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# Optimal Synthesis of Shaped Beams Through Concentric Ring Isophoric Sparse Arrays

Andrea Francesco Morabito and Pasquale Giuseppe Nicolaci

*Abstract*—**An innovative deterministic approach to the optimal power synthesis of mask-constrained shaped beams through concentric-ring isophoric sparse arrays is presented. The design procedure exploits at best the state-of-the-art techniques respectively available in the cases of circular-ring isophoric arrays radiating pencil beams and of linear isophoric arrays generating shaped beams. The technique avoids exploitation of global-optimization algorithms and allows to significantly outperform all the (few) available procedures.**

*Index Terms*—**Power synthesis, shaped beams, sparse arrays.**

## I. Introduction

Sparse array antennas, i.e., arrays whose elements are located over an aperiodic layout such to fulfill given radiation requirements, represent an important topic for antenna designers. This is shown by a large number of both 'classic' (e.g., [1]-[3]) and more recent (e.g., [4]-[17]) contributions, as well as by many research projects {e.g., the Invitation to Tenders (ITT) [18] by the European Space Agency (ESA)}.

Such a large interest is due to the advantages offered by these systems with respect to equispaced arrays. In fact, for a fixed aperture size, sparse arrays allow decreasing the number of elements without significantly affecting the beamwidth [10]. Such a lowering, in turn, mitigates mutual-coupling issues (due to the increased value of the average inter-element spacing) and implies a reduced cost, weight, and complexity of the feeding network [2]. Moreover, the aperiodicity of the layout allows reducing grating lobes in the radiation pattern and hence an improvement of performance in terms of both sidelobes level (SLL) and bandwidth [8]. Finally, aperiodic arrays may allow a SLL reduction without resorting to an excitation-amplitude tapering [13].

The last advantage above led over the years to the large diffusion of a particular kind of sparse arrays: the so-called 'Isophoric' Sparse Arrays (ISAs) [8]-[17], i.e., aperiodic arrays having a *constant* excitation amplitude over the whole aperture. This feature allows the feeding power amplifiers to operate at their point of maximum efficiency and greatly simplifies the beam forming network [10],[13].

Amongst all ISAs planar architectures, Concentric Ring Isophoric Sparse Arrays (CRISAs), i.e., ISAs whose elements are disposed onto concentric rings, appear being one the most convenient ones due to their capability of uniformly spreading the antenna energy over all azimuth directions [9],[12]-[17]. In fact, CRISAs constitute one of the usual ESA's choices to realize the satellite multibeam coverage of Earth [9],[14],[15].

Of course, ISAs adoption has also its disadvantages, the most critical one being related to the corresponding synthesis procedures. In fact, since the elements' locations are an unknown of the design problem, the synthesis is unaffordable through Convex Programming (CP) procedures of the kind presented in [19]. Therefore, as done for instance in [4]-[7], the antenna designers often recur to Global Optimization (GO) procedures. However, due to their high computational weight, GO techniques practically result unsuitable for the synthesis of ISAs composed by a large number of elements.

To overcome such difficulties, the following two-steps procedure has been recently devised for the design of Linear Isophoric Sparse Arrays (LISAs) and CRISAs [8]-[15]:

1. identify a Reference Continuous Aperture Source (RCAS) fulfilling 'at best' the radiation requirements at hand;
2. derive the array layout as a discretization of the RCAS.

This procedure allowed to outperform previous approaches [8]-[15]. In fact, a number of well-assessed methods already exist to perform step 1 (e.g., [20],[21]) and, only in the 'pencil beams' case, step 2 (e.g., [1],[8],[9],[13]-[15]).

Unfortunately, much less alternatives to perform step 2 are available in the 'shaped beams' case. The reason of such lack derives from a simple circumstance: while the RCASs required to generate sufficiently-narrow pencil beams are *real* functions [20], in the shaped beams case they result *complex* [21]. This issue, which drastically complicates step 2 [14], has been recently solved for the case of LISAs in [12] but still results unsolved for CRISAs. In fact, the unique approach currently available to perform step 2 in the CRISAs case is the 'rough' one in [14], which bypasses the problematic of the RCAS's complexity by:

a) identifying the elements' locations by applying the technique presented in [9] *only* to the RCAS's amplitude;
b) assigning to each array element an excitation phase equal to value assumed in its location by RCAS's phase.

This procedure neglects the fact that, as discussed in [12], the array elements locations must be a function of *both* the

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RCAS's amplitude and phase distributions, and hence its performance is considerably improvable (see Section III). On the other side, beyond [14], the unique contribution addressing the synthesis of CRISAs in the shaped beams case is [7] which, however, relies on GO and hence is exploitable only in case of CRISAs composed by a low number of elements.

In the attempt of filling such a gap, this paper proposes a new approach to the mask-constrained power synthesis of shaped beams through CRISAs. The technique can be seen as the extension of the approach in [12] to the case of ring-symmetric array layouts, and results fast and effective even in case of arrays composed by a large number of elements.

In the following, Section II describes the proposed synthesis procedure while Section III assesses it against *all the other* available techniques. Conclusions follow.

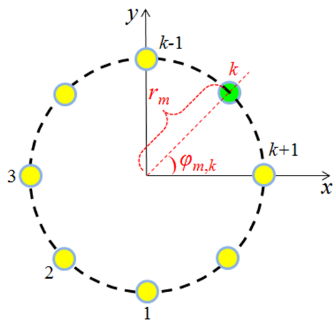


Fig. 1. Pictorial view of the $m$-th ring of a generic CRISA

## II. RATIONALE OF THE DESIGN PROCEDURE

The proposed approach consists in synthesizing the CRISA by performing a discretization of a RCAS which optimally fulfills a given circularly-symmetric power mask. Let us denote by $s(\rho)$ such a RCAS, having a circularly-symmetric distribution and covering a disk of radius $R$ over the $xy$ plane, $\rho=(x^2+y^2)^{1/2}$ being the radial coordinate spanning the aperture. Moreover, let us denote with $N$ the overall number of CRISA elements and with $\phi$ the aperture azimuth coordinate.

The CRISA is conceived as the union of $M$ concentric rings on which a given number of radiating elements is located with an uniform angular spacing (see Fig. 1). The unknowns are the elements' locations and excitation phases and the aim is to determine them in such a way to minimize the mean square difference between the far-field distributions respectively corresponding to the RCAS [let us say $F^{RCAS}(u)$] and the CRISA [let us say $F^{CRISA}(u)$]. Apart from inessential constants, these two fields can be written as:

$$F^{RCAS}(u)=\int_0^R s(\rho)J_0(\beta\rho u)\rho d\rho \tag{1}$$

$$F^{CRISA}(u)=\int_0^R s^A(\rho)J_0(\beta\rho u)\rho d\rho \tag{2}$$

with

$$s^A(\rho)=\int_0^{2\pi}\sum_{m=1}^{M}\sum_{k=1}^{N_m}\delta(\rho-r_m)\delta(\phi-\varphi_{k,m})(1/\rho)d\phi \tag{3}$$

wherein $u=\sin\theta$ ($\theta$ denoting the elevation angle with respect to boresight), $\beta=2\pi/\lambda$ ($\lambda$ denoting the wavelength), $r_m$ and $N_m$ respectively are the radius of the $m$-th CRISA's ring and the number of elements located over it, and $\varphi_{k,m}$ is the azimuth coordinate of the $k$-th element belonging to the $m$-th ring (see Fig. 1). Under these assumptions[1], it will be:

$$\int_0^{\infty}\left|\frac{F^{RCAS}(u)-F^{CRISA}(u)}{\beta u}\right|^2 du=\int_0^R\left|S(\rho)-S^A(\rho)\right|^2 d\rho \tag{4}$$

wherein $S(\rho)$ and $S^A(\rho)$ represent the cumulative distributions associated to the functions $s(\rho)$ and $s^A(\rho)$, respectively, i.e.:

$$S(\rho)=\int_0^{\rho}s(\tau)\tau d\tau \tag{5}$$

$$S^A(\rho)=\int_0^{\rho}s^A(\tau)\tau d\tau \tag{6}$$

Therefore, minimizing the mean square difference between $F^{RCAS}(u)$ and $F^{CRISA}(u)$ with $1/u^2$ weighting is equivalent to minimize the mean square difference between the functions (5) and (6). This can be done, once $M$ and $N$ have been chosen, by means of the following procedure:

1. represent $S(\rho)$ as a curve in a three-dimensional space where the first and second coordinates are its real and imaginary parts, respectively, and the third coordinate is $\rho$ (see Fig. 2). Then inscribe in this curve an *equilateral* polygonal composed by $N$ segments;
2. for $m$=1,..,$M$, determine the value of $N_m$ in such a way that the $N$ segments above can be grouped into $M$ contiguous subintervals, the $m$-th of which is composed by $N_m$ segments and guarantees that the following ratio:
$$V_m=\frac{\left|S(\rho_{m+1})-S(\rho_m)\right|}{N_m} \tag{7}$$
is constant (wherein $\rho_m$ and $\rho_{m+1}$ denote the endpoints of the $m$-th subinterval, with $\rho_1$=0 and $\rho_{M+1}=R$);
3. for $m$=1,..,$M$, determine $r_m$ in such a way to fulfil the following equation:
$$\mathrm{Re}\left[\frac{S(r_m)-S(\rho_m)}{S(\rho_{m+1})-S(\rho_m)}\right]=\frac{1}{2} \tag{8}$$
4. assign to all the elements belonging to the $m$-th ring, for $m$=1,..,$M$, an excitation phase equal to the angle subtended in the complex plane by the real axis and the segment connecting $S(\rho_m)$ and $S(\rho_{m+1})$.

The motivations underlying the procedure are in the following.

Concerning step 1, it derives from a simple principle: the fact that the CRISA must be composed by $N$ *isophoric* elements entails that $S(\rho)$ must be partitioned in $N$ segments having *the same length*.

As far as step 2 is concerned, it must be noted that $S(\rho)$ represents the volume subtended by the RCAS over the circle of radius $\rho$. Therefore, this step allows to subdivide the aperture into $M$ concentric annular sectors, the $m$-th of which contains $N_m$ 'iso-volume' sectors (see Fig. 2). This operation can be performed by exploiting the fast iterative procedure in [9], which exploits an equation analogous to (7) and provide

[1] Differently from the RCAS's far field, the CRISA's array factor depends also on the azimuth angle. However, as shown in [9], such dependence is negligible for $u \ll N_m\lambda/(2\pi r_m)$, $m$=1,…,$M$. In all examples of Section III, as well as in very many applications, these angular sectors cover the whole region wherein the power pattern is significant, and hence the circularly-symmetric representation (2) can be exploited.

an analytic way to determine the optimal $V_m$ value. On the other side, it should be noted that [9] addresses only the synthesis of pencil beams through *real and positive* RCASs, so that the additional steps 3 and 4 are necessary herein.

Step 3 allows identifying, for $m$=1,..,$M$, the optimal radius the $m$-th CRISA's ring inside the interval $[\rho_m, \rho_{m+1}]$. In fact, by virtue of steps 1 and 2 above, it will be:

$$S^A(\rho)=\sum_{m=1}^{M} H(\rho-r_m)\left[S(\rho_{m+1})-S(\rho_m)\right] \tag{9}$$

where $H$ is the unit step function. By substituting (9) into (4) one achieves:

$$\int_0^R \left|S(\rho)-S^A(\rho)\right|^2 d\rho=\sum_{m=1}^{M}\int_{\rho_m}^{\rho_{m+1}}\left|S(\rho)-S^A(\rho)\right|^2 d\rho=$$
$$=\sum_{m=1}^{M}\left[\int_{\rho_m}^{r_m}\left|S(\rho)-S(\rho_m)\right|^2 d\rho+\int_{r_m}^{\rho_{m+1}}\left|S(\rho)-S(\rho_{m+1})\right|^2 d\rho\right] \tag{10}$$

with $S(0)=S^A(0)=0$. By equating to zero the derivative of (10) with respect to $r_m$, one exactly obtains equation (8). Therefore, the radius of the $m$-th CRISA ring must be determined by projecting the midpoint of the $m$-th polygonal segment onto $S(\rho)$ and then reading the corresponding $\rho$ value.

Finally, step 4 is perfectly coherent with previous steps and allows to assign to the CRISA's elements the required circularly-symmetric excitation-phase distribution.

## III. Assessment against all previous approaches

We tested the effectiveness of the proposed approach by comparing it to the only two techniques available today for the synthesis of shaped beams through CRISAs (i.e., the deterministic procedure in [14] and the global-optimization algorithm in [7]). In the two following test cases, the synthesis of the RCAS has been performed through the method in [21], and the subsequent discretization steps respectively required 1.16 and 1.04 seconds to be performed by a calculator having an Intel Core i7 2.50 GHz CPU and a 10 GB RAM.

### *III.1 Comparison with the synthesis procedure in [14]*

In [14], considering the mission scenario of the ESA ITT [18], authors synthesized a flat-top beam covering almost all of the Earth disc as seen from a geostationary satellite. The adopted power mask is depicted in Fig. 3. It is circularly symmetric and enforces a maximum ripple equal to ±0.5 dB for $\theta \leq 8°$ and a maximum SLL of -20 dB for $11° \leq \theta < 16°$ and of -10 dB for $16° \leq \theta \leq 90°$. These requirements were fulfilled through a CRISA composed by 966 elements located over an aperture of diameter 42λ. An uniform circular aperture of diameter 0.8λ was used as elementary radiator, and the CRISA provided a minimum directivity of 18.3 dBi in the flat zone.

Notably, by exploiting the proposed approach, we have been able to fulfill the same mask through a CRISA composed by 460 elements located over a circular aperture of diameter 18.4λ. An uniform circular aperture of diameter 0.5λ has been used as elementary radiator, and the CRISA provided a minimum directivity of 19.77 dBi in the flat zone. Therefore, with respect to the technique in [14], the proposed approach allowed saving the 52% of elements. Moreover, although exploiting 0.3λ smaller feeds, the synthesized array provided a 1.47 dBi higher directivity inside the Earth disc. The synthesized power pattern and array layout are respectively shown in figures 3 and 4. Notably, the achieved maximum directivity, i.e., 20.77 dBi, is just 2.39 dBi lower than the one pertaining to an 'ideal' theoretical power pattern being constant in the region $\theta \leq 8°$ and zero elsewhere.

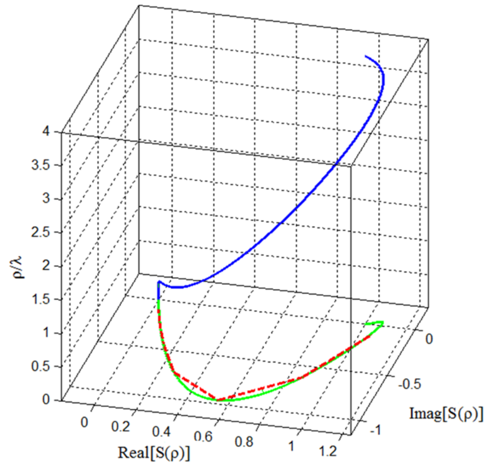


Fig. 2. Representation of $S(\rho)$ in the complex plane (green color: reference function; red color: discretized function) and as a curve in a three-dimensional space where the first and second coordinates are its real and imaginary parts, respectively, and the third coordinate is the radial coordinate (blue curve).

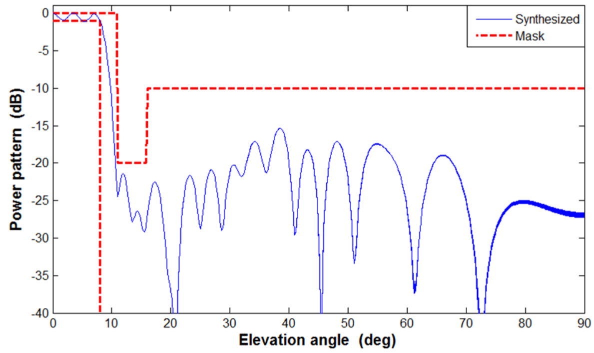


Fig. 3. Upper and lower bounds (red lines) and power pattern radiated by the synthesized array (blue curves, superposition of cuts along the azimuth angle).

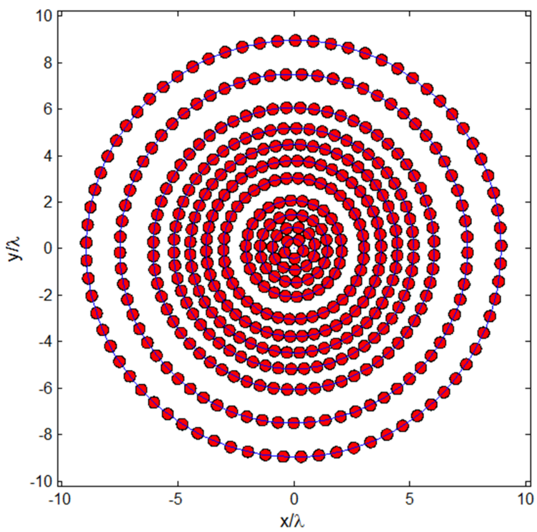


Fig. 4. CRISA radiating the power pattern in Fig. 3 (feed diameter: 0.5λ).

### *III.2 Comparison with the synthesis procedure in [7]*

In [7] authors synthesized a CRISA composed by 220 elements located over an aperture of diameter 10λ in order to fulfill the circularly-symmetric power mask depicted in Fig. 5. The latter enforces a maximum SLL equal to -12.18 dB for 19.6°≤$\theta$≤90°, an Half Power Beamwidth (HPBW) equal to 31.2°, and a maximum ripple equal to ±0.41 dB for $\theta$≤13.4°.

Although results in [7] were generated by means of a Simulated Annealing GO algorithm aimed at jointly minimizing the maximum ripple and the maximum SLL, the proposed procedure allowed a significant improvement of performance. In particular, we have been able to fulfill the same mask by exploiting a CRISA composed by 163 *isotropic* elements located over an aperture of diameter 5.86λ. Therefore, the presented approach allowed a 26% reduction in the elements' number without experiencing any radiation loss.

The synthesized array layout is shown in Fig. 6, while the corresponding square-amplitude array factor is shown on Fig. 5. The CRISA guarantees in the flat zone a minimum directivity of 15.98 dBi and a maximum directivity of 16.8 dBi (which is just 1.87 dBi lower than the directivity pertaining to a theoretical power pattern being constant in the region $\theta$≤13.4° and zero elsewhere).

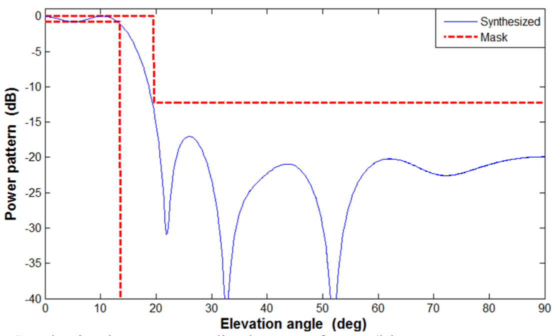


Fig. 5. Synthesized square-amplitude array factor (blue curves - superposition of cuts along the azimuth angle) and adopted mask (red lines).

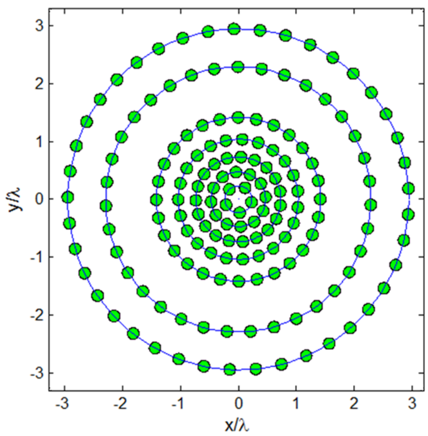


Fig. 6. CRISA radiating the power pattern in Fig. 5 (isotropic feeds).

## IV. Conclusions

The problem of the optimal synthesis of shaped beams in the presence of completely-arbitrary lower and upper bounds on the power distribution has been solved by exploiting ring-symmetric isophoric sparse arrays. The proposed technique allowed to considerably improve the performance achievable through previous approaches. In particular, for equal power-pattern masks, the required number of array elements has been significantly reduced.

The approach can be used in conjunction with the techniques in [13],[15],[17] in such a way to exploit also the feeds' shape and size as a degree of freedom of the design and hence to get a further enhancement of performance.

## References

[1] M. I. Skolnik, "Chapter 6. Nonuniform Arrays" in R. E. Collin and F.Zucker (eds), *Antenna Theory*, New York, McGraw-Hill, 1969.

[2] A. Ishimaru, "Theory of unequally-spaced arrays," *IRE Transactions on Antennas and Propagation*, vol. 10, no. 6, pp. 691–702, 1962.

[3] R. E. Wiley, "Space tapering of linear and planar arrays," *IRE Trans. on Antennas and Propagation*, vol. 10, n. 4, pp. 369–377, 1962.

[4] D. G. Kurup, M. Himdi, and A. Rydberg, "Synthesis of uniform amplitude unequally spaced antenna arrays using the differential evolution algorithm," *IEEE Transactions on Antennas and Propagation*, vol. 51, n. 9, pp. 2210-2217, 2003.

[5] N. Jin and Y. Rahmat-Samii, "Advances in particle swarm optimization for antenna design: Real-number, binary, single-objective and multiobjective implementation," *IEEE Transactions on Antennas and Propagation*, vol. 55, n. 3, pp. 557-567, 2007.

[6] E. Rajo-Iglesias and O. Quevedo-Teruel, "Linear array synthesis using an ant-colony-optimization-based algorithm," *IEEE Antennas and Propagation Magazine*, vol. 49, n. 2, pp. 70-79, 2007.

[7] A. Trastoy-Rios, M. Vicente-Lozano, and F. Ares-Pena, "Shaped beams from circular apertures and arrays with uniform amplitude," *Electronics Letters*, vol. 36, n. 14, pp. 1180-1182, 2000.

[8] O. M. Bucci, M. D'Urso, T. Isernia, P. Angeletti, and G. Toso, "Deterministic synthesis of uniform amplitude sparse arrays via new density taper techniques," *IEEE Transactions on Antennas and Propagation*, vol. 58, n. 6, pp. 1949-1958, 2010.

[9] O. M. Bucci, and S. Perna, "A deterministic two dimensional density taper approach for fast design of uniform amplitude pencil beams arrays", *IEEE Trans. on Antennas and Prop.*, vol. 59, n. 8, pp. 2852-2861, 2011.

[10] M. C. Viganó, G. Toso, G. Caille, C. Mangenot, I. E. Lager, "Sunflower array antenna with adjustable density taper", *International Journal of Antennas and Propagation*, vol. 2009, Article ID 624035, 10 pages.

[11] A. F. Morabito, T. Isernia, and L. Di Donato, "Optimal synthesis of phase-only reconfigurable linear sparse arrays having uniform-amplitude excitations," *Progress in Electromagnetics Research*, vol. 124, pp. 405-423, 2012.

[12] O. M. Bucci, T. Isernia, and A. F. Morabito, "An effective deterministic procedure for the synthesis of shaped beams by means of uniform-amplitude linear sparse arrays," *IEEE Transactions on Antennas and Propagation*, vol. 61, n. 1, pp. 169-175, 2013.

[13] P. Angeletti, G. Toso, and G. Ruggerini, "Array antennas with jointly optimized elements positions and dimensions Part II: Planar Circular Arrays," *IEEE Transactions on Antennas and Propagation*, vol. 62, n. 4, pp. 1627-1639, 2014.

[14] O. M. Bucci, T. Isernia, A. F. Morabito, S. Perna, and D. Pinchera, "Isophoric sparse arrays: a synthesis procedure for circularly symmetric shaped beams," *Proceedings of the 6th European Conference on Antennas and Propagation*, pp. 832-836, 26-30 March 2012, Prague.

[15] A. F. Morabito, A. R. Lagana, and T. Isernia, "Isophoric array antennas with a low number of control points: a 'size tapered' solution," *Progress In Electromagnetics Research Letters*, vol. 36, pp. 121-131, 2013.

[16] M. Carlin, G. Oliveri, and A. Massa, "Hybrid BCS-deterministic approach for sparse concentric ring isophoric arrays," *IEEE Transactions on Antennas and Propagation*, vol. 63, n. 1, pp. 378-383, 2015.

[17] O. M. Bucci, T. Isernia, A. F. Morabito, S. Perna, and D. Pinchera, "Density and element-size tapering for the design of arrays with a reduced number of control points and high efficiency," *Proceedings of the Fourth European Conference on Antennas and Propagation*, 12-16 April 2010, Barcelona, Spain.

[18] ESA/ESTEC Invitation to Tenders n° AO/1-5598/08/NL/ST, "Innovative architectures for reducing the number of controls of multiple beam telecommunications antennas".

[19] A. F. Morabito and P. Rocca, "Optimal synthesis of sum and difference patterns with arbitrary sidelobes subject to common excitations constraints," *IEEE Antennas Wireless and Propagation Letters*, vol. 9, pp. 623-626, 2010.

[20] O. M. Bucci, T. Isernia, and A. F. Morabito, "Optimal synthesis of directivity constrained pencil beams by means of circularly symmetric aperture fields," *IEEE Antennas Wireless and Propagation Letters*, vol. 8, pp. 1386–1389, 2009.
[21] O. M. Bucci, T. Isernia, and A. F. Morabito, "Optimal synthesis of circularly symmetric shaped beams," *IEEE Transactions on Antennas and Propagation*, vol. 62, n. 4, pp. 1954-1964, 2014.