Optimizing Power Transmission in Given Target Areas in the Presence of Protection Requirements

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Abstract—**We present a new approach to the synthesis of fields able to maximize the power radiated in an arbitrary portion of the visible space in the presence of protection requirements and constraints on the beam efficiency. As a byproduct, beam efficiency can also be optimized if desired. The devised procedure works whether the target area has a linear, square, or circular footprint. The synthesis is stated in terms of a convex programming problem, with the related advantages in terms of both convergence speed and solutions optimality. The given theory is supported by numerical results including comparisons with state-of-the-art approaches.**

Index Terms – **Power Synthesis, Beam Collection Efficiency, Beam Transmission Efficiency.**

I. Introduction

Antenna synthesis problems are often classified into problems wherein pencil beams are being sought and problems where shaped beams are instead of interest (see for instance [1]-[3]). When moving to applications where the transfer of energy (rather than of information) is of interest, a third kind of canonical problem is also important. In fact (see [4]-[11]), situations do exist where one wants to optimize the ratio between the power deposited over a given area and the whole transmitted power, i.e., the so called Beam Transmission Efficiency (BTE) [5] or Beam Collection Efficiency (BCE) [6],[7]. Notably, as the target region may correspond to angles (much) larger than λ/D, λ and D respectively being the wavelength and the maximal size of the antenna, the problem cannot be classified under the 'pencil beam' category. Moreover, the shape of the pattern also is of no concern [6], so that the problem does not belong to the 'shaped beam' class of synthesis problems as well.

In those cases where no bound is needed on the level of the fields and array antennas are considered, it has been shown in [10] how the synthesis can be solved in terms of a generalized eigenvalue problem. The same strategy can be conveniently adopted any time one deals with the maximization of the ratio amongst positive-semidefinite *quadratic* functions, and in [11] interesting connections with the prolate spheroidal functions have been identified. Contribution [10] also claims the capability to control the sidelobe levels, but it has to be noted that it only can take into account linear equality constraints on the field. As such, it does not allow enforcing possible *upper* bounds on the *power* pattern, which are instead of interest in a number of applications. For instance, they are relevant in wireless power transmission systems (wherein one may want to protect given regions or apparatuses) [8], as well as in hyperthermia problems (wherein one needs to protect healthy tissues) [9]. Obviously, consideration of upper bounds on power pattern (rather than precise *nominal field* values) equips the designer with much more degrees of freedom.

As a step towards the solution of such a class of problems, we propose an effective approach for the synthesis of fields able to maximize the power deposited over a given angular area and to contemporarily guarantee the fulfillment of arbitrary upper-bound constraints on the power distribution (outside and within the area of interest), as well as a BCE as large as possible. The approach can be applied in both the canonical cases wherein the electromagnetic source is an equispaced linear array antenna or a circularly-symmetric continuous aperture source. Results pertaining to the first situation can be easily extended to the case of generic linear sources as well as to sources having a two-dimensional factorable pattern. Hence, the approach can be exploited in a straightforward fashion to all cases wherein the target area has a linear, circular, or square footprint. As a distinguishing characteristic, the approach is able to reduce the overall antenna design to (a series of) *linear programming* problems, so that it jointly guarantees the achievement of optimal solutions and an extremely low computational burden.

In the following, Section II is devoted to present and discuss the synthesis approach while Section III shows numerical results concerning various beam footprints. Conclusions and possible extensions of the approach follow.

II. The Design Procedure

In the following, we focus on the case where the target area is in the far-field region of the sources and adopt the BCE expression used in [6], i.e.,

$$BCE_{\psi} = \frac{P_{\psi}}{P} \tag{1}$$

wherein P_{ψ} and P represent the power transmitted in the region ψ and the whole radiated power, respectively.

This is the post-print of the following article: A. F. Morabito, A. R. Laganà, and T. Isernia, "Optimizing Power Transmission in Given Target Areas in the Presence of Protection Requirements," IEEE Antennas and Wireless Propagation Letters, vol. 14, pp. 44-47, 2015. Article has been published in final form at: ieeexplore.ieee.org/document/6891155. DOI: 10.1109/LAWP.2014.2354514.

We will separately discuss the case of linear equispaced arrays and the case of circularly-symmetric aperture fields.

II.A The linear equispaced arrays case

The field is synthesized through an equispaced linear array and the unknowns of the problem are the elements' excitations. Using the same quantities as in [10] and [6], we adopt the following power expressions

$$P_\psi = \int_{u \in \psi} |F(u)|^2 du \tag{2}$$

$$P = \int_{\substack{u \in visible \\ space}} |F(u)|^2 du \tag{3}$$

being F the radiated field and $u=\beta d \sin\theta$, wherein d is the spacing amongst neighboring elements, θ denotes the angle between the boresight and observation directions, and $\beta=2\pi/\lambda$.

By exploiting the theory in [2],[12], under such hypotheses the square-amplitude array factor can be expressed as

$$T(u) = |F(u)|^2 = \sum_{p=-N+1}^{N-1} D_p e^{jpu} \tag{4}$$

wherein N is the number of radiating elements. Therefore, the problem at hand can be formulated as follows:

$$\min_{D_{-N+1},\, \ldots,\, D_{N-1}} \; -\int_\Omega \sum_{p=-N+1}^{N-1} D_p e^{jpu} du \tag{5.a}$$

subject to

$$\begin{cases} \sum_{p=-N+1}^{N-1} D_p e^{jpu} \le UB(u) & (5.b) \\ \sum_{p=-N+1}^{N-1} D_p e^{jpu} \ge 0 & (5.c) \\ D_p = D_{-p}^* & (5.d) \\ \dfrac{\int_\Omega \sum_{p=-N+1}^{N-1} D_p e^{jpu} du}{\int_{-\beta d}^{\beta d} \sum_{p=-N+1}^{N-1} D_p e^{jpu} du} \ge A & (5.e) \end{cases} \tag{5}$$

wherein Ω denotes the specific region (expressed in terms of the u variable) over which the maximum power deposition is required, the function $UB(u)$ denotes an *arbitrary* upper-bound mask for the power pattern, constraints (5.c) and (5.d) are due to the fact that $T(u)$ must be a real and non-negative function (* denoting complex conjugate), and inequality (5.e) is meant to enforce a BCE_Ω at least as large as A.

By taking into account the bandlimitedness of $T(u)$ [2], expressions (5) can be substituted with a sufficiently fine discretization, so that all constraints can be seen as a system of ordinary *linear* inequalities in the D_p variables, and the optimization problem above can be interpreted as a *linear programming* problem [13].

By taking advantage from such a circumstance, implying that one can find the solution to the problem (5) in a simple and fast fashion, one can maximize the BCE in terms of the auxiliary D_p variables. In fact, it suffices to solve the problem (5) with increasing values of A, until no solution exists anymore because of the lack of intersection amongst the set of solutions to the different constraints.

The last step of the procedure amounts to determine the actual array excitations by factorizing the $T(u)$ polynomial. Such a factorization, which practically has a negligible computational burden, can be performed along the guidelines of [2] or [14]. As shown in Section III, the overall synthesis procedure can be easily adapted to the case of square footprints.

Finally, as a crucial circumstance, it must be stressed that without the adoption of representation (4) the problem at hand would have implied the maximization of a positive-semidefinite *quadratic* functional under *non-convex* constraints, which is much harder to solve with respect to the present constrained optimization.

II.B The circularly-symmetric aperture sources case

In order to adapt the above strategy to the case where the target area is circular, a convenient strategy is that of looking for a circularly-symmetric continuous aperture distribution. In fact, this kind of source has a two-fold utility. First, it can be exploited as a reference to quickly design direct radiating arrays [15]. Second, it allows stating the *ultimate* theoretical radiation performance achievable by any aperture antenna of fixed size [14].

In order to extend the approach described in the previous Subsection to the present case, advantage can be taken from the design procedure in [14], which amounts in:

i. synthesizing a reference one-dimensional power pattern, conceived as the square-amplitude array factor of a one-dimensional 'virtual' equispaced array fulfilling all the radiation constraints at hand;
ii. determining a circularly-symmetric source such that the difference between the azimuth cut of its power pattern and the pattern coming out form step (i) is negligible in the visible part of the spectrum.

As far as step (i) is concerned, it can be performed along the same guidelines of the previous Subsection by substituting (5.e) with the constraint

$$\frac{\int_\Omega \sum_{p=-N+1}^{N-1} D_p e^{jpu} u\,du}{\int_0^{\beta d} \sum_{p=-N+1}^{N-1} D_p e^{jpu} u\,du} \ge A \tag{6}$$

and requiring that the location of the roots of $T(u)$ in the complex Schelkunoff z plane (with $z=e^{ju}$) is such that

$$T(u) = E(u)E^*(u) \qquad with\ E(u) even \tag{7}$$

$E(u)$ representing the far field distribution generating the desired power pattern.

As $T(u)$ is now devoted to represent the power pattern of a *continuous* source, a value of d smaller than 0.5λ has to be used (see [14] for more details). Hence, in order to avoid superdirective solutions, the function UB in (5.b) must be chosen in such a way to keep under control the source's energy in the invisible part of the spectrum [14].

The aim of the additional condition (7), which can be expressed as *convex* constraints [14], is that of making $E(u)$ suitable for step (ii), which essentially consists in the following inverse zero-order Hankel transform:

$$f(\rho)=\int_0^\infty E(u)J_0(\rho u)u\,du \tag{8}$$

wherein f is the sought source, while J_0 is the zero-order Bessel function of first kind and ρ is the radial coordinate spanning the antenna aperture, which will have a diameter equal to the length of the 'virtual' array, i.e., $L=d(N-1)$.

III. NUMERICAL ASSESSMENT

As a first test case, we used the procedure to maximize, through a planar array on a square grid, the power deposition over a square Ω region having the two spectral widths equal to $0.4\beta d$ (see Fig. 1, where θ and ϕ respectively denote the elevation and azimuth angles). In particular, the same constraints and the performance parameters as in [6] have been adopted.

To this aim, we looked for a square planar array having factorable excitations, which makes sense by virtue of the strong energy decay induced at large u values by the sought high BCE performance. It could be discussed how a direct optimization of the two-dimensional array would be in principle a more rigorous and general approach with respect to the adoption of 'auxiliary' one-dimensional sources. Unfortunately, such a strategy would have the considerable draw-back of making impossible formulating the problem at hand as a convex one[1]. Therefore, in that case global optimization techniques should be used and, because of the limitations of these latter in dealing with large scale-problems, the actual optimality of the final result would not be granted.

With the above in mind, we first synthesized a one-dimensional power pattern by exploiting an equispaced linear array composed by 8 elements with d=0.5λ. In this case, we solved problem (5) by enforcing a protection bound of 0 dB over the whole target region and recursively increasing the A value until the requirements became so strict to prevent the existence of a solution. By so doing, we achieved a BCE_Ω equal to 0.9994. The rationale of the approach guarantees that *no electrically smaller* linear equispaced array can fulfill analogous upper-bound constraints and, at the same time, provide a higher BCE value.

Then, we factorized over a square layout the excitations coming out from the solution of the one-dimensional problem above and determined the source amplitude distribution depicted in Fig. 2. The latter corresponds to a 64-elements square array, having an inter-element distance along the main axes of 0.5λ and able to generate the pattern shown in Fig. 1.

The BCE_Ω of the planar array, computed by means of the formula in [6], turned out being equal to 0.9987. Such a result agrees with the result achieved in [6] (BCE=0.9990) wherein, however, 100 half-a-wavelength spaced elements have been used. Therefore, the present approach allowed significant savings in terms of both number of elements (-36.0%) and antenna size (-39.5%).

As a second test case, the synthesis has been aimed at solving again a problem described in [6], i.e., to exploit a circular aperture antenna of diameter 5λ in order to maximize the BCE in the circular region given by $0.3\leq(u/\beta d)\leq0.6$. In particular, in [6] a BCE equal to 0.9503 has been achieved by exploiting an array antenna composed by 76 elements.

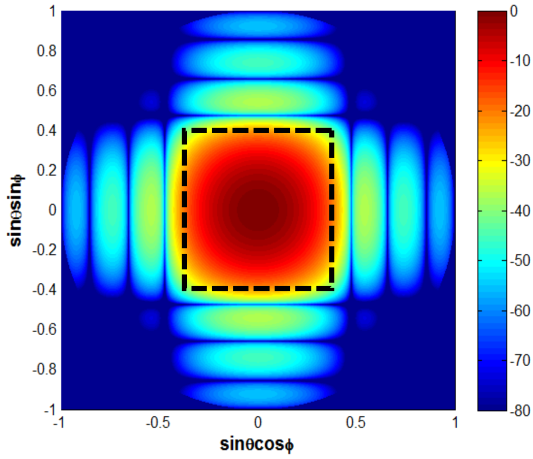


Fig. 1. The square target area case: synthesized power pattern. Target region also depicted (black dashed line). Isotropic element pattern embedded.

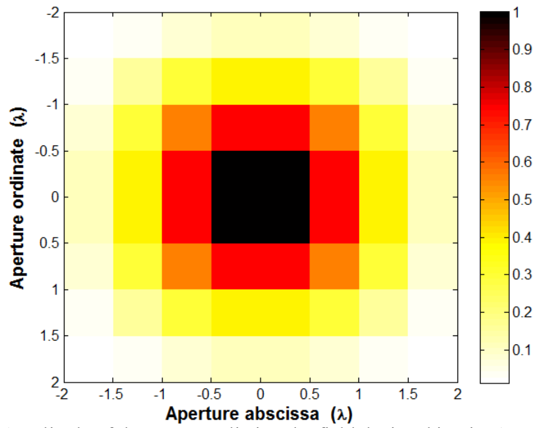


Fig. 2. Amplitude of the source radiating the field depicted in Fig. 1.

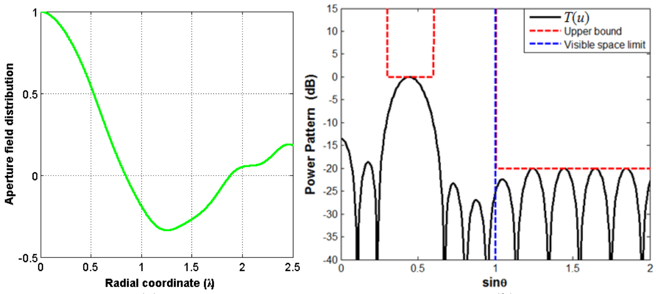
To solve such a problem, we have performed two operations. First, we synthesized a circularly-symmetric continuous source of diameter 5λ maximizing the BCE. To this end, a recursive increasing of the A parameter has been performed in the same spirit of the first test case. Also, we enforced a 0 dB power upper bound in the target area and, in order to avoid the occurrence of superdirective fields, we required that in the invisible part of the spectrum the maximum power pattern value is at least 20 dB lower than its maximum value in the visible part of the spectrum. The synthesized source and power pattern are shown in Fig. 3. The BCE resulted equal to 0.9706.

As a second operation, we discretized this continuous source into an array composed by 64 elements. The array excitations have been determined by sampling the continuous source over a grid whose 'elementary brick' is an equilateral

[1] In fact, the proposed approach takes decisive advantage from the Fundamental Theorem of Algebra, which does not hold true in case of two-dimensional polynomials [2].

triangle of side 0.5λ. By so doing, a BCE equal to 0.9398 has been achieved. Therefore, with respect to [6], the present approach provided a 1.1% lower BCE while saving the 15.8% of array elements and fulfilling a larger number of constraints.

The power pattern of the designed array is shown in Fig. 4. Notably, larger BCE values could have been achieved by identifying them through a method more sophisticated than a standard sampling procedure. In this respect, contribution [15] is worth to be mentioned. As a suitable alternative, local optimization technique aimed at refining the excitations or the locations could be used.



(a) (b)

Fig. 3. The circular target area case: synthesized continuous source (a) and corresponding power pattern (b). Power mask (red curve) and edge of visible part of the spectrum (blue dashed line) also shown.

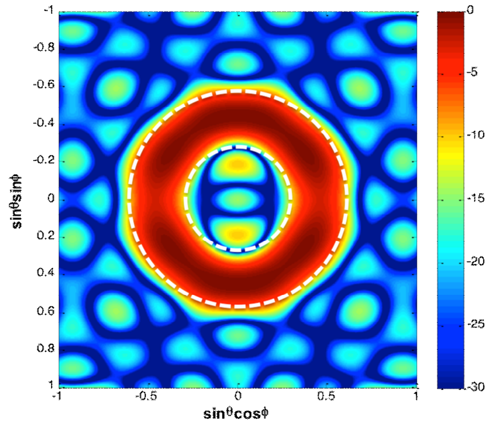


Fig. 4. Power pattern corresponding to the equispaced array derived from the source shown in Fig. 3. Target region also depicted (white dashed lines). Isotropic element pattern embedded.

IV. Further Potentialities and Conclusions

The problem of the synthesis of field distributions jointly maximizing the power transmitted in a given portion of the visible space and the ratio between it and the whole radiated power (in the presence of arbitrary upper-bound constraints for protection purposes) has been formulated and solved in many instances of actual interest.

In fact, a fast and effective procedure based on the solution of *linear programming* problems has been devised for the design of array antennas and continuous sources for the optimal coverage of one-dimensional, square and circular target areas (also conceivable as unions of different non-connected intervals). Notably, besides allowing to solve in a globally optimal fashion such a class of problems (which was still lacking in the literature), the approach also favorably compares with state-of-the-art approaches available for the case where *just the BCE* (i.e., excluding further constraints) is of interest. Moreover, while the other techniques available for the beam efficiency maximization can only take into account *equality* constraints on the *field* distribution, the present one allows enforcing *inequality* constraints on the *power* pattern (thus exploiting *all* the available degrees of freedom).

Interestingly, further scenarios do exist in which one can take decisive advantage from the proposed approach. In fact, by exploiting the power pattern representation introduced in [12], the devised strategy can be adopted *whenever* the power pattern can be expanded as a function of a single variable. This is indeed possible for planar arrays having a rhombic shape (with elements disposed over an uniform quadrangular grid and able to radiate circularly-symmetric field distributions) [16] and in the case of fields exhibiting elliptical symmetries [17]. Finally, by exploiting the 'reduced field' concept described in [18], the proposed approach can be used even if the target region is located in the near-field region of the source.

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