix from the q-ROPFS linguistic scale Table 4.

Step 2: Using Eqn. (8), the degree of entropy is calculated as $E(\phi_1) = 0.3953$, $E(\phi_2) = 0.4223$, $E(\phi_3) = 0.3818$, $E(\phi_4) = 0.4472$, $E(\phi_5) = 0.4152$, $E(\phi_6) = 0.4492$, $E(\phi_7) = 0.4130$, $E(\phi_8) = 0.3573$, $E(\phi_9) = 0.2392$, $E(\phi_{10}) = 0.3936$, $E(\phi_{11}) = 0.4355$.

Step 3: Based on the algorithm phase - (iv), the objective entropy weight $\bar{\beta}_{j}$ is calculated utilizing the equation (10) as $\bar{\beta}_{1} = 0.0909$, $\bar{\beta}_{2} = 0.0869$, $\bar{\beta}_{3} = 0.0930$, $\bar{\beta}_{4} = 0.0831$, $\bar{\beta}_{5} = 0.0879$, $\bar{\beta}_{6} = 0.0828$, $\bar{\beta}_{7} = 0.0883$, $\bar{\beta}_{8} = 0.0966$, $\bar{\beta}_{9} = 0.1144$, $\bar{\beta}_{10} = 0.0912$, $\bar{\beta}_{11} = 0.0849$.

Step 4: After changing the non-beneficial to beneficial attributes by engaging normalization Eqn. (11), the square matrix obtained by Eqn. (12) is shown in the Table 6.

Step 5: The relative impact loss matrix (Eqn. (13)) is shown in the Table 7.

Step 6: Based on the algorithm, Weighted system matrix is computed in the Table 8 through relative impact loss solution.

Step 7: The weight of CILOS methodology is determined through the Eqn. (14) and solutions are $\tilde{\rho}_1 = 0.0779$, $\tilde{\rho}_2 = 0.1311$, $\tilde{\rho}_3 = 0.0544$, $\tilde{\rho}_4 = 0.1998$, $\tilde{\rho}_5 = 0.0753$, $\tilde{\rho}_6 = 0.2038$, $\tilde{\rho}_7 = 0.0510$, $\tilde{\rho}_8 = 0.0408$, $\tilde{\rho}_9 = 0.0398$, $\tilde{\rho}_{10} = 0.0455$, $\tilde{\rho}_{11} = 0.0816$.

Step 8: The combined IDOCRIW weight is calculated through Eqn. (15) as $\beta_1 = 0.0809$, $\beta_2 = 0.1299$, $\beta_3 = 0.0577$, $\beta_4 = 0.1866$, $\beta_5 = 0.0755$, $\beta_6 = 0.1926$, $\beta_7 = 0.0514$, $\beta_8 = 0.0450$, $\beta_9 = 0.0579$, $\beta_{10} = 0.0474$ and $\beta_{11} = 0.0791$. An importance of attributes is displayed in Fig. 3.

Step 9: After calculating the compromise ranking normalization technique using the formula Eqn. (16), the vectors X_i and Y_i are initiated using the equations (17) and (18) respectively. The obtained results of X_i and Y_i are shown in Tables 9 and 10.

Step 10: The aggregation ranking operator results are necessary for finding the solution to the final result. At this time, the solutions of Ξ_{a_i} , Ξ_{b_i} , and Ξ_{c_i} were calculated. From the Eqns. (19), (20), and (21), the ranking value ($\hat{\mathbf{x}}_i$) of COCOSO is calculated by employing the Eqn. (22). All the results are represented in Table 11.

Reliability is given a higher importance than other attributes, with a weighting of 19.3%, despite battery demand having a lower priority. The alternative hydro-metallurgical reclamation (Z_2) was identified as the most beneficial EVLB recycling method. This solution can save harmful mining practices, secure a steady lithium supply, and obtain valuable resources such as nickel, lithium, and cobalt. Minimising the ecological effects of hazardous waste has been identified as an eco-friendly recycling process. The main advantage is the lower energy consumption in the higher recovery process and the use of a small amount of electricity. The alternative Direct Recycling (Z_3) is the second-best option for the EVLB recycling process with the use of sustainable production, lower cost, and prevention of battery disposal. The alternative pyrometallurgical recovery (Z_1) is the worst alternative for the recycling process. This solves the problem of large feed-size batteries, reduces energy demand by 6-56%, and reduces greenhouse gas emissions by 23%. Based on these approaches, investment in hydrometallurgical reclamation is strongly recommended for greater adaptation of EVs from multi-directive perspectives.

6.2. Comparative study

In this section, integrated IDOCRIW and CoCoSo results are compared with existing MADM methods. In decision science, each MADM method has a unique, distinctive way of thinking. In this analysis, we employed score additive methods and distance-based methods, namely TOPSIS [51], WASPAS [52], COPRAS [53], EDAS [54], and VIKOR [55]. The TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) ranks the alternatives based on any of the distance measures through positive and negative ideal solutions used in many applications. The VIKOR (VIekriterijumsko KOmpromisno Rangiranje) means multi-criteria optimization and compromise solution, which is analogous to the TOPSIS approach. This appraises options with the pertinence of attributes and orders the alternatives based on the utility value, which mainly measures between ideal and nadir solutions, with the proposed compromise solution holding an advantageous parameter. The WASPAS (weighted aggregated sum product assessment) methodology is an aggregated weighted sum model and weighted product model. The technique is calculated through joint generalised attribute solutions

Table 5

q-rung orthopair j	picture score	function	matrix.
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Z_i/ϕ_j	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7	ϕ_8	ϕ_9	ϕ_{10}	ϕ_{11}
Z_1	0.7279	0.7279	0.7279	0.5038	0.1515	0.5038	0.5038	0.7279	0.0745	0.0745	0.3157
Z_2	0.5038	0.3157	0.3157	0.3157	0.3157	0.5038	0.5038	0.5038	0.0745	0.3157	0.5038
Z_3	0.1515	0.3157	0.1515	0.3157	0.5038	0.3157	0.1515	0.0745	0.7279	0.1515	0.7279

Та	ble	6	

Table 6	
Square matrix.	

ϕ_j	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7	ϕ_8	ϕ_9	ϕ_{10}	ϕ_{11}
ϕ_1	0.5262	0.1782	0.1233	0.2386	0.5616	0.3807	0.1878	0.0819	0.0850	0.1376	0.2040
ϕ_2	0.3642	0.4109	0.2843	0.3807	0.2695	0.3807	0.1878	0.1183	0.0850	0.5828	0.3256
ϕ_3	0.1095	0.4109	0.5924	0.3807	0.1689	0.2386	0.6245	0.7998	0.8300	0.2797	0.4704
ϕ_4	0.3642	0.4109	0.2843	0.3807	0.2695	0.3807	0.1878	0.1183	0.0850	0.5828	0.3256
ϕ_5	0.5262	0.1782	0.1233	0.2386	0.5616	0.3807	0.1878	0.0819	0.0850	0.1376	0.2040
ϕ_6	0.5262	0.1782	0.1233	0.2386	0.5616	0.3807	0.1878	0.0819	0.0850	0.1376	0.2040
ϕ_7	0.1095	0.4109	0.5924	0.3807	0.1689	0.2386	0.6245	0.7998	0.8300	0.2797	0.4704
ϕ_8	0.1095	0.4109	0.5924	0.3807	0.1689	0.2386	0.6245	0.7998	0.8300	0.2797	0.4704
ϕ_9	0.1095	0.4109	0.5924	0.3807	0.1689	0.2386	0.6245	0.7998	0.8300	0.2797	0.4704
ϕ_{10}	0.3642	0.4109	0.2843	0.3807	0.2695	0.3807	0.1878	0.1183	0.0850	0.5828	0.3256
ϕ_{11}	0.1095	0.4109	0.5924	0.3807	0.1689	0.2386	0.6245	0.7998	0.8300	0.2797	0.4704

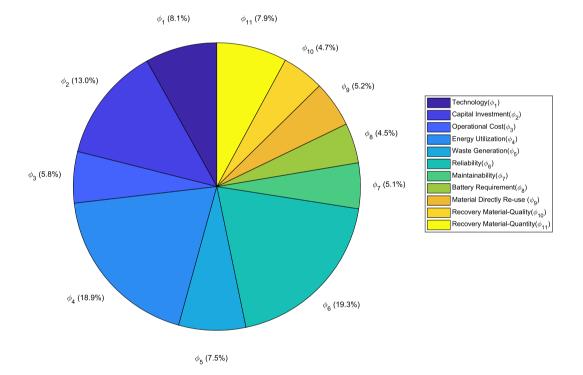


Fig. 3. Priority of attributes.

Table 7
Relative impact loss matrix.

ϕ_{j}	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7	ϕ_8	ϕ_9	ϕ_{10}	ϕ_{11}
ϕ_1	0	0.5663	0.7919	0.3734	0	0	0.6993	0.8976	0.8976	0.7639	0.5663
ϕ_2	0.3079	0	0.5201	0	0.5201	0	0.6993	0.8521	0.8976	0	0.3079
ϕ_3	0.7919	0	0	0	0.6993	0.3734	0	0	0	0.5201	0
ϕ_4	0.3079	0	0.5201	0	0.5201	0	0.6993	0.8521	0.8976	0	0.3079
ϕ_5	0	0.5663	0.7919	0.3734	0	0	0.6993	0.8976	0.8976	0.7639	0.5663
ϕ_6	0	0.5663	0.7919	0.3734	0	0	0.6993	0.8976	0.8976	0.7639	0.5663
ϕ_7	0.7919	0	0	0	0.6993	0.3734	0	0	0	0.5201	0
ϕ_8	0.7919	0	0	0	0.6993	0.3734	0	0	0	0.5201	0
ϕ_9	0.7919	0	0	0	0.6993	0.3734	0	0	0	0.5201	0
ϕ_{10}	0.3079	0	0.5201	0	0.5201	0	0.6993	0.8521	0.8976	0	0.3079
ϕ_{11}	0.7919	0	0	0	0.6993	0.3734	0	0	0	0.5201	0

Table 8Weight system matrix.

ϕ_{j}	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7	ϕ_8	ϕ_9	ϕ_{10}	ϕ_{11}
ϕ_1	-4.8830	0.5663	0.7919	0.3734	0	0	0.6993	0.8976	0.8976	0.7639	0.5663
ϕ_2	0.3079	-1.6990	0.5201	0	0.5201	0	0.6993	0.8521	0.8976	0	0.3079
ϕ_3	0.7919	0	-3.9359	0	0.6993	0.3734	0	0	0	0.5201	0
ϕ_4	0.3079	0	0.5201	-1.1203	0.5201	0	0.6993	0.8521	0.8976	0	0.3079
ϕ_5	0	0.5663	0.7919	0.3734	-5.0568	0	0.6993	0.8976	0.8976	0.7639	0.5663
ϕ_6	0	0.5663	0.7919	0.3734	0	-1.8671	0.6993	0.8976	0.8976	0.7639	0.5663
ϕ_7	0.7919	0	0	0	0.6993	0.3734	-4.1958	0	0	0.5201	0
ϕ_8	0.7919	0	0	0	0.6993	0.3734	0	-5.2491	0	0.5201	0
ϕ_9	0.7919	0	0	0	0.6993	0.3734	0	0	-5.3857	0.5201	0
ϕ_{10}	0.3079	0	0.5201	0	0.5201	0	0.6993	0.8521	0.8976	-4.8922	0.3079
ϕ_{11}	0.7919	0	0	0	0.6993	0.3734	0	0	0	0.5201	-2.6226

Table 9Normalization matrix multiply with weight.

Z_i/ϕ_j	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7	ϕ_8	ϕ_9	ϕ_{10}	ϕ_{11}	X_i
Z_1	0.0809	0	0	0	0.0755	0.1926	0	0	0	0	0	0.3490
Z_2	0.0494	0.1299 0.1299	0.0413 0.0577	0.1886 0.1886	0.0403	0.1926	0 0.0514	0.0154 0.0450	0 0.0519	0.0474 0.0151	0.0361 0.0791	0.7411 0.6187
Z_3	0	0.1299	0.05//	0.1880	0	0	0.0514	0.0450	0.0519	0.0151	0.0791	0.018/

Table 10	
Normalization matr	ix power of weight.

Z_i/ϕ_j	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7	ϕ_8	ϕ_9	ϕ_{10}	ϕ_{11}	Y_i
Z_1	1.0000	0	0	0	1.0000	1.0000	0	0	0	0	0	3.0000
Z_2	0.9610	1.0000	0.9808	1.0000	0.9537	1.0000	0	0.9530	0	1.0000	0.9399	8.7884
Z_3	0	1.0000	1.0000	1.0000	0	0	1.0000	1.0000	1.0000	0.9473	1.0000	7.9473

Table 11	
Final aggregation ranking value of CoCoSo methodology.	

Z_i	Ξ_{a_i}	Ranks	Ξ_{b_i}	Ranks	Ξ_{c_i}	Ranks	Ξ_i	Ranks
Z_1	0.1562	3	2.0000	3	0.3514	3	1.3147	3
Z_2	0.4444	1	5.0527	1	1.0000	1	3.4752	1
Z_3	0.3479	2	4.4219	2	0.8989	2	3.0734	2

determined from the results of the described methods. Based on the experts' decision, WASPAS is adequate, computationally relaxed, and efficacious. The Complex Proportional Assessment (COPRAS) determines the result through a stepwise ranking system in terms of alternative significance and utility appraisal. COPRAS, having a lower computational and performance index, also evolved for ordering alternatives. The EDAS (Evaluation based on Distance from Average Solution) evaluates alternative ranking through averaging distance measures based on each attribute measure. The positive and negative ideal solutions are calculated, and the utility result is obtained at a higher level from the nadir solution and a lower level from the ideal solution. This frequently utilised MCDM method has a high computation level. The proposed IDOCRIW and CoCoSo algorithm results Z_1 , Z_2 , and Z_3 are related to the above-discussed MADM methodology. From all existing methodological results, the most acceptable priority is hydrometallurgical metal reclamation. The proposed IDOCRIW-COCOSO method was evaluated through a comparative analysis, which highlighted its advantages. Firstly, the IDOCRIW approach was used to evaluate the attributes' weights, reducing partial uncertainty in MADM methods. Secondly, the weighted method helped identify the priority of attributes in decision science problems. Thirdly, the COCOSO approach was differentiated from other MADM methods as it provides both outer values and identity ranking values, which are visually represented in Fig. 4. (See Table 12.)

Table 12		
Comparative	analysis	results.

Algorithm	Order of Alternative	Priority		
		High	Low	
Proposed approach	$Z_2 > Z_3 > Z_1$ 3.4752 > 3.0734 > 1.3147	Z_2	Z_1	
TOPSIS method	$\begin{array}{l} Z_2 > Z_3 > Z_1 \\ 0.4056 > 0.6260 > 0.5782 \end{array}$	Z_2	Z_1	
WASPAS approach	$Z_2 > Z_3 > Z_1$ 0.6064 > 0.7606 > 0.7866	Z_2	Z_1	
COPRAS method	$Z_3 > Z_2 > Z_1$ 0.3755 > 0.3545 > 0.2920	Z_3	Z_1	
EDAS method	$Z_3 > Z_2 > Z_1$ 0.7610 > 0.6766 > 0.2008	Z_3	Z_1	
VIKOR method	$Z_2 > Z_3 > Z_1$ 0 < 0.65603 < 0.9858	Z_2	Z_1	

6.3. Sensitivity test

To verify the reliability and effectiveness of the proposed integrated approach by varying the parameters of the obtained results by way of changing the priority of attributes in multiple dimension aspects of decision expert thoughts, varying the qth value of the proposed set, and altering the aggregation function parameter values, we employed attribute weight-based analysis through various appearances, as below mentioned:

Case-(i): Suppose each attribute has equal importance, then the importance of the attributes is $\beta_1 = 0.0909$, $\beta_2 = 0.0909$, $\beta_3 = 0.0909$,

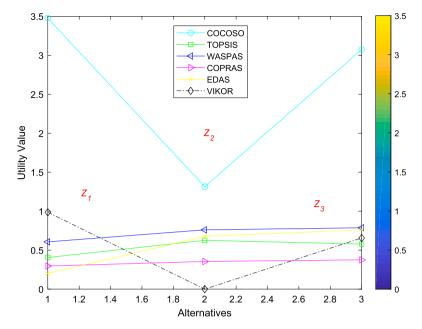


Fig. 4. Illustration of comparative test.

 $\beta_4 = 0.0909, \beta_5 = 0.0909, \beta_6 = 0.0909, \beta_7 = 0.0909, \beta_8 = 0.0909, \beta_9 = 0.0909, \beta_{10} = 0.0909, and \beta_{11} = 0.0909.$ Hence, the result of the proposed approach is $Z_1 = 1.3109, Z_2 = 3.4943$, and $Z_3 = 3.3644$. The best ranking alternative is Z_2 .

Case-(ii): If beneficial and non-beneficial attributes carry 50% weight, then the weight of attributes is $\beta_1 = 0.1$, $\beta_2 = 0.0833$, $\beta_3 = 0.0833$, $\beta_4 = 0.0833$, $\beta_5 = 0.0833$, $\beta_6 = 0.1$, $\beta_7 = 0.0833$, $\beta_8 = 0.0833$, $\beta_9 = 0.1$, $\beta_{10} = 0.1$, and $\beta_{11} = 0.1$. Therefore, the obtained results are $Z_1 = 1.3129$, $Z_2 = 3.4629$, and $Z_3 = 3.2980$.

Case-(iii): Suppose advantageous attributes have 65% weight and non-advantageous attributes have 35% weight, then the weight vector $\beta_j = [0.13\ 0.0583\ 0$

Case-(iv): Assuming favourable and unfavourable attributes carry 25% and 75% importance, the importance of attributes is $\beta_1 = 0.05$, $\beta_2 = 0.125$, $\beta_3 = 0.125$, $\beta_4 = 0.125$, $\beta_5 = 0.125$, $\beta_6 = 0.05$, $\beta_7 = 0.125$, $\beta_8 = 0.125$, $\beta_9 = 0.05$, $\beta_{10} = 0.05$, and $\beta_{11} = 0.05$. As a result of the proposed methodology, $Z_1 = 1.3014$, $Z_2 = 3.6737$, and $Z_3 = 3.7321$. This analysis is performed based on an investigation of how the criterion weights affect the ranking order. This type of study can be extremely useful for achieving a broader spectrum of criteria weights to test the practicality of the proposed approach. We estimate the final value of the alternatives for each scenario. Finding the places of alternative that were found as the alternative Hydro-metallurgical reclamation (Z_2) is 90% best in all cases. Consequently, the priority ranking of the possibilities generated using the proposed approach is reliable.

7. Conclusion

This article presents a comprehensive examination of the EVLB recycling method in an uncertain environment, which is a novel contribution. In addition to identifying the optimal recycling approach, the study also provides a ranking of the recycling methods. To determine potential alternatives and select the most effective EVLB recycling technique, we utilized q-ROPFNs to overcome vagueness. Secondly, we calculated the importance of attributes using the extended IDOCRIW method in the q-ROPFS environment to alleviate the subjective stochasticity known as the extended IDOCRIW method. Third, we developed the q-ROPFS-COCOSO method to rank the alternatives. Finally, we demonstrated the effectiveness of the q-ROPFS-IDOCRIW-COCOSO methodology through comparative and sensitivity investigations. The case study showed that hydrometallurgical reclaiming represents the best recycling process from a socio-economic consumer perspective. However, the proposed algorithm has limitations, including: (1) the IDOCRIW method is only applicable for positive values, failing to attend to the final utility value if negative values occur in the decision matrix; (2) the solo decision-expert is used in the article, hence the g-ROPFS aggregation operator is not defined; (3) a five-level-linguistic q-ROPFS scale is employed to generate the judgmental matrix; and (4) only single normalization is used in the proposed algorithm. Future directions could focus on: (1) combining dual objective and subjective methods for weight finding; (2) integrating complex fuzzy sets with q-ROPFS to deal with amplitude and phase periods; (3) developing innovative aggregation operators for q-ROPFS; and (4) solving problems without linguistic scales to provide exact, precise values for each entry of the initial matrix. Additionally, future research could address the Geographical Information System (GIS)-based renewable energy decision problem.

CRediT authorship contribution statement

Thirumalai Nallasivan Parthasarathy: Writing – review & editing, Writing – original draft, Validation, Resources, Methodology, Formal analysis. Samayan Narayanamoorthy: Writing – original draft, Visualization, Validation, Supervision, Software, Methodology, Investigation, Formal analysis, Conceptualization, Writing – review & editing. Chakkarapani Sumathi Thilagasree: Investigation, Methodology, Resources, Software, Visualization, Writing – review & editing. Palanivel Rubavathi Marimuthu: Resources, Software, Validation, Visualization. Soheil Salahshour: Resources, Software, Validation, Visualization, Writing – review & editing. Massimiliano Ferrara: Visualization, Validation, Software. Ali Ahmadian: Writing – review & editing, Visualization, Validation, Software, Resources, Project administration, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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